Available online at http://www.journalcra.com

International Journal of Current Research
Vol. 8, Issue, 02, pp.25932-25942, February, 2016

INTERNATIONAL JOURNAL
OF CURRENT RESEARCH

ISSN: 0975-833X

RESEARCH ARTICLE

TIME VARYING AND DYNAMIC CORRELATION AMONG OIL AND NATURAL GAS PRICES: MULTIVARIATE LONG MEMORY APPROACH

*Riadh El Abed

University of Tunis El Manar, Faculté des Sciences Economiques et de Gestion de Tunis, Laboratoire d'Ingénierie Financière et Economique (LIFE), 65 Rue Ibn Sina, Moknine 5050, Tunisia

ARTICLE INFO

Article History:
Received 25th November, 2015
Received in revised form
22nd December, 2015
Accepted 05th January, 2016
Published online 14th February, 2016

Key words:

DCC-FIAPARCH, Asymmetries, Long memory, Natural gas prices and oil prices.

JEL classification:
C13, C22, C32, C52.

ABSTRACT

This study examines the interdependence of natural gas price (HENRYHUB) and oil prices (WTI and BRENT). The aim of this paper is to examine how the dynamics of correlations between the markets evolved from January 01, 2004 to February 26, 2015. To this end, we adopt a dynamic conditional correlation (DCC) model into a multivariate Fractionally Integrated Asymmetric Power ARCH (FIAPARCH) framework, which accounts for long memory, power effects, leverage terms and time varying correlations. The empirical findings indicate the evidence of time-varying comovement, a high persistence of the conditional correlation and the dynamic correlations revolve around a constant level and the dynamic process appears to be mean reverting. Moreover, the univariate FIAPARCH models are particularly useful in forecasting market risk exposure for synthetic portfolios of stocks and currencies.

Citation: **Riadh El Abed, 2016.** "Time varying and dynamic correlation among oil and natural gas prices: Multivariate long memory approach", *International Journal of Current Research,* 8, (02), 25932-25942.

INTRODUCTION

Modeling volatility is an important issue of research in financial markets. Leptokurtosis and volatility clustering are common observation in financial time series (Mandelbrot, 1963). It is well known that financial returns have non-normal distribution which tends to have fat-tailed. Mandelbrot (1963) strongly rejected normal distribution for data of asset returns, conjecturing that financial return processes behave like non-Gaussian stable processes (commonly referred to as "Stable Paretian" distributions). many high-frequency financial time series have been shown to exhibit the property of long-memory and Financial time series are often available at a higher frequency than the other time series (Harris & Sollis, 2003). The long range dependence or the long memory implies that the present information has a persistent impact on future counts. Note that the long memory property is related to the sampling frequency of a time series. Natural gas and crude oil prices are among the most important fuels in the modern economy because of their extensive use by many economic sectors. They are complements and substitutes in consumption, as well as rivals, in production of electricity. Economic theory suggests that crude oil and natural gas prices should be related because natural gas and crude oil are complements in consumption and also substitutes, as well as rivals, in production. Economic variables link natural gas and oil prices through both supply and demand. Market behavior argument that past changes in the oil price drove changes in the natural gas price, but the converse did not appear to occur. Economic factors link oil and natural gas prices through both supply and demand. The crude oil price and natural gas price are characterized by asymmetries. One reason for the asymmetric interaction is the relative size of each market. Market behavior suggests that past changes in the oil price drove changes in the natural gas price. In the economic theory, increases in oil prices may affect the natural gas market in several ways. Natural gas and crude oil are competitive substitutes primarily in the industrial sectors of the economy and electric generation. According to the National Petroleum Council (NPC) in its 2003 report estimated that approximately 5 percent of industrial boilers can switch between petroleum fuels and natural gas (Costello, Huntington, and Wilson, (2005)). In the

***Corresponding author: Riadh El Abed,**
University of Tunis El Manar, Faculté des Sciences Economiques et de Gestion de Tunis, Laboratoire d'Ingénierie Financière et Economique (LIFE), 65 Rue Ibn Sina, Moknine 5050, Tunisia.

other hand, an increase in crude oil prices resulting from an increase in crude oil demand may lead to increased costs of natural gas production and development, putting upward pressure on natural gas prices. The empirical evidence on the natural gas price – crude oil relationships has been document by numerous studies. For example, Pangiotidis and Rutledge (2004) found evidence for co-integration between gas and oil prices in the United Kingdom in the period spam from 1996 to 2003. Barcella (1999) suggest the existence of co-integrated relationship between oil and gas prices in the US which was attributed to long-run economic factors. In addition, there was a high correlation of 0.916 between yearly prices of natural gas and oil price.

The dynamic relationship between gas markets and crude oil has been investigated in the extant literature. For example, Alexander (2004) finds strong correlation between returns on natural gas futures contracts and crude oil. Villar and Joutz (2006) using the cointegration techniques and find a long-run (equilibrium) relationship between the Henry Hub natural gas price and the WTI oil price. To model the evolution of electricity and natural gas prices in the United Kingdom, Benth and Kettler (2011) use a bivariate non-symmetric copula and find that options prices are significantly influenced by the marginal distributions and the copula, along with the seasonality of the underlying prices. Some recent studies showed that a separation between oil prices and natural gas prices had occurred (see, Ramberg and Parsons, 2012). Erdos (2012) found that the existing long-term equilibrium relationship between oil prices and US natural gas prices disappeared after 2009. In addition, Loungani and Matsumoto (2012) found that the separation between US natural gas prices and oil prices occurred as a result of the oversupply of natural gas due to increases in the natural gas production brought on by the US shale gas revolution. To analyze the dynamic relationships between crude oil and natural gas prices, authors chose the co-integration methodology, the ECM (error correction model) and the causality of Granger (1969), (see, Jabir, Imad. (2006)). In this paper, we empirically investigate the time-varying linkages of daily crude oil (WTI and BRENT) and natural gas prices (HENRYHUB) from January 01, 2004 until February 26, 2015. We use a DCC model into a multivariate fractionally integrated APARCH framework (FIAPARCH-DCC model), which provides the tools to understand how financial volatilities move together over time and across markets. Conrad *et al*. (2011) applied a multivariate fractionally integrated asymmetric power ARCH (FIAPARCH) model that combines long memory, power transformations of the conditional variances, and leverage effects with constant conditional correlations (CCC) on eight national stock market indices returns. The long-range volatility dependence, the power transformation of returns and the asymmetric response of volatility to positive and negative shocks are three features that improve the modeling of the volatility process of asset returns.

The flexibility feature represents the key advantage of the FIAPARCH model of Tse (1998) since it includes a large number of alternative GARCH specifications. Specifically, it increases the flexibility of the conditional variance specification by allowing an asymmetric response of volatility to positive and negative shocks and long-range volatility dependence. In addition, it allows the data to determine the power of returns for which the predictable structure in the volatility pattern is the strongest (see Conrad *et al*., 2011). Although many studies use various multivariate GARCH models in order to estimate DCCs among markets during financial crises (see Chiang *et al*., 2007; Celic, 2012; Kenourgios *et al*., 2011), the forecasting superiority of FIAPARCH on other GARCH models is supported by Conrad *et al*. (2011), Chkili *et al*. (2012) and Dimitriou and Kenourgios (2013). The present study investigates dynamics correlations among oil and natural gas prices from January 01, 2004 until February 26, 2015. We provide a robust analysis of dynamic linkages among their markets that goes beyond a simple analysis of correlation breakdowns. The time-varying DCCs are captured from a multivariate student-t-FIAPARCH-DCC model which takes into account long memory behavior, speed of market information, asymmetries and leverage effects. The rest of the paper is organized as follows. Section 2 presents the econometric methodology. Section 3 provides the data and a preliminary analysis. Section 4 displays and discusses the empirical findings and their interpretation, while section 5 provides our conclusions.

