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RESEARCH ARTICLE

DEFINITION OF FEATURES OF INTERNAL FORCE FACTORS FUNCTIONALLY GRADED PLATES

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ABSTRACT

The behavior of a plate of the functionally graded material under concentrated loads was under consideration. Using the method of decomposition of the desired functions in series by Legendre polynomials on the transverse coordinate, three-dimensional problem for plates was reduced to a two-dimensional one. This approach has allowed us to take into account the transverse shear and normal stresses. On the basis of received equations, using the two-dimensional Fourier integral transform and the generalization method that was built with a special G-function, the fundamental solution was found. Numerical studies demonstrated the behavior patterns of the stress-strain state components depending on the elastic constants of functionally graded material were performed.

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INTRODUCTION

Functionally graded materials (FGM) are materials that have a continuous spatial variation of mechanical properties, such as a density and elastic modules. These materials cause considerable recent interest from experts in the field of theoretical and applied mechanics and engineering community. By varying the volume fractions of materials it is possible to obtain a smooth change in properties from one surface to another. This change in material properties can significantly reduce stress concentrations in the structure elements by the action of various factors. Production of thin-walled elements of designs from FGM leads to the need to study their behavior. These materials with spatially inhomogeneous microstructure are important in promising areas such as energy, gas turbines, nuclear fusion, etc.

The first works devoted to the study of the properties of functionally graded materials have been proposed (Yamanoushi *et al.*, 1990) and (Koizumi, 1993). Theory of plates and shells made of FGM is rapidly developing in recent years that is confirmed by a large number of papers (Alijani *et al.*, 2011; Zenkour, 2006; Chorfi & Houmat, 2010; Altenbach & Eremeyev, 2008, Matsunaga, 2008), and others. According to the literature review, a detailed study was done to the questions of linear and non-linear dynamic and static analysis of functionally graded plates and membranes. The greatest attention is given to objects rectangular or round shape.

The large number of publications is dedicated to development of methods for constructing fundamental solutions (solutions corresponding to concentrated impacts) of equations of the theory of thin elastic plates and shells. Problem statement, methods for their solutions and a number of concrete results are presented in monographs and scientific articles of (Ambartsumyan, 1974; Gol'denveizer, 1944) as well as in a number of reviews of (Darevsky, 1966; Zhigalko, 1966) and others. Fundamental solutions of the static equations for isotropic plates and for transversely isotropic plates were obtained in the papers of (Bokov & Strelnikova, 2015).

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From the analysis of these studies we can conclude that there are two approaches to the construction of fundamental solutions of equations of thin elastic plates and shells. The first is the study of singular solutions of homogeneous differential equations corresponding to a specific concentrated action.Such an approach has been successfully used for the spherical shell by (Gol'denveizer, 1954), for shallow spherical and cylindrical shells by (Vekua, 1970; E. Reissner, 1985) and then for shallow shells of double curvature – (Kiel, 1973; Ganowicz, 1973; Jahanshahi, 1963). A major shortcoming of this approach is that for the definition of a singular solution, corresponding to a particular concentrated action, it is necessary to satisfy the system of geometric and static conditions in the vicinity of the singular point. Sometimes, especially for non-spherical shells, it leads to erroneous results, or to solutions containing excess regular solutions.

The second approach leads to the solution of differential equations with right-hand sides of the Dirac delta function. This is used in a variety of methods for constructing fundamental solutions. The most common of them - the Fourier integral method - developed in the works of (Velichko *et al.*, 1971; Velichko *et al.*, 1968; Velichko *et al.*, 1975; Sanders, 1970). All of these methods have been developed to study the nature of the component features of the stress-strain state of plates and shells, or to study the strength of plates and shells under local loads.

MATERIALS AND METHODS

Basic relations and mathematical formulation of the problem

In a rectangular Cartesian coordinate system x, y, z we consider plate in thickness of $2h$, made of a mixture of cast iron and copper. Mechanical properties of the composite vary continuously across the thickness of the plate so that the upper portion is copper, and the bottom – cast iron. Suppose that on the plate a concentrated force $\bar{F}$ is applied at the coordinate origin (singular point). Concentrated force can be represented as an abstraction (end-largest force acting on a small area of the surface (Han, 1988)).

