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RESEARCH ARTICLE

INTEGRATED SUPPLY CHAIN MODEL IN DETERIORATING INVENTORY ITEMS AND WASTE REDUCTION CONTEMPLATIONS THROUGH JIT WITH FUZZY APPROACH

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ABSTRACT

In inventory strategy companies employ to increase efficiency and decrease waste by receiving goods only as they are needed in the production process, thereby reducing inventory costs. Normally, the vendor and the buyer problems treated separately. In this paper, we discuss a joint integrated vendor buyer model for both in crisp and in fuzzy sense. This integrated approach is the success of supply chain management to minimize the joint total cost. This proposed model is based on the joint total cost of both the vendor and the buyer. Holding cost of the vendor, holding cost of the buyer, production cost of the vendor and purchasing cost of the buyer are taken as triangular fuzzy numbers. In this model, we reduce the inventory and waste considerations through Just-In-Time. Finally, a numerical example and sensitivity analysis are given to illustrate this model. Graded mean integration representation method is used for defuzzification.

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1. INTRODUCTION

A supply chain management consists of different number of entities (eg. Raw material supplier, manufacturer or producer, purchaser, transporter, distributor, retailer, customer or consumer etc.) who are liable to convert the raw materials into refined products and make them to gratify the consumer's demand in time by minimum cost. Inventory improves the cash flow by timely shipment of consumers order. The vendor must determine the most economical number of shipments to supply a buyer's order quantity. For this reason we use the integrated inventory model which can help businesses to compute the best economic order quantity and shipment policy. Cooperation between the vendor and buyer is vital because it reduces the joint annual inventory total cost and the response time of the vendor-buyer system. Inventory items are categorized as four types. (1) Deterioration items (2) Obsolescence items (3) Damage or Pilferage items (4) No Obsolescence / Deterioration items. Deterioration items refers to the items that become decayed, damaged, evaporative, expired, invalid, devaluation and so on through time or physical characteristics (color, consistency, odor, etc.) of a substance due to faulty packaging or abnormal storage conditions.

Deteriorating items can be classified into two categories. The first category refers to the items that become decayed, damaged, evaporative, or expired through time, like meat, vegetables, fruit, medicine, flowers, film and so on; the other category refers to the items that lose part or total value through time because of new technology or the introduction of alternatives, like computer chips, mobile phones, fashion and seasonal goods, and so on. Obsolescence items are referred to the outdated and antiquated products lose their value because of rapid changes of technology. Damage or Pilferage items are referred to the crime of shoplifting involving employees who steal items from their place of employment, especially in a manufacturing plant. No obsolescence / Deterioration items are referred to the life cycle of some products are indefinite in nature.

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Just – In – Time (JIT) manufacturing eliminates waste, as out of date or expired products. Under this technique, only essential stocks are obtained, less working capital is required to business procurement. Just – In – Time (JIT) manufacturing encourages the 'right first time' concept, so that screening costs and reworking costs is minimized. Just – In – Time (JIT) production mainly focused on the purchasing and manufacturing items would be instantly supplied. A single vendor that supplies products to the single buyer always creating a decision problem.

Goyal [1977] first introduced the idea of a joint total cost for a single vendor and a single buyer assuming an infinite production rate for the vendor and lot policy for the shipments from the vendor to the buyer. Banerjee [1986] developed a joint economic lot size model for a single vendor with finite production rate. Ha and Kim [1997] derived the JIT system between the vendor and the buyer using geometric programming. Yang and Wee [2000] derived an integrated approach on economic order policy of deteriorated item for vendor and buyer. Nagoor Gani and Sabarinathan [2013] made an optimal policy for vendor-buyer integrated model in Fuzzy environment. Miltenburg [2001] investigated that the term JIT could be adopted to signify techniques, improving quality products and reduce costs by eliminating all waste from the production system. Hwang et al. [2001] suggests that the climate of increasingly strict regulations for energy efficiency, material composition, waste reduction and product recycling. In today's economic environment, there is a critical problem to success for the waste reduction and process improvement resourcefulness such as Total Quality Management (TQM), Business Process Reengineering (BPR) and integrated vendor and buyer management. A new struggle is now undergone in corporate strategic planning board rooms involving the globally – cognizant, ecological design of a 'green product'.