Econometric methodology

Univariate FIAPARCH model

The AR(1) process represents one of the most common models to describe a time series r_t of crude oil and natural gas returns. Its formulation is given as

$$(1-\xi L)r_t = c + \varepsilon_t, \ \ t \in \mathbb{N} \qquad \text{.... (1)}$$

with

$$\varepsilon_t = z_t\sqrt{h_t} \qquad \text{.....(2)}$$

where $|c| \in [0, +\infty[$, $|\xi| < 1$ and $\{z_t\}$ are independently and identically distributed $(i.i.d.)$ random variables with $E(z_t) = 0$. The variance h_t is positive with probability equal to unity and is a measurable function of Σ_{t-1}, which is the σ −algebra generated by $\{r_{t-1}, r_{t-2}, \ldots\}$. Therefore, h_t denotes the conditional variance of the returns $\{r_t\}$, that is:

$$E[r_t/\Sigma_{t-1}] = c + \xi r_{t-1} \qquad \text{.....(3)}$$

$$Var[r_t/\Sigma_{t-1}] = h_t \qquad \text{.... (4)}$$

Tse (1998) uses a FIAPARCH(1,d,1) model in order to examine the conditional heteroskedasticity of the yen-dollar exchange rate. Its specification is given as

$$(1-\beta L)\left(h_t^{\delta/2} - \omega\right) = [(1-\beta L) - (1-\phi L)(1-L)^d](1 + \gamma s_t)|\varepsilon_t|^\delta \qquad \text{............(5)}$$

where $\omega \in [0, \infty[$, $|\beta| < 1$, $|\phi| < 1$, $0 \leq d \leq 1$, $s_t = 1$ if $\varepsilon_t < 0$ and 0 otherwise, $(1-L)^d$ is the financial differencing operator in terms of a hypergeometric function (see Conrad *et al*., 2011), γ is the leverage coefficient, and δ is the power term parameter (a Box-Cox transformation) that takes (finite) positive values. A sufficient condition for the conditional variance h_t to be positive almost surely for all t is that $\gamma > -1$ and the parameter combination (ϕ, d, β) satisfies the inequality constraints provided in Conrad et Haag (2006) and Conrad (2010).When $\gamma > 0$, negative shocks have more impact on volatility than positive shocks. The advantage of this class of models is its flexibility since it includes a large number of alternative GARCH specifications. When $d = 0$, the process in Eq. (5) reduces to the APARCH(1,1) one of Ding *et al*. (1993), which nests two major classes of ARCH models. In particular, a Taylor/Schwert type of formulation (Taylor, 1986; Schwert, 1990) is specified when $\delta = 1$, and a Bollerslev(1986) type is specified when $\delta = 2$.When $\gamma = 0$and $\delta = 2$, the process in Eq. (5) reduces to the $FIGARCH(1, d, 1)$ specification (see Baillie *et al*., 1996; Bollerslev and Mikkelsen, 1996) which includes Bollerslev's (1986) GARCH model (when $d = 0$) and the IGARCH specification (when $d = 1$) as special cases.

Multivariate FIAPARCH model with dynamic conditional correlations

In what follow, we introduce the multivariate FIAPARCH process (M-FIAPARCH) taking into account the dynamic conditional correlation (DCC) hypothesis (see Dimitriou *et al*., 2013) advanced by Engle (2002). This approach generalizes the Multivariate Constant Conditional Correlation (CCC) FIAPARCH model of Conrad *et al*. (2011). The multivariate DCC model of Engle (2002) and Tse and Tsui (2002) involves two stages to estimate the conditional covariance matrix H_t. In the first stage, we fit a univariate FIAPARCH(1,d,1) model in order to obtain the estimations of $\sqrt{h_{iit}}$. The daily crude oil and natural gas returns are assumed to be generated by a multivariate AR(1) process of the following form:

$$Z(L)r_t = \mu_0 + \varepsilon_t \qquad \text{……..(6)}$$

where

-$\mu_0 = [\mu_{0,i}]_{i=1,\dots,n}$: the $N-$dimensional column vector of constants;
-$|\mu_{0,i}| \in [0, \infty[$;
-$Z(L) = diag\{\psi(L)\}$: an $N \times N$ diagonal matrix ;
-$\psi(L) = [1 - \psi_i L]_{i=1,\dots,n}$;
-$|\psi_i| < 1$;
-$r_t = [r_{i,t}]_{i=1,\dots,N}$: the $N-$dimensional column vector of returns;
-$\varepsilon_t = [\varepsilon_{i,t}]_{i=1,\dots,N}$: the $N-$dimensional column vector of residuals.

The residual vector is given by

$$\varepsilon_t = z_t \odot h_t^{\wedge 1/2} \qquad \text{….. (7)}$$

where

-$\odot$: the Hadamard product;
-$\wedge$: the elementwise exponentiation.

$h_t = [h_{it}]_{i=1,\dots,N}is\Sigma_{t-1}$ measurable and the stochastic vector $z_t = [z_{it}]_{i=1,\dots,N}$ is independent and identically distributed with mean zero and positive definite covariance matrix $\rho = [\rho_{ijt}]_{i,j=1,\dots,N}$ with $\rho_{ij} = 1$ for $i = j$.Note that $E(\varepsilon_t/\mathcal{F}_{t-1}) = 0$and $H_t = E(\varepsilon_t \varepsilon_t'/\mathcal{F}_{t-1}) = diag(h_t^{\wedge 1/2})\, \rho\, diag(h_t^{\wedge 1/2})$. $h_t$is the vector of conditional variances and $\rho_{i,j,t} = h_{i,j,t}/\sqrt{h_{i,t}h_{j,t}} \forall\, i,j = 1,\dots,N$ are the dynamic conditional correlations.

The multivariate FIAPARCH(1,d,1) is given by

$$B(L)\left(h_t^{\wedge \delta/2} - \omega\right) = [B(L) - \Delta(L)\Phi(L)][\mathrm{I}_N + \Gamma_t]|\varepsilon_t|^{\wedge \delta} \qquad (8)$$

where $|\varepsilon_t|$ is the vector ε_t with elements stripped of negative values.

Besides, $B(L) = diag\{\beta(L)\}$ with $\beta(L) = [1 - \beta_i L]_{i=1,\dots,N}$and $|\beta_i| < 1$. Moreover, $\Phi(L) = diag\{\phi(L)\}$ with $\phi(L) = [1 - \phi_i L]_{i=1,\dots,N}$ and $|\phi_i| < 1$. In addition, $\omega = [\omega_i]_{i=1,\dots,N}$ with $\omega_i \in [0, \infty[$ and $\Delta(L) = diag\{d(L)\}$with $d(L) = [(1 - L)^{d_i}]_{i=1,\dots,N}$ $\forall\, 0 \le d_i \le 1$. Finally, $\Gamma_t = diag\{\gamma \odot s_t\}$ with $\gamma = [\gamma_i]_{i=1,\dots,N}$ and $s_t = [s_{it}]_{i=1,\dots,N}$ where $s_{it} = 1$ if $\varepsilon_{it} < 0$ and 0 otherwise.

In the second stage, we estimate the conditional correlation using the transformed crude oil and henryhub return residuals, which are estimated by their standard deviations from the first stage. The multivariate conditional variance is specified as follows:

$$H_t = D_t R_t D_t \qquad \text{…(9)}$$

where$D_t = diag\left(h_{11t}^{1/2}, \dots, h_{NNt}^{1/2}\right)$ denotes the conditional variance derived from the univariate AR(1)-FIAPARCH(1,d,1) model and $R_t = (1 - \theta_1 - \theta_2)R + \theta_1 \psi_{t-1} + \theta_2 R_{t-1}$ is the conditional correlation matrix[1]. In addition, θ_1 and θ_2 are the non-negative parameters satisfying $(\theta_1 + \theta_2) < 1$, $R = \{\rho_{ij}\}$ is a time-invariant symmetric $N \times N$ positive definite parameter matrix with $\rho_{ii} = 1$ and ψ_{t-1} is the $N \times N$ correlation matrix of ε_τ for $\tau = t - M, t - M + 1, \dots, t - 1$. The $i, j - th$ element of the matrix ψ_{t-1} is given as follows:

$$\psi_{ij,t-1} = \frac{\sum_{m=1}^{M} z_{i,t-m} z_{j,t-m}}{\sqrt{\left(\sum_{m=1}^{M} z_{i,t-m}^2\right)\left(\sum_{m=1}^{M} z_{j,t-m}^2\right)}}, \qquad 1 \le i \le j \le N \quad \text{….(10)}$$

where $z_{it} = \varepsilon_{it}/\sqrt{h_{iit}}$ is the transformed henry hub and oil return residuals by their estimated standard deviations taken from the univariate AR(1)-FIAPARCH(1,d,1) model.