In solving problems about the action of concentrated forces, required SSS is considered as local that does not extend to the outer contour line of the plate. Therefore, we consider the plate as an infinite and assume that the required components of the SSS tend to zero at infinity. The validity of this assumption is checked after solving the problem. The mathematical formulation of the problem contains a complete system of equations of the theory of elasticity without considering the boundary conditions on the edges of a real plate. The system of equilibrium equations of isotropic plates on the basis of the theory of S.P. Timoshenko describing the SSS at bending consists of (Pelekh & Lazko, 1982):

– geometrical relationships

$$e_{x1} = h\frac{\partial \gamma_x}{\partial x}, e_{xy1} = h\left(\frac{\partial \gamma_x}{\partial y} + \frac{\partial \gamma_y}{\partial x}\right), e_{xz0} - \frac{e_{xz2}}{5} = \gamma_x + \frac{\partial w_0}{\partial x} \; (x \to y) \tag{1}$$

– Hooke's law relationships

$$M_x = D\left(e_{x1} + \nu e_{y1}\right), \; M_y = D\left(e_{y1} + \nu e_{x1}\right), \; H = \frac{1-\nu}{2} D e_{xy1}, Q_x = \Lambda\left(e_{xz0} - \frac{e_{xz2}}{5}\right) (x \to y), \tag{2}$$

where $D = \frac{2h^2}{3}\frac{E}{1-\nu^2}, \Lambda = \frac{5hG}{3}$.

– equilibrium equations

$$\frac{\partial M_x}{\partial x} + \frac{\partial H}{\partial y} - Q_x + m_x = 0, \frac{\partial M_y}{\partial y} + \frac{\partial H}{\partial x} - Q_y + m_y = 0, (3) \frac{\partial Q_x}{\partial x} + \frac{\partial Q_y}{\partial y} + q_z = 0. \tag{3}$$

To find a fundamental solution of the system (1) – (3), the vector components of volumetric forces in formulas (3) should be taken in the form

$$m_x(x, y) = h^2 m_x^* \delta(x, y), m_y(x, y) = h^2 m_y^* \delta(x, y), \tag{4}$$

$$q_z(x, y) = h^2 q_z^* \delta(x, y) \; (x \to y),$$

where $\overset{*}{m}_x, \overset{*}{m}_y, \overset{*}{q}_z = const$, $\delta(x, y)$ – two-dimensional Dirac delta function (Vladimirov, 1976).

Statement of research method

Substituting geometrical ratio (1) in the ratio of elasticity (2) and going to the dimensionless coordinate system $x_1 = x/h$, $x_2 = y/h$, $x_3 = z/h$ we obtain:

$$M_1 = D_0\left(\frac{\partial\gamma_1}{\partial x_1} + \nu\frac{\partial\gamma_2}{\partial x_2}\right), M_2 = D_0\left(\frac{\partial\gamma_2}{\partial x_2} + \nu\frac{\partial\gamma_1}{\partial x_1}\right),$$

$$H = \frac{1-\nu}{2}D_0\left(\frac{\partial\gamma_1}{\partial x_2} + \frac{\partial\gamma_2}{\partial x_1}\right), \tag{5}$$

$$Q_1 = \Lambda_0\left(\gamma_1 + \frac{\partial w_0}{\partial x_1}\right), Q_2 = \Lambda_0\left(\gamma_2 + \frac{\partial w_0}{\partial x_2}\right),$$

where $D_0 = \frac{D}{Eh^2} = \frac{2}{3}\frac{1}{1-\nu^2}$, $\Lambda_0 = \frac{5G}{3E}$.