Li - Hsing Ho and Wei – Feng Kao [2013] investigates that a supply chain model with inventory and waste reduction considerations. This paper considers a simple and practical situation for deteriorating items which integrates inventory and quality assertion in a JIT supply chain and derives the optimal solution. This model assumes that manufacturing will produce some defective items and those products will not impact the buyer's purchase policy. The vendor absorbs all the inspection cost. This approach deals with a single batch. At the end of the 100% screening process, if defective items are found, duplicate costs must be paid by the vendor.

2. MATERIALS AND METHODS

2.1 Fuzzy Numbers

Any fuzzy subset of the real line R, whose membership function «$_A$ satisfied the following conditions, is a generalized fuzzy number $\tilde{A}$.

«$_A$ is a continuous mapping from R to the closed interval [0, 1].

«$_A$ = 0, $-\infty < x \le a_1$,

«$_A$ = L(x) is strictly increasing on [a1, a2]

«$_A$ = wA, $a_2 \le x \le a_3$

«$_A$ = R(x) is strictly decreasing on [a3, a4]

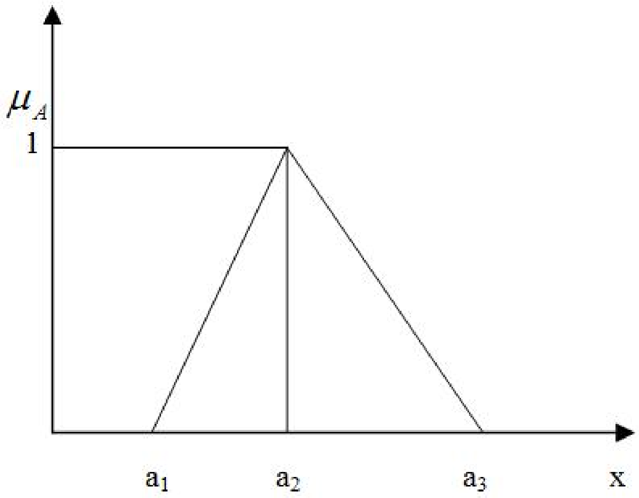
«$_A$ = 0, $a_4 \le x < \infty$

Where $0 < w_A \le 1$ and a1, a2, a3 and a4 are real numbers. Also this type of generalized fuzzy number be denoted as $\tilde{A} = (a_1, a_2, a_3, a_4 : w_A)_{LR}$; When wA = 1, it can be simplified as $\tilde{A} = (a_1, a_2, a_3, a_4)_{LR}$

2.2 Triangular Fuzzy Number

The fuzzy set $\tilde{A} = (a_1, a_2, a_3)$ where a1 < a2 < a3 and defined on R, is called the triangular fuzzy number, if the membership function of $\tilde{A}$ is given by «$_A$

$$
«_A = \begin{cases} \dfrac{x-a_1}{a_2-a_1}, & a_1 \le x \le a_2 \\ \dfrac{a_3-x}{a_3-a_2}, & a_2 \le x \le a_3 \\ 0, & otherwise \end{cases}
$$



2.3 The Function Principle

The function principle is used for the operation of addition, subtraction, multiplication and division of fuzzy numbers. Suppose $\tilde{A} = (a_1, a_2, a_3)$ and $\tilde{B} = (b_1, b_2, b_3)$ are two triangular fuzzy numbers. Then

(i) The addition of $\tilde{A}$ and $\tilde{B}$ is
$\tilde{A} + \tilde{B}$ = (a1 + b1, a2 + b2, a3 + b3) where a1, a2, a3, b1, b2, b3 are any real numbers.

(ii) The subtraction of $\tilde{B}$ if $-\tilde{B}$ = (-b1, - b2, - b3) is
$\tilde{A} - \tilde{B}$ = (a1 – b3, a2 - b2, a3 – b1) where a1, a2, a3, b1, b2, b3 are any real numbers.

