

Available online at http://www.journalcra.com

International Journal of Current Research Vol. 7, Issue, 10, pp.21218-21222, October, 2015 INTERNATIONAL JOURNAL OF CURRENT RESEARCH

RESEARCH ARTICLE

CHIRAL SCHRÖDINGER SOLITON FOR A SEISMIC CHANNEL

^{1,*}Héctor Torres-Silva and ²Diego Torres Cabezas

¹Escuela Universitaria de Ingeniería Eléctrica-Electrónica, Universidad de Tarapacá, Avda, 18 de Septiembre 2222, Casilla Postal 6-D. Arica, Chile

²Departamento Tecnologías de Información, Dirección del Trabajo, Agustinas 1253, Of. 509. Santiago, Chile

ARTICLE INFO	ABSTRACT
<i>Article History:</i> Received 28 th July, 2015 Received in revised form 09 th August, 2015 Accepted 25 th September, 2015 Published online 20 th October, 2015	Although seismic waves have been studied for many years, their soliton nature has only recently come to wide notice. Deformation solitons propagate along earthquake faults and induce earthquakes. Rotation solitons are generated in earthquake sources and propagate throughout the Earth. The conclusion to be reached from our example is that the research on seismic solitons is essential for investigating the propagation of seismic waves and helps understand mechanisms triggering earthquakes. This paper discusses the development of elastodynamics equations similar to Maxwell's
Key words:	 equations in a chiral single-mode which is applied to a seismic channel, which is dispersive and nonlinear. The chirality is described in terms of the formalism proposed by Born-Fedorov. The
Earthquake, Soliton, Channel, Chirality.	nonlinearity is Kerr-type, and dispersion of the medium is taken into account explicitly through the Taylor series expansion. Through numerical calculations these theoretical results would allow analyze the effects of chirality on the soliton equation for propagation of S-seismic pulses of strong earthquakes as happened recently in Japan Chili and Nepal.

Copyright © 2015 Héctor Torres-Silva and Diego Torres Cabezas. This is an open access article distributed under the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

Citation: Héctor Torres-Silva and Diego Torres Cabezas, 2015. "Chiral schrödinger soliton for a seismic channel", *International Journal of Current Research*, 7, (10), 21218-21222.

INTRODUCTION

Rotational motions in earthquake sources naturally generate rotational seismic waves. The goal of this study is to describe rotational seismic waves excited in earthquake sources (Majewski, 2008; Teisseyre, 2007). We consider a continuum with nonlinear microstructure. Such a continuum allows the propagation of nonlinear rotational seismic waves. We consider the rotational seismic waves obtaining a nonlinear equation describing rotational seismic waves propagating in the solid Earth modelled as the continuum with nonlinear microstructure (Teisseyre, 2009). This paper is about the seismic waves in a strong earthquake. The difference in speed of travel of P-waves and S-wave is vital to transmit energy of seismic wave. The P wave is a longitudinal wave or a compression wave. Force is applied in the direction that the wave is travelling. Ground or earth is pretty incompressible, so the energy is transferred pretty quickly. In the S wave, the medium is displaced in a transverse (up and down - compared to the line of travel) way, and the medium must shear or "move away" from the material right next to it to cause the shear and transmit the wave.

*Corresponding author: Héctor Torres-Silva

Escuela Universitaria de Ingeniería Eléctrica-Electrónica, Universidad de Tarapacá, Avda,18 de Septiembre 2222, Casilla Postal 6-D. Arica, Chile This takes more time, and this is why the S wave moves more slowly than the P wave in seismic events. Here we model the interplay between the nonlinearity and dispersion using a chiral approach. There are two types of tectonic solitons excited in the earthquake source and propagating along the fault: longitudinal self-distortion (plastic) solitons and shear selfdistortion solitons The Earth's interior is modeled as an elastoplastic continuum and elastoplastic waves were investigated by Erofevev (2003). He derived two soliton equations that describe elastic longitudinal and plastic shear distortion solitons. The problem of sine-Gordon solitons propagating along the fault was considered by Teisseyre and Yamashita (1999). We apply similar method in order to determine the solitons propagating along the fault. The type of solitons studied here are the seismic shear self-distortion Schrödinger's solitons that propagate along the fault and describes the tectonic wave that can be excited by past earthquakes and may propagate slowly along the fault to trigger new earthquakes.