Stretch bending and torque are determined in relation to the value Eh^2, shear forces– in relation to the value Eh.
Going to the dimensionless coordinates, we obtain:

$$\frac{\partial M_1}{\partial x_1} + \frac{\partial H}{\partial x_2} - Q_1 + m_1 = 0, \quad \frac{\partial M_2}{\partial x_2} + \frac{\partial H}{\partial x_1} - Q_2 + m_2 = 0, \tag{6}$$

$$\frac{\partial Q_1}{\partial x_1} + \frac{\partial Q_2}{\partial x_2} + q_3 = 0,$$

where $m_1 = \overset{*}{m}_1\delta(x_1, x_2)$, $m_2 = \overset{*}{m}_2\delta(x_1, x_2)$, $q_3 = \overset{*}{q}_3\delta(x_1, x_2)$.

Solving specified system, we obtain the transform of generalized displacements:

$$\tilde{\gamma}_1 = \frac{1}{2\pi}\left[\frac{\overset{*}{m}_1}{D_0}\frac{\xi_1^2}{p^4} + 3(1+\nu)\overset{*}{m}_1\frac{\xi_2^2}{p^2\left(p^2 + 2{,}5\right)} + \frac{\overset{*}{q}_3}{D_0}\frac{i\xi_1}{p^4} + \frac{\overset{*}{m}_2}{D_0}\frac{\xi_1\xi_2}{p^4} - \right.$$

$$\left. - 3(1+\nu)\overset{*}{m}_2\frac{\xi_1\xi_2}{p^2\left(p^2 + 2{,}5\right)}\right], \tag{7}$$

$$\tilde{\gamma}_2 = \frac{1}{2\pi}\left[\frac{\overset{*}{m}_2}{D_0}\frac{\xi_2^2}{p^4} + 3(1+\nu)\overset{*}{m}_2\frac{\xi_1^2}{p^2\left(p^2 + 2{,}5\right)} + \frac{\overset{*}{q}_3}{D_0}\frac{i\xi_2}{p^4} + \frac{\overset{*}{m}_1}{D_0}\frac{\xi_1\xi_2}{p^4} - \right.$$

$$\left. - 3(1+\nu)\overset{*}{m}_1\frac{\xi_1\xi_2}{p^2\left(p^2 + 2{,}5\right)}\right],$$

$$\tilde{w}_0 = \frac{1}{2\pi}\left[-\frac{\overset{*}{m}_1}{D_0}\frac{i\xi_1}{p^4} - \frac{\overset{*}{m}_2}{D_0}\frac{i\xi_2}{p^4} + \frac{\overset{*}{q}_3}{D_0}\frac{1}{p^4} + \frac{\overset{*}{q}_3}{\Lambda_0}\frac{1}{p^2}\right],$$

where $p^2 = \xi_1^2 + \xi_2^2$; (ξ_1, ξ_2) – point coordinates in the space of transform.
Applying the Fourier transform to equations of Hooke's law (5):

$$\tilde{M}_1 = -D_0\left(i\xi_1\tilde{\gamma}_1 + i\nu\xi_2\tilde{\gamma}_2\right),\ \tilde{M}_2 = -D_0\left(i\xi_2\tilde{\gamma}_2 + i\nu\xi_1\tilde{\gamma}_1\right),$$
$$\tilde{H} = \frac{1-\nu}{2}D_0\left(i\xi_2\tilde{\gamma}_1 + i\xi_1\tilde{\gamma}_2\right), \tag{8}$$
$$\tilde{Q}_1 = \Lambda_0\left(\tilde{\gamma}_1 - i\xi_1\tilde{w}_0\right),\ \tilde{Q}_2 = \Lambda_0\left(\tilde{\gamma}_1 - i\xi_2\tilde{w}_0\right).$$

Substitute the previously obtained transforms of generalized displacements (7) to the transforms of torque (8)

$$\tilde{H} = -\frac{1}{2\pi}\left[(1-\nu)m_1^*\frac{i\xi_1^2\xi_2}{p^4} + m_1^*\frac{i\xi_2^3}{p^2\left(p^2+2{,}5\right)} - \right. \tag{9}$$
$$-(1-\nu)q_3^*\frac{\xi_1\xi_2}{p^4} + (1-\nu)m_2^*\frac{i\xi_1\xi_2^2}{p^4} - m_2^*\frac{i\xi_1\xi_2^2}{p^2\left(p^2+2{,}5\right)} +$$
$$\left. + m_2^*\frac{i\xi_1^3}{p^2\left(p^2+2{,}5\right)} - m_1^*\frac{i\xi_1^2\xi_2}{p^2\left(p^2+2{,}5\right)}\right].$$