The matrix ψ_{t-1} could be expressed as follows:

[1] Engle (2002) derives a different form of DCC model. The evolution of the correlation in DCC is given by: $Q_t = (1 - \alpha - \beta)\bar{Q} + \alpha z_{t-1} + \beta Q_{t-1}$, where $Q = (q_{ijt})$ is the $N \times N$ time-varying covariance matrix of z_t, $\bar{Q} = E[z_t z_t']$ denotes the $n \times n$ unconditional variance matrix of z_t, while α and β are nonnegative parameters satisfying $(\alpha + \beta) < 1$. Since Q_t does not generally have units on the diagonal, the conditional correlation matrix R_t is derived by scaling Q_t as follows: $R_t = (diag(Q_t))^{-1/2} Q_t (diag(Q_t))^{-1/2}$.

$$\psi_{t-1} = B_{t-1}^{-1} L_{t-1} L_{t-1}^{'} B_{t-1}^{-1} \qquad \ldots(11)$$

where B_{t-1} is a $N \times N$ diagonal matrix with $i-th$ diagonal element given by $\left(\sum_{m=1}^{M} z_{i,t-m}^{2}\right)$ and $L_{t-1} = (z_{t-1}, \ldots, z_{t-M})$ is a $N \times N$ matrix, with $z_t = (z_{1t}, \ldots, z_{Nt})'$.

To ensure the positivity of ψ_{t-1} and therefore of R_t, a necessary condition is that $M \leq N$.Then, R_t itself is a correlation matrix if $R_{t-1}$is also a correlation matrix. The correlation coefficient in a bivariate case is given as:

$$\rho_{12,t} = (1-\theta_1-\theta_2)\rho_{12} + \theta_2\rho_{12,t} + \theta_1 \frac{\sum_{m=1}^{M} z_{1,t-m} z_{2,t-m}}{\sqrt{\left(\sum_{m=1}^{M} z_{1,t-m}^{2}\right)\left(\sum_{m=1}^{M} z_{2,t-m}^{2}\right)}} \qquad \ldots(12)$$

Data and preliminary analysis

The data comprises daily crude oil prices (WTI and BRENT) and natural gas (Henryhub). All data are sourced from the (http//www.eia.com). The sample covers a period from January 01, 2004 until February 26, 2015, leading to a sample size of 4075 observations. For each series, the continuously compounded return is computed as $r_t = 100 \times \ln(p_t/p_{t-1})$for $t = 1,2, \ldots, T$, where p_t is the price on day t. The chosen period permits to analyse the sensitivity of crude oil returns and natural gas return. Summary statistics for crude oil and natural gas returns are displayed in Table 1 (Panel A). From these tables, Henryhub is the most volatile, as measured by the standard deviation of 4.1855%, while BRENT is the least volatile with a standard deviation of 2.1019%. Besides, we observe that Henryhub has the highest level of excess kurtosis, indicating that extreme changes tend to occur more frequently for the natural gas price. In addition, all crude oil and gas returns exhibit high values of excess kurtosis. To accommodate the existence of “fat tails”, we assume student-t distributed innovations. Furthermore, the Jarque-Bera statistic rejects normality at the 1% level for all crude oil and gas. Moreover, all return series are stationary, I(0), and thus suitable for long memory tests. Finally, they exhibit volatility clustering, revealing the presence of heteroskedasticity and strong ARCH effects. In order to detect long-memory process in the data, we use the log-periodogram regression (GPH) test of Geweke and Porter-Hudak (1983) on two proxies of volatility, namely squared returns and absolute returns. The test results are displayed in Table 1 (Panel D). Based on these tests’ results, we reject the null hypothesis of no long-memory for absolute and squared returns at 1% significance level. Subsequently, all volatilities proxies seem to be governed by a fractionally integrated process. Thus, FIAPARCH seem to be an appropriate specification to capture volatility clustering, long-range memory characteristics and asymmetry.

Fig. 1 illustrates the evolution of oil prices (WTI, BRENT) and natural gas (raw series and returns) during the period from January 1, 2004 until February 26, 2015. the observed pattern of crude oil and natural gas prices tend to support this theory. However, there have been periods in which natural gas and crude oil prices have appeared to move independently of each other. The figure shows significant variations in the levels during the turmoil, especially at the time of Lehman Brothers failure (September 15, 2008). Specifically, when the global financial crisis triggered, there was a decline for all prices. Moreover, **Fig. 1** plots the evolution of natural gas returns and oil returns over time. The figure shows that all natural gas and crude oil trembled since 2008 with different intensity during the global financial and European sovereign debt crises. Moreover, the plot shows a clustering of larger return volatility around and after 2008.

Table 1. Descriptive statistics

	WTI	BRENT	HENRYHUB
Panel A: descriptive statistics			
Mean	2.07E-02	0.0248	-0.0234
Maximum	16.414	18.13	39.007
Minimum	-12.827	-16.832	-27.844
Std. Deviation	2.3366	2.1019	4.1855
Skewness	-0.0100	0.0706	0.6734***
	-0.8244	-0.1192	0.0000
ExcessKurtosis	4.9438***	5.9262***	12.179***
	0.0000	0.0000	0.0000
Jarque-Bera	2964.6***	4262.2***	18212***
	0.0000	0.0000	0.0000
Panel B: Serial correlation and LM-ARCH tests			
$LB(20)$	69.4576***	46.3902***	156.296***
	0.0000	-0.0007	0.0000
$LB^2(20)$	2833.19***	100.985***	1455.83***
	0.0000	0.0000	0.0000
ARCH 1-10	61.576***	23.711***	50.185***
	0.0000	0.0000	0.0000
Panel C: Unit Root tests			
ADF test statistic	-30.0369*	-30.9284*	-35.7354*
	(-1.9409)	(-1.9409)	(-1.9409)
Panel D: long memory tests (GPH test- d estimates			
Squared returns			
$m = T^{0.5}$	0.4152	0.4991	0.1943
	[0.0996]	[0.0888]	[0.0674]
$m = T^{0.6}$	0.477	0.4788	0.3559
	[0.0631]	[0.0804]	[0.0484]
Absolute returns			
	0.4713	0.4434	0.3607
$m = T^{0.5}$			
	[0.0910]	[0.0803]	[0.0763]
$m = T^{0.6}$	0.4478	0.3763	0.446
	[0.0556]	[0.0562]	[0.0526]

Notes: Crude oil and natural gas returns are in daily frequency. $r^2$and$|r|$ are squared log return and absolute log return, respectively. $m$denotes the bandwith for the Geweke and Porter-Hudak’s (1983) test. Observations for all series in the whole sample period are 4075. The numbers in brackets are t-statistics and numbers in parentheses are p-values. ***, **, and * denote statistical significance at 1%, 5% and 10% levels, respectively. $LB(20)$and$LB^2(20)$ are the 20th order Ljung-Box tests for serial correlation in the standardized and squared standardized residuals, respectively.

This means that markets are characterized by volatility clustering, i.e., large (small) volatility tends to be followed by large (small) volatility, revealing the presence of heteroskedasticity. This market phenomenon has been widely recognized and successfully captured by ARCH/GARCH family models to adequately describe natural gas and crude oil returns dynamics. This is important because the econometric model will be based on the interdependence of the markets in the form of second moments by modeling the time varying variance-covariance matrix for the sample.