Equation (19) of this paper describe nonlinear Schrödinger's equation that with shear self-distortion solitons. This type of equation usually has multi-soliton solutions. This means that many solitons can be excited and may propagate slowly along the fault. Our main result (equation 19) is obtained under the chiral approach (Lakhtakia *et al.*, 1985; Torres Silva *et al.*, 1997; Torres Silva *et al.*, 1998; Torres Silva *et al.*, 1996). The chirality was first observed as optical activity and corresponds

to the rotation of the plane of polarization in linear isotopic materials. The Born-Fedorov equations for elastodynamic system are given by (Torres Silva *et al.*, 1996; Torres-Silva and Souza de Assis, 2010; Torres-Silva and Lopez-Bonilla, 2011).

 $D = \varepsilon \left(\vec{E} + T_c \nabla \times \vec{E} \right) \tag{1}$

 $B = \mu \left(\vec{H} + T_c \nabla \times \vec{H} \right) \tag{2}$

These equations are symmetric under duality transformations and temporal reversibility. The pseudoscalar T_c represents the measure of chirality and has units of length. The validity of equations (1) and (2) has been demonstrated in studies of optically active molecules (Lakhtakia et al., 1985), and of the propagation of light in optically active crystals (Majewski, 2008). Although from an electromagnetic point of view, chiral homogeneous material can be described by different specific equations (Torres Silva et al., 1997; Torres Silva et al., 1998; Torres Silva et al., 1996). By analogy between elastodynamics and electrodynamics in this work we use the Born-Fedorov equations as the most suitable for applications of our interest. Here the density of matter is equivalent to the electric permittivity $\rho \rightarrow \varepsilon$, and the Lamé parameter $\mu_s \rightarrow 1/\mu$. The speed of transversal seismic S- waves is given by $\sqrt{\rho/\mu_s}$ [10.12]. A theoretical investigation is made of the changes of the polarization of transverse seismic waves during their propagation through the Earth. The consequences of these results for earthquake mechanism studies, based on transverse waves, are discussed.

Basic equation for rotational propagation

Using equations (1) and (2), we obtain in the analogous framework under this section the nonlinear Schrödinger equation for a chiral seismic channel. (S-type waves)

$$D_{s} = \rho_{n} \vec{E}_{s} + \rho T_{c} \left(\nabla \times \vec{E}_{s} \right)$$
(3)

Where ρ_n is the density and T_c the chiral rotational seismic coefficient. The corresponding like Maxwell equations are

$$\nabla \times \vec{H}_{s} = \frac{\partial \left(\rho_{n}\vec{E}_{s}\right)}{\partial t} + \sigma \vec{E}_{s} + \frac{\partial}{\partial t}T_{c}\left(\nabla \times \vec{E}_{s}\right) = \frac{\rho_{n}\partial \vec{E}_{s}}{\partial t} + \sigma \vec{E}_{s} + \rho T_{c}\nabla \times \frac{\partial \vec{E}_{s}}{\partial t} \quad \dots \quad (5)$$
$$\nabla \times \vec{E}_{s} = -\frac{\partial \vec{B}_{s}}{\partial t} = -(1/\mu_{s})\frac{\partial \vec{H}_{s}}{\partial t} - (1/\mu_{s})T_{c}\frac{\partial \left(\nabla \times \vec{H}_{s}\right)}{\partial t} \quad \dots \dots \quad (6)$$

Taking the rotational of the equation (6) and whereas

 $\nabla \cdot D_{S} \Box 0 = \rho_{n} \nabla \cdot \vec{E}_{S} + \rho T_{c} \nabla \cdot \nabla \times \vec{E}_{S}; \ \nabla \cdot \vec{E}_{S} \cong 0 \rightarrow \nabla \rho_{n} \rightarrow 0 \text{ small}$ $\nabla \cdot B_{S} = 0 \Longrightarrow \nabla \cdot \vec{H}_{S} = 0$