Let us denote

$$\tilde{\Phi}_1(\xi_1,\xi_2) = \frac{i\xi_1^2\xi_2}{p^4},\ \tilde{\Phi}_2(\xi_1,\xi_2) = \frac{i\xi_1^2\xi_2}{p^2\left(p^2+2{,}5\right)}, \tag{10}$$
$$\tilde{\Phi}_3(\xi_1,\xi_2) = \frac{i\xi_1^3}{p^2\left(p^2+2{,}5\right)},\ \tilde{\Phi}_5(\xi_1,\xi_2) = \frac{\xi_1\xi_2}{p^4}.$$

Then, torque in the space of transforms are written as

$$\tilde{H} = -\frac{1}{2\pi}\left[(1-\nu)m_1^*\tilde{\Phi}_1(\xi_1,\xi_2) + m_1^*\tilde{\Phi}_3(\xi_2,\xi_1) - \right. \tag{11}$$
$$-(1-\nu)q_3^*\tilde{\Phi}_5(\xi_1,\xi_2) + (1-\nu)m_2^*\tilde{\Phi}_1(\xi_2,\xi_1) - m_2^*\tilde{\Phi}_2(\xi_2,\xi_1) +$$
$$\left. + m_2^*\tilde{\Phi}_3(\xi_1,\xi_2) - m_1^*\tilde{\Phi}_2(\xi_1,\xi_2)\right],$$

We must now turn ratio (11). First, we find the originals of functions (10) using a Fourier integral (Sneddon, 1955)

$$F^{-1}\left[\tilde{f}(\xi_1,\xi_2)\right] = f(x_1,x_2) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\tilde{f}(\xi_1,\xi_2)e^{-i(\xi_1x_1+\xi_2x_2)}d\xi_1 d\xi_2 \tag{12}$$

Obtain

$$\Phi_1(x_1,x_2) = -\frac{x_2\left(x_1^2+3x_2^2\right)}{2\left(x_1^2+x_2^2\right)^2},\ \Phi_2(x_1,x_2) = \frac{x_2}{2\left(x_1^2+x_2^2\right)}\sqrt{2{,}5}G_{0,1}\sqrt{x_1^2+x_2^2} +$$
$$+\frac{x_2\left(3x_1^2-x_2^2\right)}{2\left(x_1^2+x_2^2\right)^2}\sqrt{2{,}5}G_{1,2}\sqrt{x_1^2+x_2^2}, \tag{13}$$

$$\Phi_3(x_1,x_2)=\frac{3x_1}{2\left(x_1^2+x_2^2\right)}G_{0,1}\sqrt{2,5}\sqrt{x_1^2+x_2^2}+\frac{x_1\left(x_1^2-3x_2^2\right)}{2\left(x_1^2+x_2^2\right)^2}G_{1,2}\sqrt{2,5}\sqrt{x_1^2+x_2^2},$$

$$\Phi_5(x_1,x_2)=-\frac{1}{2}\frac{x_1x_2}{x_1^2+x_2^2}.$$

where $G_{n,\nu}(rz)$ – a special G-function (Khizhnyak & Shevchenko, 1980).

Applying inversion formula for the two-dimensional Fourier integral (12) to the transform of the internal power factor (11) and taking into account the expression (13), we obtain

$$H=-\frac{1}{2\pi\left(x_1^2+x_2^2\right)}\left[-(1-\nu)m_1^*\frac{x_2\left(x_1^2+3x_2^2\right)}{2\left(x_1^2+x_2^2\right)}+\sqrt{2,5}\sqrt{x_1^2+x_2^2}\,x_2m_1^*\times\right.$$
$$\times\left\{G_{0,1}-\frac{3x_1^2-x_2^2}{x_1^2+x_2^2}G_{1,2}\right\}+(1-\nu)q_3^*\frac{x_1x_2}{2}-(1-\nu)m_2^*\frac{x_1\left(3x_1^2+x_2^2\right)}{2\left(x_1^2+x_2^2\right)}+ \quad (14)$$
$$\left.+\sqrt{2,5}\sqrt{x_1^2+x_2^2}\,x_1m_2^*\left\{G_{0,1}+\frac{x_1^2-3x_2^2}{x_1^2+x_2^2}G_{1,2}\right\}\right].$$

We received expression for H in the space of originals.