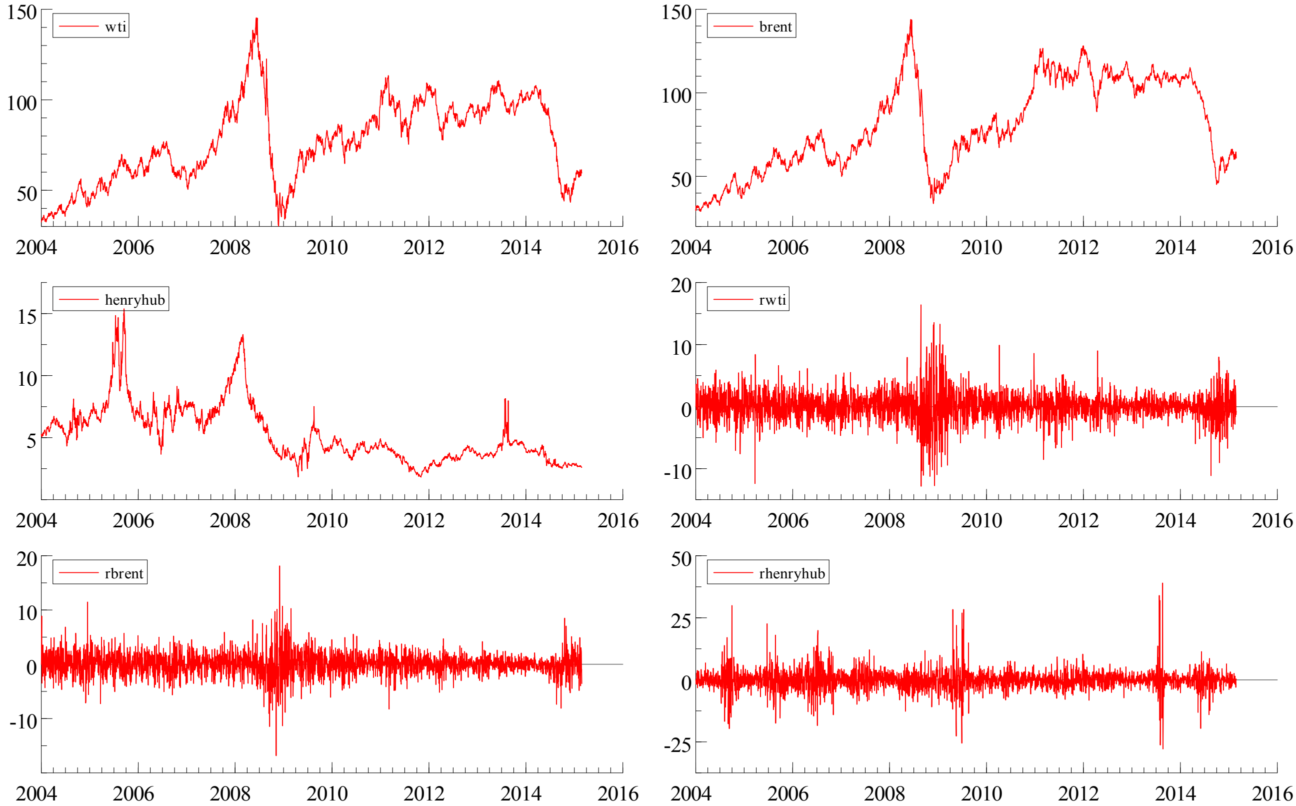

Fig. 1. Oil prices (WTI, BRENT) and natural gas (Henryhub) behavior over time

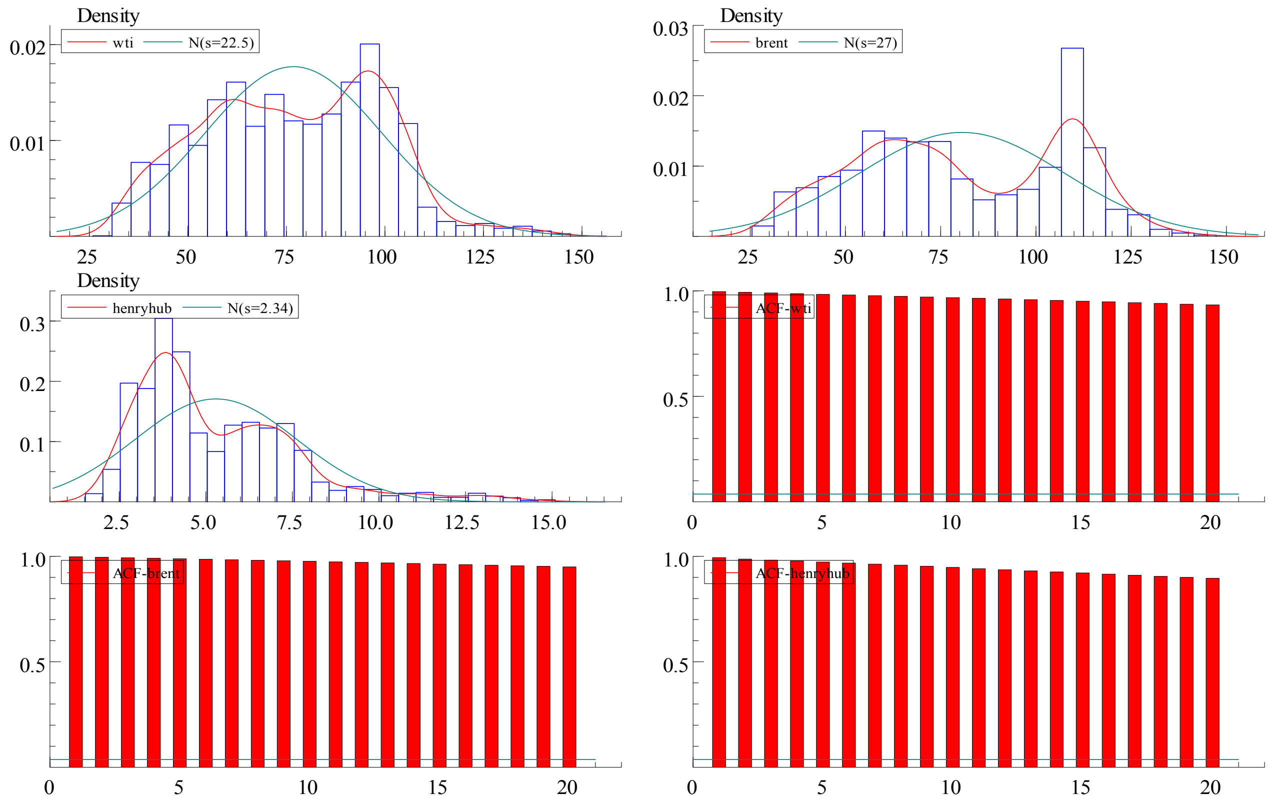

Figure 2. Henry Hub, West Texas Intermediate and BRENT Prices Histogram and Autocorrelogram Functions