(iii) The multiplication of $\tilde{A}$ and $\tilde{B}$ is
$\tilde{A} \times \tilde{B}$ = (c1 , c2 , c3) where T = (a1b1, a1b3 , a3b1, a3b3) c1 = min T, c2 = a2 + b2
c3 = max T. If a1, a2, a3, b1, b2, b3 are all non zero positive real numbers, then
$\tilde{A} \times \tilde{B}$ = (a1b1, a2b2, a3b3).

(iv) $\frac{1}{\tilde{B}} = \tilde{B}^{-1} = \left(\frac{1}{b_3}, \frac{1}{b_2}, \frac{1}{b_1}\right)$ where b1, b2, b3 are all non zero positive real numbers,
then the division of $\tilde{A}$ and $\tilde{B}$ is $\frac{\tilde{A}}{\tilde{B}} = \left(\frac{a_1}{b_3}, \frac{a_2}{b_2}, \frac{a_3}{b_1}\right)$.

(v) For any real number K, K$\tilde{A}$ = (Ka1, Ka2, Ka3) , if K > 0

K$\tilde{A}$ = (Ka3, Ka2, Ka1) , if K < 0

2.4 Graded Mean Integration Representation Method

If $\tilde{A} = (a_1, a_2, a_3)$ is a triangular fuzzy number then the graded mean integration representation method of $\tilde{A}$ is,

$$p(\tilde{A}) = \frac{a_1 + 4a_2 + a_3}{6}$$

2.5 Notations and Assumptions

The mathematical model in this paper is developed on the basis of the following notations and assumptions.

2.5.1 Notations

Q - Order Quantity of the purchaser

R	-	Annual demand rate
P	-	Production rate
n	-	Total number of shipments per lot from the vendor to the buyer
HV	-	Vendor's holding cost per unit per unit time
HB	-	Buyer's holding cost per unit per unit time
SV	-	Production cost paid by the vendor
SB	-	Purchasing cost paid by the purchaser
r	-	Annual inventory holding cost
TCV	-	Total annual cost of the vendor
TCB	-	Total annual cost of the buyer
JTC	-	Joint annual total cost of the vendor and the buyer
~	-	Screening cost per unit
—	-	Reworking cost per unit
	-	Percentage of defective items
$\tilde{H}_V$	-	Vendor's fuzzy holding cost per unit per unit time
$\tilde{H}_B$	-	Buyer's fuzzy holding cost per unit per unit time
$\tilde{S}_V$	-	Fuzzy Production cost paid by the vendor
$\tilde{S}_B$	-	Fuzzy Purchasing cost paid by the purchaser
$T\tilde{C}_V$	-	Fuzzy Total annual cost of the vendor
$T\tilde{C}_B$	-	Fuzzy Total annual cost of the buyer
$J\tilde{T}C$	-	Joint fuzzy integrated total annual cost of the vendor and the buyer

2.5.2 Assumptions

Single vendor and single buyer for a single product are considered.

- Production rate is uniform and finite.
- Demand rate is constant over time.
- Shortages are not allowed.
- Lead time is zero.
- Inventory is continuously reviewed.
- Transportation cost is not considered.
- Sequential conveyances are delivered so that the next one arrives at the buyer when stock from previous freight has just been finished.
- Screening cost and reworking cost is constant.
- In screening process, if defective items are found, then the duplicate costs is paid by the vendor.
- The vendor transferred the extra cost to the purchaser if shortened lead time is entreated.
- Holding cost of the vendor (HV), holding cost of the buyer (HB), production cost of the vendor (SV) and purchasing cost of the buyer (SB) are taken as a triangular fuzzy numbers.

3. Model Formulation

3.1 Proposed Joint Integrated Inventory Model in Crisp Sense

From the above notations and assumptions, the joint total annual cost for the both vendor and buyer, vendor's total annual cost and buyer's total annual cost are determined.