We obtain the following wave equation

$$\nabla^{2} \vec{E}_{s} + (1/\mu_{s}) \rho T_{c}^{2} \frac{\partial^{2}}{\partial t^{2}} \nabla^{2} \vec{E}_{s} = (1/\mu_{s}) \rho_{n} \frac{\partial^{2} \vec{E}_{s}}{\partial t^{2}} + (1/\mu_{s}) \sigma \frac{\partial \vec{E}_{s}}{\partial t} + ((1/\mu_{s}) \rho_{n} T_{c} + (1/\mu_{s}) \rho) T_{c} \nabla \times \frac{\partial^{2} \vec{E}_{s}}{\partial t^{2}} + (1/\mu_{s}) \sigma T_{c} \nabla \times \frac{\partial \vec{E}_{s}}{\partial t}$$

$$(7)$$

Here we assume that the chiral seismic medium is of a Kerr nonlinearity type, described by a refractive index such that the seismic permittivity is

$$\rho_n = \rho_S + \rho_2 \left| \vec{E}_S \right|^2 \tag{8}$$

Where ρ_s is the linear part and ρ_2 is the nonlinear component respectively of $\rho_n \sigma$ is the rock conductivity loss. From equation (8) we can be inferred easily the expression for the index of refraction as in (Torres Silva *et al.*, 1998). Replacing ρ_n into equation (7) we obtain

$$\nabla^{2}\vec{E}_{s} + (1/\mu_{s})\rho_{s}T_{c}^{2}\frac{\partial^{2}\vec{E}_{s}}{\partial t^{2}}\nabla^{2}\vec{E}_{s} = (1/\mu_{s})\rho_{s}\frac{\partial^{2}\vec{E}_{s}}{\partial t^{2}}$$

$$+ (1/\mu_{s})\sigma\frac{\partial\vec{E}_{s}}{\partial t} + (1/\mu_{s})\rho_{2}\left|\vec{E}_{s}\right|^{2}\frac{\partial^{2}\vec{E}_{s}}{\partial t^{2}} + (1/\mu_{s})T_{c}\rho_{2}\left|\vec{E}_{s}\right|^{2}\nabla\times\frac{\partial^{2}\vec{E}_{s}}{\partial t^{2}}$$

$$+ (1/\mu_{s})T_{c}\sigma\nabla\times\frac{\partial\vec{E}_{s}}{\partial t}$$

$$(9)$$

Assuming that \vec{E}_s represents a localized waveform, which propagates in the z direction, it has

$$\vec{E}_{s}(\vec{r},t) = (\hat{x} + j\hat{y})\Psi(\vec{r},t)e^{-j(kz-\omega_{0}t)} = \vec{\Psi}e^{-j(kz-\omega_{0}t)} \qquad \dots \dots (10)$$

Where Ψ represents the complex envelope. To solve the equation (9) the property of the Fourier transform $\partial^2/\partial t^2 \leftrightarrow -\omega_0^2$ is applied, and then we determine ∇^2 and $\nabla \times$.

After several algebraic manipulations the result is as follows

$$\left(1 - T_c^2 k_0^2\right) \left(-2 j k_0 \frac{\partial \vec{\Psi}}{\partial z} - k_0^2 \vec{\Psi}\right)$$
(3)
$$- \frac{1}{v^2} \left(2 j \omega_0 \frac{\partial \vec{\Psi}}{\partial t} - \omega_0^2 \vec{\Psi}\right) =$$
(4)
$$2 \zeta k_0^3 \vec{\Psi} + j \omega_0 \alpha \left(1 - T_c k_0\right) \vec{\Psi}$$
(11)
$$- \beta \omega_0^2 \left|\vec{\Psi}\right|^2 \left(1 - T_c k_0\right) \vec{\Psi}$$

Where $v_s^2 = \mu_s \frac{1}{\rho_s}$; $\alpha = (1/\mu_s)\sigma$; $k_0 = \frac{\omega_0}{v}$; $\beta = (1/\mu_s)\rho_2$, $j = \sqrt{-1}$. To get to the equation (13) the approximation of small amplitudes is also considered, given by

$$\left|\frac{\partial^{2}\vec{\Psi}}{\partial z^{2}}\right| \Box \left|j2k\frac{\partial\vec{\Psi}}{\partial z}\right|, \left|\frac{\partial\vec{\Psi}}{\partial t}\right| \Box \left|j\omega_{0}\vec{\Psi}\right|, \left|\frac{\partial^{2}\left|\vec{\Psi}\right|^{2}\vec{\Psi}}{\partial t^{2}}\right| \Box \left|j\omega_{0}\frac{\partial\left|\vec{\Psi}\right|^{2}\vec{\Psi}}{\partial t}\right| \Box \left|j\omega_{0}\left|\vec{\Psi}\right|^{2}\vec{\Psi}\right|$$