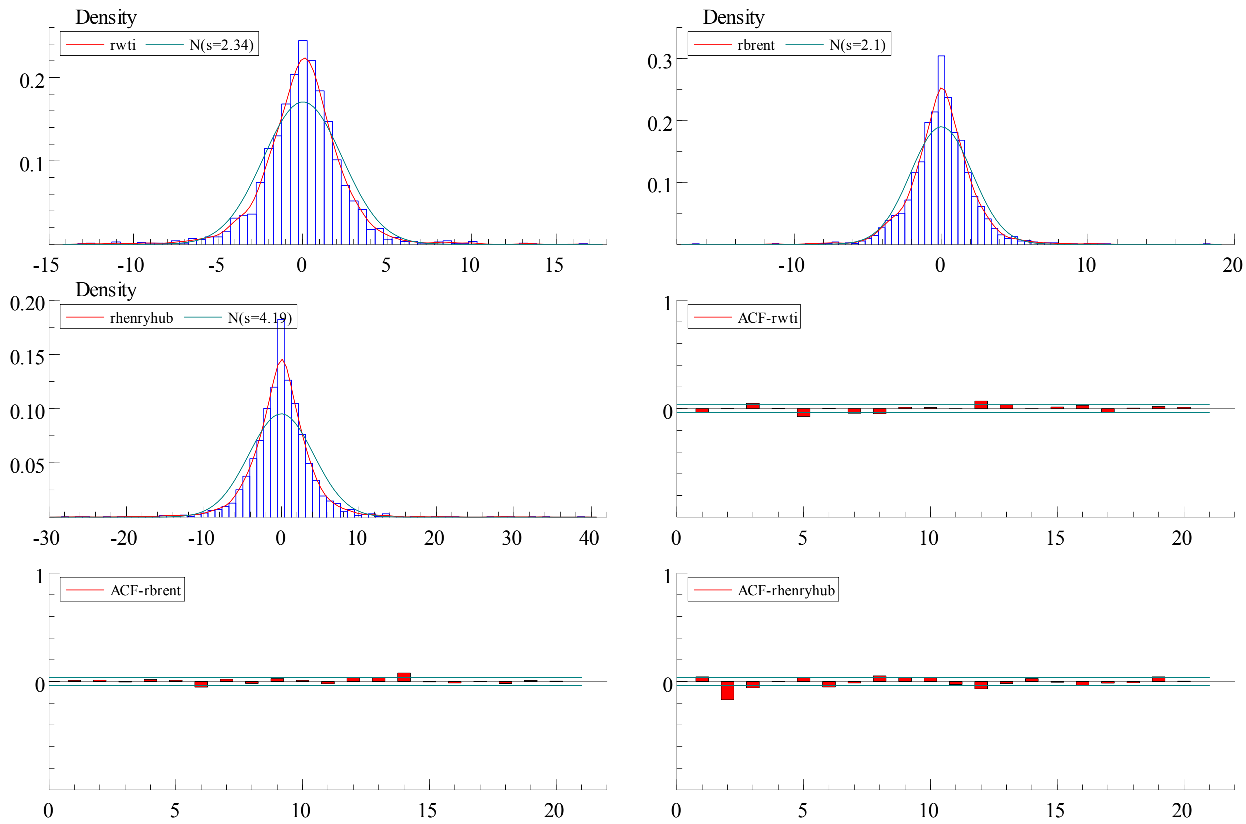

Figure 3. Returns of Henry Hub, West Texas Intermediate and BRENT Prices Histogram and Autocorrelogram Functions