RESULTS

The results of studies of the influence of the elastic parameters on the SSS of the plate

To study the features of the SSS of functionally graded plates under concentrated force action we set:

$m_1^*=m_2^*=m_3^*=1$.

Results are presented in a dimensionless Cartesian coordinate system x_1, x_2. Numerical studies have been conducted for the following composite materials: copper and cast iron. Poisson's ratios (ν) for these materials are: 0,348 and 0,17 respectively (Dementyev *et al.*, 2002).

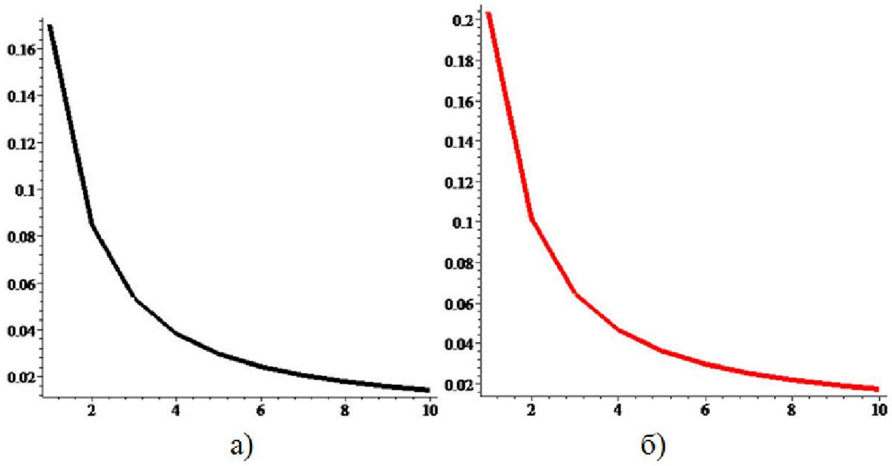

Fig. 1. Torque H: a) – copper; b) – cast iron

Fig. 1 shows graphs of changes of generalized moment H along the abscissa axis ($x_2 = 0$) for individual materials. These graphs show that with decreasing the Poisson's ratio values of torque increases.