The effect of dispersion is included heuristically. Making the change of variables $\phi = 2k_0 \vec{\Psi}$ y $z^* = \frac{(7) z}{1 - T_c^2 k_0^2}$ and rearranging the terms we obtain

$$j\left[\frac{\partial\phi}{\partial z^*} + \frac{1}{v}\frac{\partial\phi}{\partial t}\right] = -(1 - k_0 T_c)\frac{j\omega\alpha}{2k}\phi$$

$$-\frac{\beta\omega_0^2}{(2k)^3}(1 - k_0 T_c)|\phi|^2\phi$$

$$+k_0^2 T_c\left(1 - \frac{k_0 T_c}{2}\right)\phi$$
 (12)

As the envelope $\Psi(z,t)$ is a slowly varying function of in zand t, the dispersion relation $k = k(\omega)$ can be transformed to the domain of spatial variations by means of $\Delta \omega = \omega - \omega_0$, which is a small deviation of the sideband frequency with respect to ω_0 , and through $\Delta k = k - k_0$, which represent the corresponding wave number. Using the Fourier transform for Δk , $\Delta \omega$, approximating $\frac{1}{\nu} \Box \frac{\Delta k}{\Delta \omega}$, and using the Taylor series we obtain

$$\Delta k = \frac{1}{v} \frac{\partial}{\partial t} = \frac{\partial k}{\partial \omega} \frac{\partial}{\partial t} - j \frac{1}{2} \frac{\partial^2 k}{\partial \omega_0^2} \frac{\partial^2}{\partial t^2} + \dots = \frac{k_0}{\omega_0} \frac{\partial}{\partial t}$$

By substituting this operator in equation (14), we obtain

Where

$$k' = \frac{\partial k}{\partial \omega} = \frac{1}{v_g}; \ k'' = \frac{\partial^2 k}{\partial \omega^2};$$

Equation (13) describes the propagation of pulses in a chiral, rotational, dispersive and non linear channel. The analysis of each term is as follows (Torres Silva *et al.*, 1998):

The first term represents the evolution of the pulse with distance. The second, and third terms represents the scattering of a seismic chiral channel $k' \left(= \frac{1}{v_{c}} \right)$ and k'' corresponds to the chromatic dispersion k', indicating that the pulse moves with the group velocity, while the dispersion of the group velocity (GVD) is represented by k'', which alters the relative phases of the frequency components of pulse widening, producing its temporal expansion. The fourth is associated with attenuation (α) , in this case such losses are weighted by the seismic chirality. The fifth term $|\phi|^2 \phi$ represents nonlinear effect, like a Kerr effect, which is characterized by having a refractive index dependent on the seismic field intensity. An index of this type, means that we have a phase shift dependent on the intensity and as temporal changes of phase are also temporal changes of frequency, we have that type Kerr nonlinearities can alter and widen the spectrum of frequency of the pulse. This term also depends on the chiral factor of the channel. Finally, the last term is clearly associated with the chirality of the seismic channel.

Analysis

In order to ease up the solution of the propagation equation the following changes of variables is introduced:

$$t^* = t - \frac{Z}{v_g}, \ z^* = Z$$

Defining

$$v_{s}^{2} = \mu_{s} \frac{1}{\rho_{s}}; \ \alpha = (1/\mu_{s})\sigma; \ k_{0} = \frac{\omega_{0}}{\nu}; \ \beta = (1/\mu_{s})\rho_{2}; C = 1 - k_{0}T_{c}$$
.....(14)

(12)

We have

$$j\frac{\partial q}{\partial Z} = -\frac{1}{2}k''\frac{\partial^2 q}{\partial t^{*2}} - j\frac{\omega_0\alpha C}{2k_0}q + \frac{\beta\omega_0^2\alpha}{(2k_0)^3}|q|^2q \qquad (16)$$

Finally, is useful normalize the Eq. (16) by introducing

$$\tau = \frac{t^*}{t_0}$$
.....(17)