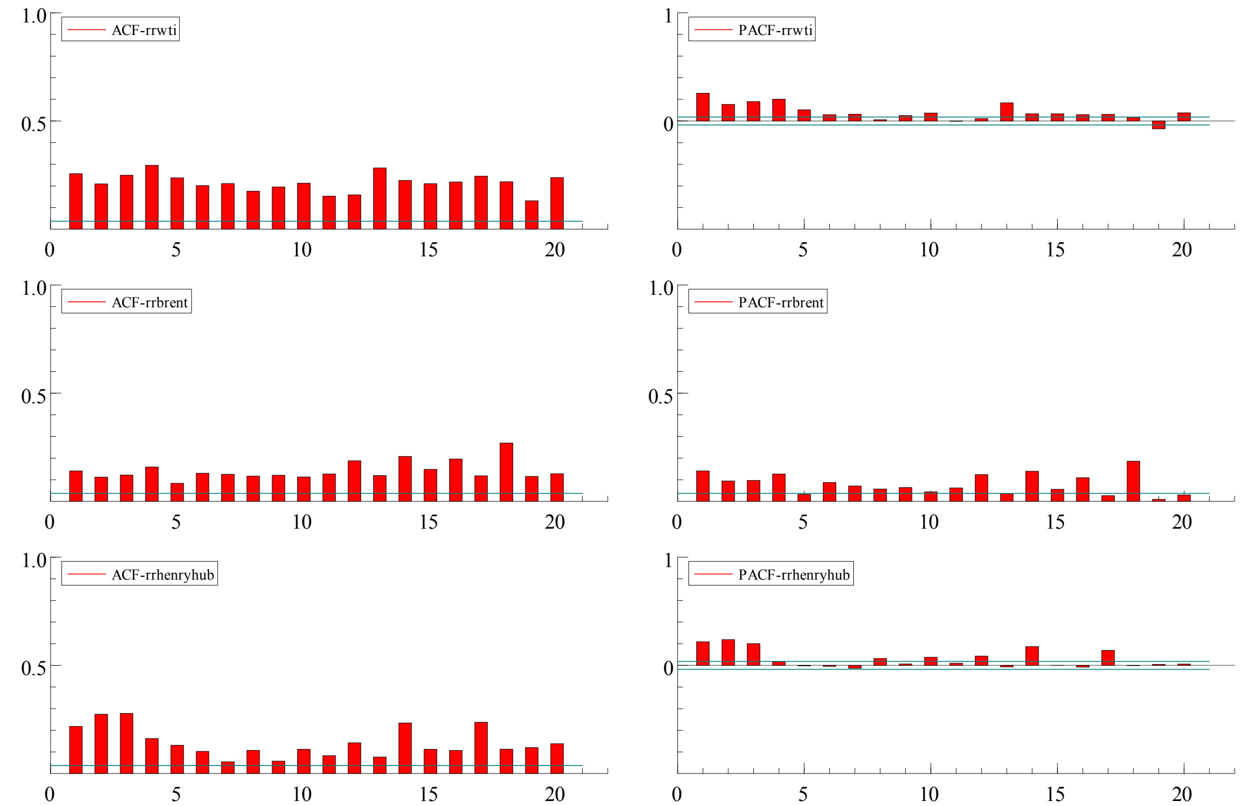

Figure 4. ACF and PACF functions of squared return series

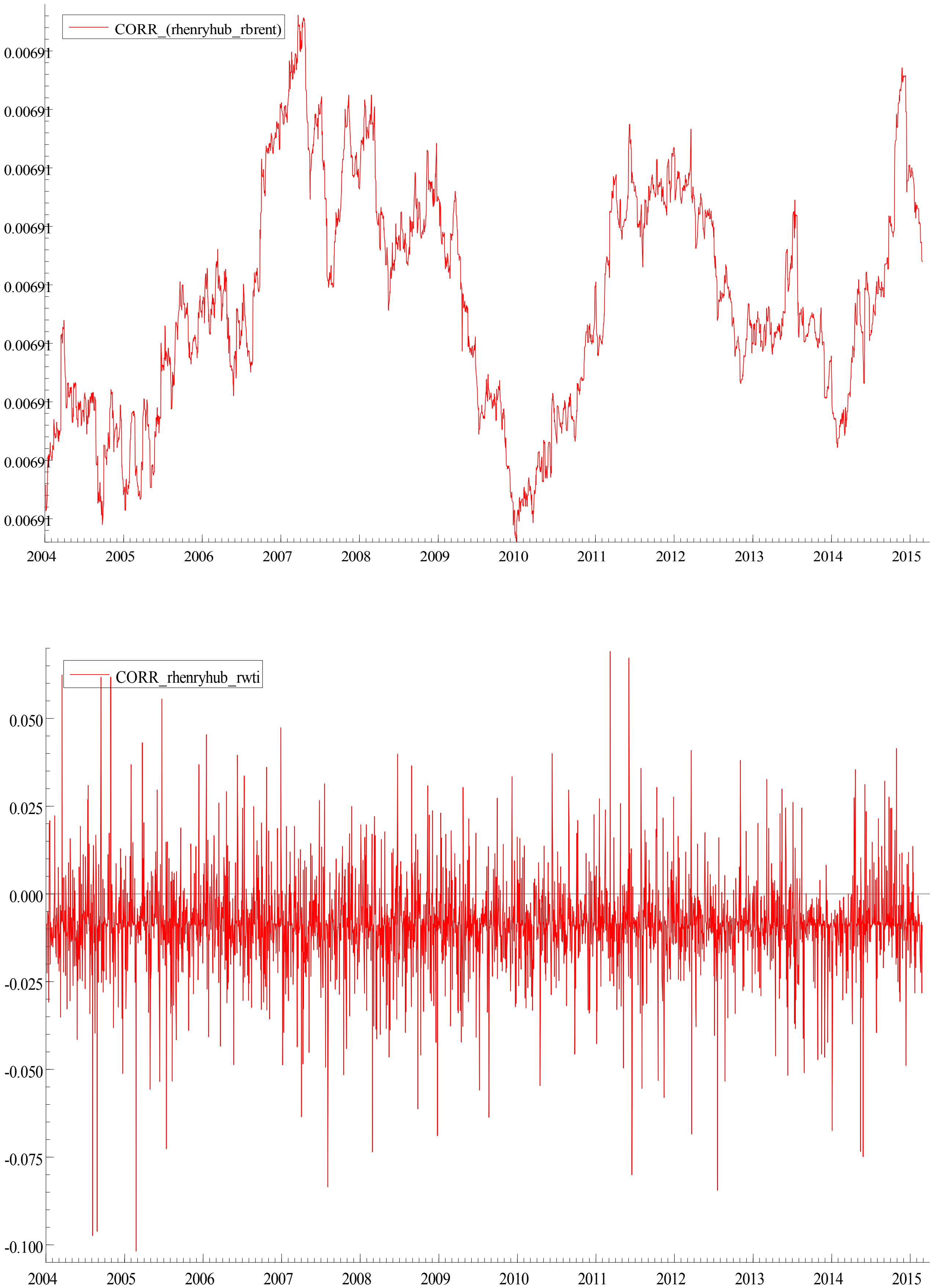

Figure 5. The DCC behavior over time

Table 2. Univariate FIAPARCH(1,d,1) models (MLE)

	WTI		BRENT		HENRYHUB	
	Coefficient	t-prob	Coefficient	t-prob	Coefficient	t-prob
Estimate						
c	0.0475	0.1368	0.0254	0.3947	-0.0480	0.3360
ω	0.0672*	0.063	-0.0211	0.6440	0.1031***	0.0016
d	0.4760***	0.0000	0.3234***	0.0001	0.4158***	0.0000
ϕ	0.3891***	0.0000	0.3504***	0.0000	-0.0700	0.1582
β	0.7463***	0.0000	0.6323***	0.0000	0.8958***	0.0000
γ	0.3590***	0.0036	0.3347**	0.0154	0.3318**	0.0745
δ	1.6992***	0.0000	2.0874***	0.0000	1.1486***	0.0000
v	8.6049***	0.0000	8.0966***	0.0000	5.9632***	0.0000
Diagnostics						
$LB(20)$	14.8132	0.7869	20.4115	0.4324	69.0225***	0.0000
$LB^2(20)$	16.4316	0.5624	23.852	0.1599	47.2973***	0.0001

Notes: For each of the five exchange rates, Table 2 reports the Maximum Likelihood Estimates (MLE) for the student-t-FIAPARCH(1,d,1) model. $LB(20)$and$LB^2(20)$ indicate the Ljung-Box tests for serial correlation in the standardized and squared standardized residuals, respectively. $v$denotes the the t-student degrees of freedom.parameter ***, ** and * denote statistical significance at 1%, 5% and 10% levels, respectively.

Table 3. Estimation results from the bivariate FIAPARCH(1,d,1)-DCC model

	HENRYHUB-WTI		HENRYHUB-BRENT	
	coefficient	t-prob	coefficient	t-prob
Panel A: Estimates of Multivariate DCC				
a	0.4305**	0.0517	0.0054	0.7594
b	0.5100***	0.0099	0.9038***	0.0000
v	8.8107***	0.0000	8.8394***	0.0000
Panel B : Diagnostic tests				
$Hosking(20)$	106.116**	0.0226	85.7250***	0.0000
$Hosking^2(20)$	92.2487	0.1291	46.1517	0.1708
$Li-McLeod(20)$	106.106**	0.0226	85.6810***	0.0000
$Li-McLeod^2(20)$	92.1857	0.1300	46.1393	0.1711

Notes: The superscripts ***, ** and * denote the statistical significance at 1%, 5% and 10% levels, respectively.v indicates the student's distribution's degrees of freedom. $Hosking\ (20)$and$Hosking^2(20)$ denote the Hosking's Multivariate Portmanteau Statistics on both standardized and squared standardized Residuals. $Li-McLeod\ (20)$and$Li-McLeod^2(20)$ indicate the Li and McLeod's Multivariate Portmanteau Statistics on both Standardized and squared standardized Residuals.

The autocorrelation functions (ACFs) and probability distributions for the series are presented in Figure 2. Histograms summarize the frequency of occurrence that a variable falls within a certain range of values. Histograms provide insights into the standard error, the probability distribution and the mean of a given variable. Histograms of Henry Hub, WTI and BRENT prices are presented in the top panels of Figure 2, with a plot of a normal distribution superimposed over the histogram and a smooth line generated from the histogram. Figure 3 contains histograms and ACFs for the return of both price series. In each case, the differencing transformation appears to have resolved the non stationarities in the three series for the most part. In both cases, the probability distributions of the differenced series appear to be approximately normally distributed. The autocorrelations are statistically insignificant at most lags. These results support the concept that the levels of the price series are integrated processes of order one, I(1) and unit root. In order to examine whether the natural gas returns and oil (RWTI, RBRENT) exhibit a long memory (persistence), autocorrelation and partial autocorrelation functions of the squared returns are plotted in Figure 4. As can be seen in Figure 4, squared values of the returns are positive an significant up to 20 lags. They also exhibit a very slow decay with a hyperbolic rate, implying the volatility of gas returns and crude oil have a long memory.

Empirical results

The univariate FIAPARCH estimates

In order to take into account the serial correlation and the GARCH effects observed in our time series data, and to detect the potential long range dependence in volatility, we estimate the student[2]-t-AR(0)-FIAPARCH(1,d,1)[3] model defined by Eqs. (1) and (5). Table 2 reports the estimation results of the

[2] The z_t random variable is assumed to follow a student distribution (see Bollerslev, 1987) with $v > 2$ degrees of freedom and with a density given by:
$D(z_t, v) = \frac{\Gamma(v+\frac{1}{2})}{\Gamma(\frac{v}{2})\sqrt{\pi(v-2)}}(1+\frac{z_t^2}{v-2})^{\frac{1}{2}-v}$
where$\Gamma(v)$ is the gamma function and v is the parameter that describes the thickness of the distribution tails. The Student distribution is symmetric around zero and, for $v > 4$, the conditional kurtosis equals $3(v-2)/(v-4)$, which exceeds the normal value of three. For large values of v, its density converges to that of the standard normal.
For a Student-*t* distribution, the log-likelihood is given as: $L_{Student} = T\left\{log\Gamma\left(\frac{v+1}{2}\right) - log\Gamma\left(\frac{v}{2}\right) - \frac{1}{2}log[\pi(v-2)]\right\} - \frac{1}{2}\sum_{t=1}^{T}\left[\log(h_t) + (1+v)log\left(1+\frac{z_t^2}{v-2}\right)\right]$
where$T$ is the number of observations, v is the degrees of freedom, $2 < v \leq \infty$ and $\Gamma(.)$ is the gamma function.
[3] The lag orders$(1, d, 1)$and (0,0) for FIAPARCH and ARMA models, respectively, are selected by Akaike (AIC) and Schwarz (SIC) information criteria. The results are available from the author upon request.

univariate FIAPARCH(1,d,1) model for each gas and oil return series of our sample. The estimates of the constants in the mean are statistically no significant at 1% level or better for all the series. Besides, the constants in the variance are significant except for the BRENT. In addition, for all Crude oil and gas prices, the estimates of the leverage term (γ) are statistically significant, indicating an asymmetric response of volatilities to positive and negative shocks. This finding confirms the assumption that there is negative correlation between returns and volatility. Moreover, the estimates of the power term (δ) are highly significant for oil and gas prices and ranging from 1.1486 to 2.0874. Conrad *et al.* (2011) show that when the series are very likely to follow a non-normal error distribution, the superiority of a squared term (δ = 2) is lost and other power transformations may be more appropriate. Thus, these estimates support the selection of FIAPARCH model for modeling conditional variance of oil and gas returns. Besides, all crude oil and gas display highly significant differencing fractional parameters(d), indicating a high degree of persistence behavior. This impliesthat the impact of shocks on the conditional volatility of returns consistently exhibits a hyperbolic rate of decay. Interestingly, the highest power term is obtained for WTI, one is characterized by the highest degree of persistence. In all cases, the estimated degrees of freedom parameter (v) is highly significant and leads to an estimate of the Kurtosis which is equal to $3(v-2)/(v-4)$ and is also different from three.