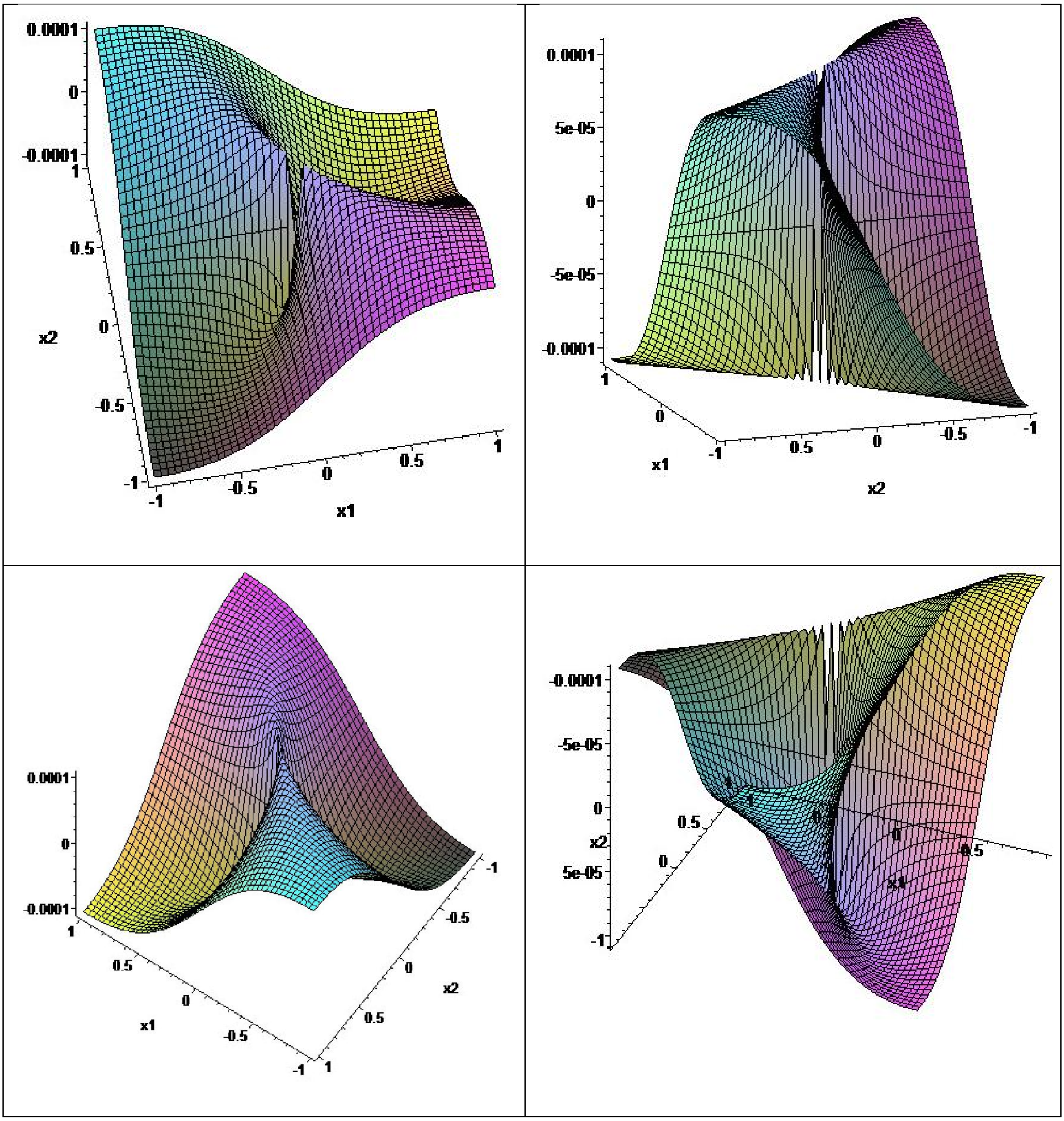

Fig. 2. Torque H. The composite of copper and iron

Fig. 2 shows graphs of changes of torque H for functionally graded material that is the mixture of copper and cast iron. Poisson's ratio for this material varies linearly in the interval $\left[\frac{1}{3};\frac{2}{3}\right]$ of thickness the plate. The dependence of the torque value from the Poisson's ratio is the same as shown in Fig. 1.

DISCUSSION AND CONCLUSION

Although useful in its own frame and offering a practical tool of investigating the structure elements of FGM materials this method will provide a valuable mean for solution of a number of new tasks of bending the plates of medium thickness.

In the presence of concentrated dislocations fundamental decisions - Green's functions - are the foundation for building potential representations as integrals of displacement jumps distributed with unknown density. Such integral representations will be used in solving the problems of bending the plates with different kinds of defects, cuts and incisions. The proposed method allowed us to take into account the transverse shear and normal stresses, so we overcome the shortcoming of the classical Kirchhoff-Love theory. Numerical studies of the SSS of functionally graded material are carried out. These studies allowed revealing patterns of behavior of the SSS components, depending on the elastic constants of composite materials.

The practical significance of these results is the possibility of using the developed methods for solving problems in the calculations associated with the design and definition of the operating parameters of thin-walled structural elements of functionally graded materials under the action of concentrated force impacts. Due to the large depreciation of the energy, refinery and chemical equipment at the moment, relevant issue is the problem of extending the working life of equipment, even in the presence of micro-defects in it. Calculation of the strength of such elements by using the theory of shells and plates of medium thickness in the presence of different kinds of defects based on the obtained fundamental solutions will facilitate the adjustment of the terms of reserve maintenance periods, priority of replacement of worn-out equipment.

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