Let us model heuristically the relationship between the wave amplitude and the initial seismic power as follows

The normalized amplitude q (equation 16) is proportional to

$$\sqrt{\frac{P_s}{P_{ref}}}$$
 or $\sqrt{\frac{P_s t_s}{P_{ref} t_{ref}}} = \sqrt{\frac{E_s}{E_{ref}}}$

Where P_s is the power earthquake, P_{ref} is the power reference and t_s is the time duration of the earthquake, $t_s = t_{ref}$, so using the empirical equation of Gutemberg and Richer (Gutemberg and Richer, 1956)

$$\log E_{\rm s} = 11.8 + 1.5M$$
(18)

We have

$$\frac{E_S}{E_{ref}} = e^{\sqrt{0.12(M_S - M_{ref})}}$$

Thus we can model the S-wave q through the transformation

$$q(Z,\tau) = e^{\sqrt{0.12(M_s - M_{ref})}} e^{-\frac{\alpha \omega_0 C}{2k_0} Z} U(Z,\tau)$$
 (19)

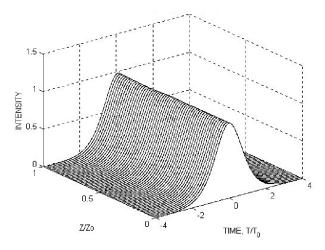
Where M_s is the peak magnitude of the soliton and M_{ref} is the reference magnitude for a weak seism which acts as standard reference. Finally the expression (16) can be scaled as

$$j\frac{\partial U}{\partial Z} = -\frac{1}{2}\frac{k''}{\iota_0^2}\frac{\partial^2 U}{\partial \tau^2} + \frac{\beta\omega_0^2}{(2k_0)^3}e^{\sqrt{0.12(M_s - M_{ref})}}e^{-\frac{\alpha\omega_0}{k_0}CZ} |U|^2 U \quad \dots \dots (20)$$

To our knowledge this is the first time that the magnitude M_s of earthquake is considered into the nonlinear Schrodinger equation. Through numerical calculations these theoretical results (eqs 14-20), would allow analyze the effects of chirality on the attenuation of equation (20) for propagation of S-seismic pulses of strong earthquakes as happened recently in Japan Chili and Nepal. Also we take into account the magnitude M_s of the earthquake.

Simulations

To solve the pulse propagation in nonlinear dispersive media we use the split-step Fourier method. The relative speed of this method compared with most FD methods can be attributed in part to the use of finite-Fourier-transform (FFT) algorithm (Agrawal, 1995). In general, linear and nonlinear parts of equation (19) act together along the length of the earthquake. The split-step method obtains an approximate solution by assuming that in propagation the seismic field over a small distance Δz , is carried out in two steps. In the first step, from $Z = Z_0$ to $Z = Z_0 + \Delta Z/2$, the linear part acts alone, and the non linear part is zero. In the second step, the nonlinearity acts alone in the point $\Delta Z/2$, (linear part is null).



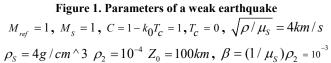


Figure 1. Shows the Intensity U calculated numerically from equation (20). The parameters correspond to a weak earthquake with $M_{ref} = 1$, so M_{ref} is the reference magnitude for a weak seism which acts as the reference standard to nonlinear Schrodinger equation with $M_s > 1$. Here the chiral effect is not considered $T_c = 0$. The equation (20) is reduced to

$$j\frac{\partial U}{\partial Z} = -\frac{1}{2}\frac{k''}{\iota_0^2}\frac{\partial^2 U}{\partial \tau^2} + \frac{\beta\omega_0^2}{(2k_0)^3}\left|U\right|^2 U$$

Which is the typical result obtained by other authors (Erofeyev, 2003; Gordon, 1983). Figure 2. Shows the envelope of U for a strong earthquake with $C = 1 - k_0 T_c = 0.3$, Nearly of Z = 100 km we have a peak of U, which it indicates that the maximum of U can trigger a major earthquake. From equation (20) we can obtain $M_s - M_{ref} \approx 6.4$.