In addition, all the ARCH parameters (ϕ) satisfy the set of conditions which guarantee the positivity of the conditional variance. Moreover, according to the values of the Ljung-Box tests for serial correlation in the standardized and squared standardized residuals, there is no statistically significant evidence, at the 1% level, of misspecification in almost all cases except for the natural gas price (HENRYHUB). Numerous studies have documented the persistence of volatility in stock and exchange rate returns (see Ding *et al.*, 1993; Ding et Granger, 1996, among others).The majority of these studies have shown that the volatility process is well approximated by an IGARCH process. Nevertheless, from the FIAPARCH estimates reported in Table 2, it appears that the long-run dynamics are better modeled by the fractional differencing parameter.

To test for the persistence of the conditional heteroskedasticity models, we examine the Likelihood Ratio (LR) statistics for the linear constraints $d = 0$(APARCH(1,1) model) and $d \neq 0$ (FIAPARCH(1,d,1) model). We construct a series of LR tests in which the restricted case is the APARCH(1,1) model ($d = 0$) of Ding *et al.* (1993). Let l_0 be the log-likelihood value under the null hypothesis that the true model is APARCH(1,1) and l the log-likelihood value under the alternative that the true model is FIAPARCH(1,d,1). Then, the LR test,$2(l - l_0)$, has a chi-squared distribution with 1 degree of freedom when the null hypothesis is true. For reasons of brevity, we omit the table with the test results, which are available from the author upon request. In summary, the LR tests provide a clear rejection of the APARCH(1,1) model against the FIAPARCH(1,d,1) one for natural gas and oil prices. Thus, purely from the perspective of searching for a model that best describes the volatility in the series, the FIAPARCH(1,d,1) model appears to be the most satisfactory representation. This finding is important since the time series behavior of volatility could affect asset prices through the risk premium (see Christensen and Nielsen, 2007; Christensen *et al.*, 2010; Conrad *et al.*, 2011). With the aim of checking for the robustness of the LR testing results discussed above, we apply the Akaike (AIC), Schwarz (SIC), Shibata (SHIC) or Hannan-Quinn (HQIC) information criteria to rank the ARCH type models. According to these criteria, the optimal specification (i.e., APARCH or FIAPARCH) for all crude oil and gas prices is the FIAPARCH one. The two common values of the power term (δ) imposed throughout much of the GARCH literature are $\delta = 2$ (Bollerslev's model) and $\delta = 1$ (the Taylor/Schwert specification). According to Brooks *et al.* (2000), the invalid imposition of a particular value for the power term may lead to sub-optimal modeling and forecasting performance. For that reason, we test whether the estimated power terms are significantly different from unity or two using Wald tests (results not reported).

The bivariate FIAPARCH(1,d,1)-DCC estimates

The analysis above suggests that the FIAPARCH specification describes the conditional variances of the oil and natural gas prices well. Therefore, the multivariate FIAPARCH model seems to be essential for enhancing our understanding of the relationships between the (co)volatilities of economic and financial time series. In this section, within the framework of the multivariate DCC model, we analyze the dynamic adjustments of the variances for the oil and gas prices. Overall, we estimate two bivariate specifications for our analysis. Table 3(Panels A and B) reports the estimation results of the bivariate student-t-FIAPARCH(1,d,1)-DCC model. The ARCH and GARCH parameters (a and b) of the DCC(1,1) model capture,respectively, the effects of standardized lagged shocks and the lagged dynamic conditional correlations effects on current dynamic conditional correlation. They are statistically significant at the 5% level, except for the AR parameter between (HENRYHUB-BRENT), indicating the existence of time-varying correlations. Moreover, they are non-negative, justifying the appropriateness of the FIAPARCH model. When $a = 0$and $b = 0$, we obtain the Bollerslev's (1990) Constant Conditional Correlation (CCC) model. As shown in Table 3, the estimated coefficients a and b are significantlypositive and satisfy the inequality $a + b < 1$ in each of the pairs of gas and oil prices. Besides, the t-student degrees of freedom parameter (v)is highly significant, supporting the choice of this distribution. The statistical significance of the DCC parameters (a and b) reveals a considerable time-varying comovement and thus a high persistence of the conditional correlation. The sum of these parameters is close to unity. This implies that the volatility displays a highly persistent fashion. Since $a + b < 1$, the dynamic correlations revolve around a constant level and the dynamic process appears to be mean reverting. The multivariate FIAPARCH-DCC model is so important to consider in our analysis since it has some key advantages. First, it captures the long range dependence property. Second, it allows obtaining all possible pair-wise conditional correlation coefficients for the gas and oil returns in the sample. Third, it's possible to investigate their behavior during periods of particular interest, such as periods of the global financial and European sovereign debt crises. Fourth, the model allows looking at possible financial contagion effects between international foreign exchange markets. Finally, it is crucial to check whether the selected crude oil and gas price series display evidence of multivariate Long Memory ARCH effects and to test ability of the Multivariate FIAPARCH specification to capture the volatility linkages among market prices. Kroner and Ng (1998) have confirmed the fact that only few diagnostic tests are kept to the multivariate GARCH-class models compared to the diverse diagnostic tests devoted to univariate counterparts. Furthermore, Bauwens *et al*. (2006) have noted that the existing literature on multivariate diagnostics is sparse compared to the univariate case. In our study, we refer to the most broadly used diagnostic tests, namely the Hosking's and Li and McLeod's Multivariate Portmanteau statistics on both standardized and squared standardized residuals. According to Hosking (1980), Li and McLeod (1981) and McLeod and Li (1983) autocorrelation test results reported in Table 3 (Panel B), the multivariate diagnostic tests allow accepting the null hypothesis of no serial correlation on squared standardized residuals and thus there is no evidence of statistical misspecification.

Fig. 5 illustrates the evolution of the estimated dynamic conditional correlations dynamics among oil and natural gas prices. The different path of the estimated DCCs displays fluctuations for all pairs of natural gas and oil prices across the phases of the global financial and European sovereign debt crises, suggesting that the assumption of constant correlation is not appropriate. The above findings motivate a more extensive analysis of DCCs, in order to capture contagion dynamics during different phases of the two crises.

Conclusion

This study examines the dynamic correlations among oil prices namely WTI and BRENT and natural gas price (HENRYHUB). Specifically, we employ a multivariate FIAPARCH-DCC model, during the period from January 01, 2004 to February 26, 2015, focusing on the estimated dynamic conditional correlations among the oil and gas markets. This approach allows investigating the second order moments dynamics of crude oil and gas prices taking into account long range dependence behavior, asymmetries and leverage effects.
The FIAPARCH model is identified as the best specification for modeling the conditional heteroscedasticity of individualtime series. We then extended the above univariate GARCH models to a bivariate framework with dynamic conditional correlation parameterization in order to investigate the interaction between oil and gas prices. Our results document strong evidence of time-varying comovement, a high persistence of the conditional correlation (the volatility displays a highly persistent fashion) and the dynamic correlations revolve around a constant level and the dynamic process appears to be mean reverting. More interestingly, the univariate FIAPARCH models are particularly useful in forecasting market risk exposure for synthetic portfolios of stocks and currencies. Our out-of-sample analysis confirms the superiority of the univariate FIAPARCH model and the bivariate DCC-FIAPARCH model over the competing specifications in almost all cases. Economic theory suggests that there is a relation between oil prices and natural gas, because the influence of an increase in oil prices may conflict in its effects on natural gas supply, and therefore, prices. Production of natural gas may increase as a co-product of oil, or may decrease as a result of higher-cost productive resources. While the net effect of an increase in oil prices on natural gas supply may be ambiguous, the effect on natural gas demand is clear, resulting in a positive relation between oil and natural gas prices.

REFERENCES

Alexander, C., 2004. Correlation in crude oil and natural gas markets, Managing Energy Price Risk: The New Challenges and Solutions, 3rd edition. Risk Books, London, pp. 573–604 (Chapter 17).

Baillie, R.T., Bollerslev, T., and Mikkelsen, H.O. 1996. Fractionally integrated generalized autoregressive conditional heteroskedasticity. *Journal of Econometrics*, 74, 3-30.

Barcella, M. L. 1999. The pricing of gas. Oxford Energy Forum. Oxford Institute for Energy Studies, Oxford. May.

Bauwens, L., Laurent, S., and Rombouts, J.V.K. (2006). Multivariate GARCH: a survey. *Journal of Applied Econometrics*, 21, 79-109.

Benth, F.E., Kettler, P.C., 2011. Dynamic copula models for the spark spread. Quant. Finan. 11, 407–421.

Bollerslev, T. 1986. Generalized autoregressive conditional heteroskedasticity *Journal of Econometrics*, 31, 307-327.

Bollerslev, T. 1987. A conditionally heteroskedastic time series model for speculative prices and rates of return. *Review of Economics and Statistics*, 69, 542-547.

Bollerslev, T., and Mikkelsen, H.O. 1996. Modeling and pricing long memory in stock market volatility. *Journal of Econometric,* 73, 151-184.

Celic, S. 2012. The more contagion effect on emerging markets: The evidence of DCC-GARCH model. *Economic Modelling*., 29, 1946-1959.

Chiang, T.C., Jeon, B.N., and Li, H. 2007. Dynamic Correlation Analysis of Financial Contagion: Evidence from Asian Markets. *Journal of International Money and Finance,* 26, 1206-1228.

Chkili, W., Aloui, C., and Ngugen, D. K. 2012. Asymmetric effects and long memory in dynamic volatility relationships between stock returns and exchange rates. *Journal of International Markets, Institutions and Money,* 22, 738-757.

Christensen, B.J., and Nielsen, M.Ø. 2007. The effect of long memory in volatility on stock market fluctuations. *Review of Economics and Statistics*, 89, 684-700.

Christensen, B.J., Nielsen, M.Ø. and Zhu, J. 2010. Long memory in stock market volatility and the volatility-in-mean effect: the FIEGARCH-M model. *Journal of Empirical Finance,* 17, 460-470.

Cointegration Approach." Economics Dept, Loughborough University. Discussion Paper Series: November 2004.

Conrad, C. 2010. Non-negativity conditions for the hyperbolic GARCH model. *Journal of Econometrics*, 157, 441-457.

Conrad, C., and Haag, B. 2006. Inequality constraints in the fractionally integrated GARCH model. *Journal of Financial Econometrics,* 3, 413-449.

Conrad, C., Karanasos, M., and Zeng, N. 2011. Multivariate fractionally integrated APARCH modeling of stock market volatility: A multi-country study. *Journal of Empirical Finance*, 18, 147-159.

Costello, Ken, H.G.Huntington, and J.F. Wilson 2005. “After the Natural Gas Bubble: An Economic Evaluation of the Recent U.S. National Petroleum Council Study.” *Energy Journal*, Vol. 26, No.2, pp 89-110.

Dimitriou, D., and Kenourgios, D. 2013. Financial crises and dynamic linkages among international currencies. *Journal of International Financial Markets, Institutions and Money,* 26, 319-332.

Dimitriou, D., Kenourgios, D., and Simos, T. 2013. Global financial crisis and emerging stock market contagion: a multivariate FIAPARCH-DCC approach. *International Review of Financial Analysis*, 30, 46–56.

Ding, Z., and Granger, C.W.J. (1996).Modeling volatility persistence of speculative returns: a new approach. *Journal of Econometrics,* 73, 185-215.

Ding, Z., Granger, C.W.J., and Engle, R.F. 1993. A long memory property of stock market returns and a new model. *Journal of Empirical Finance*, 1, 83-106.

Engle, R.F. 2002. Dynamic conditional correlation: a simple class of multivariate generalized autoregressive conditional heteroskedasticity models. *Journal of Business and Economic Statistics,* 20 (3), 339-350.

Erdős, P. 2012, August 5. Have oil and gas prices got separated. Energy Policy, pp. 707-718.

Geweke, J. and Porter-Hudak, S. 1983. The estimation and application of long-memory time series models. *Journal of Time Series Analysis*, 4, 221–238.

Granger, C.W.J. 1969. Investigating causal relations by econometric models and cross spectral methods. *Econometrica*., 37, 424-438.

Harris, R. & Sollis, R. 2003. Applied time series modelling and forecasting. England: John Wiley and SonsLtd.

Hosking, J.R.M. 1980. The multivariate portmanteau statistic. *Journal of American Statistical Association,* 75, 602-608.

Jabir, Imad. 2006. “Examining the Long-Run Relation Among Spot Prices of Crude oil and Natural Gas.” Working paper. IAEE International Conference, Potsdam, Germany: June 2006.

Kenourgios, D., Samitas, A., and Paltalidis, N. 2011. Financial crises and stock market contagion in a multivariate time-varying asymmetric framework. International Financial Markets, Institutions and Money 21, 92-106.

Kroner, K.F. and Ng, V.K. 1998. Modeling Asymmetric Comovements of Asset Returns. *The Review of Financial Studies*, 11(4), 817-844.

Li, W.K., and McLeod, A.I. 1981. Distribution of the residual autocorrelations in multivariate ARMA time series models. Journal of the Royal Statistical Society, series B (Methodological) 43(2), 231-239.

Loungani, P. and A. Matsumoto, 2012. Oil and Natural Gas Prices: Together Again. Working Paper, International Monetary Fund (http://www.usaee.org/usaee2012/ submissions/OnlineProceedings/ USAEE_PP.pdf).

Mandelbrot, B. 1963. The variation of certain speculative prices. *Journal of Business*, 36(4), 394–419. http://dx.doi.org/10.1086/294632.

McLeod, A.I., and Li, W.K. 1983. Diagnostic checking ARMA time series models using squared residual autocorrelations. *Journal of Time Series Analysis,* 4, 269-273.

Panagiotidis, T., and E. Rutledge, 2004. "Oil and Gas Market in the UK: Evidence From a Cointegration Approach." Economics Dept, Loughborough University. *Discussion Paper Series*: November, 2004.

Ramberg, D.J. and J.E. Parsons, 2012. “The weak tie between natural gas and oil prices” *The Energy Journal,* 33(2): 13-35.

Schwert, W. 1990. Stock volatility and the crash of '87. *The Review of Financial Studies*, 3, 77-102.

Taylor, S. 1986. Modeling Financial Time Series. Wiley, New York.

Tse, Y.K. 1998. The conditional heteroscedasticity of the Yen-Dollar exchange rate. *Journal of Applied Econometrics*, 193, 49-55.

Tse, Y.K. and Tsui, A.K.C. 2002. A multivariate generalized autoregressive conditional heteroscedasticity model with time-varying correlations. *Journal of Business and Economic Statistics*, 20(3), 351-362.

Villar, J.A. and Joutz, F.R. 2006. The Relationship Between Crude Oil and Natural Gas. Energy Information Administration, Office of Oil and Natural Gas, Washington, D.C.

****