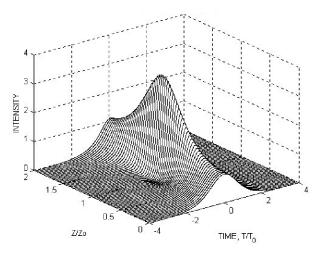


Figure 2. Parameters of a strong earthquake, Peak of $M_s > 7$,

$$\sqrt{\rho/\mu_s} = 4km/s , \rho_s = 4g/cm^3 , \rho_2 = 10(-4) Z_0 = 100km$$

As we can see, the slow rotational tectonic waves propagate along the fractured tectonic fault with a speed of about kilometers per second. These waves may have a form of rotational seismic solitons and they can trigger major earthquakes. Thus, the research on rotational seismic solitons is essential for investigating the propagation of seismic waves and helps understand mechanisms triggering earthquakes. Thus chiral rotational seismic waves propagate faster in solid rocks and much slower in fractured media along tectonic faults.

Conclusion

In this paper we have obtained the Schrödinger nonlinear equation for a channel whose core is chiral, dispersive and has nonlinear behavior. The effect of chirality is manifested by the terms associated with the loss of the chiral channel and the nonlinear coefficient. The phenomena that produces the dispersive and non-linear effects in a non-chiral seismic channel is the factor C, (which produce the soliton propagation for example). The most significant result of our work is that to use the chirality of S waves can cancel out the losses and nonlinearities of the channel, which would allow us to modify radically their behavior as channel of seismic soliton. To advance our work we hope to make a deeper theoretical study of the effect of chirality in seismic channels considering the spread of Gaussian pulses. Numerical calculation of the equations can be obtained, so as to analyze the behavior of seismic waves modelled by soliton waves. Through numerical calculations these theoretical results would allow analyze the effects of chirality on the equation for propagation of Sseismic pulses of strong earthquakes as happened recently in Japan Chili and Nepal

REFERENCES

- Agrawal, G., Nonlinear Fiber Optics, Academic Press, Inc., 1995.
- Erofeyev, V. I. 2003. Wave processes in solids with microstructure. World Scientific, Singapore
- Gordon, J. P. 1983. Opt. Lett., 8, 596.
- Gutemberg, B. and C. F. Richer, 1956. Ann. Geoph.9,1.
- Hector Torres-Silva, Diego Torres Cabezas, D. 2013. "Chiral Seismic Attenuation with Acoustic Metamaterials", *Journal of Electromagnetic Analysis and Applications*, Vol.5 No.1,
- Lakhtakia, A., Varadan, V. K. and Varadan, V. V. 1985. "Time-Harmonic Electromagnetic Fields in Chiral Media" Lecture Notes in Physics 335, Springer-Verlag.
- Majewski, E., 2008. Canonical approach to asymmetric continua, in Physics of Asymmetric Continuum: Extreme and Fracture Processes—Earthquake Rotation and Soliton Waves, R. Teisseyre, H. Nagahama, and E. Majewski (Editors), Springer, Berlin, 209–218.
- Teisseyre, K. P., 2007. Analysis of a group of seismic events using rotational components, Acta Geophys. 55, 535–553.

- Teisseyre, R. 2009. New developments in the physics of rotational motions, *Bull. Seismol. Soc. Am.*, 99, no. 2B, 1028–1039.
- Teisseyre, R., Yamashita, T. 1999. Splitting stress motion equations into seismic wave and fault-related fields. *Acta Geophys Pol.*, 47: 2, 135-147
- Torres Silva, H., Sakanaka P. H. and Reggiani, N. 1996. Revista Mexicana de Física, 42, 989.
- Torres Silva, H., Sakanaka, P. H. and Reggiani, N. 1997. "Electromagnetic properties of a chiral-plasma", *Pramana Journal of Physics*, 48, 1.
- Torres Silva, H., Sakanaka, P. H. and Reggiani, N. 1998. Journal of Physics Society of Japan, 850.
- Torres-Silva, H. and Souza de Assis, A. 2010. Efectos de velocidad y de gravitación en GPS satelitales: un esquema para la predicción y detección temprana de fuertes terremotos, Ingeniare, Vol. 18, pp 286-294.
- Torres-Silva, H. and Lopez-Bonilla, J. L. 2011. "Early Prediction and detection of Strong Earthquakes through Chiral Radiation Waves", *Journal of Vectorial Relativity*, 6, 2, 1-11.