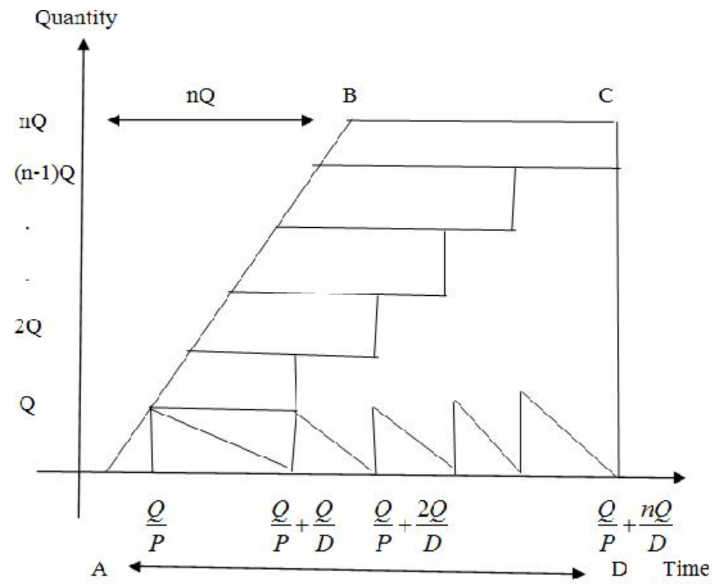
Then, the joint total annual cost is given by,

JTC (Q, n) = ordering cost + set up cost + holding cost + screening cost + reworking cost

In vendor's inventory model, the total annual cost of the vendor is given by,

TCV (Q, n) = set up cost + holding cost + screening cost + reworking cost

In buyer's inventory model, the total annual cost for the buyer is given by, TCB (Q) = ordering cost + holding cost



The average inventory for vendor IV is determined as follows,

$$I_V = \frac{\frac{nQ^2}{2P} + Q^2\left(\frac{1}{R}-\frac{1}{P}\right) + 2Q^2\left(\frac{1}{R}-\frac{1}{P}\right) + 3Q^2\left(\frac{1}{R}-\frac{1}{P}\right) + \ldots + nQ^2\left(\frac{1}{R}-\frac{1}{P}\right)}{nQ/R}$$

$$= \frac{R}{nQ}\left[\frac{nQ^2}{2P} + \frac{n\,(n-1)\,Q^2}{2}\left(\frac{1}{R}-\frac{1}{P}\right)\right]$$

$$I_V = \frac{Q}{2}\left[n\left(1-\frac{R}{P}\right) - 1 + \frac{2R}{P}\right] \tag{1}$$

Vendor's total annual cost is,

$$TC_V\,(Q,n) = \frac{S_V R}{nQ} + \frac{rQH_V}{2}\left[n\left(1-\frac{R}{P}\right) - 1 + \frac{2R}{P}\right] + \sim nQ + {}^{-}\, nQ \tag{2}$$

Buyer's total annual cost is,

$$TC_B\,(Q) = \frac{S_B\,R}{Q} + \frac{rQH_B}{2} \tag{3}$$

Then, the joint total annual cost is given by,

$JTC\,(Q,n)$ = Vendor's total annual cost + Buyer's total annual cost

$= TC_V\,(Q,N) + TC_B\,(Q)$

$$JTC\,(Q,n) = \frac{S_V R}{nQ} + \frac{rQH_V}{2}\left[n\left(1-\frac{R}{P}\right) - 1 + \frac{2R}{P}\right] + \sim nQ + {}^{-}\, nQ + \frac{S_B\,R}{Q} + \frac{rQH_B}{2} \tag{4}$$

Partially differentiate equation (4) with respect to Q, we get

$$\frac{\partial JTC(Q,n)}{\partial Q} = -\frac{S_V R}{nQ^2} + \frac{rH_V}{2}\left[n\left(1-\frac{R}{P}\right)-1+\frac{2R}{P}\right]+\sim n + \;^{-}\; n - \frac{S_B R}{Q^2} + \frac{rH_B}{2} \tag{5}$$

Again, differentiate equation (5) with respect to Q, we get,

$$\frac{\partial^2 JTC(Q,n)}{\partial^2 Q} = \frac{2S_V R}{nQ^3} + \frac{2S_B R}{Q^3} \;>\; 0 \tag{6}$$

Now, set the equation (5) to zero and we compute for Q, then,

$$Q^* = \left[\frac{2R\left(\frac{S_V}{n}+S_B\right)}{rH_B \;+\; rH_V\left[n\left(1-\frac{R}{P}\right)-1+\frac{2R}{P}\right]+2\sim n+2\;^{-}\; n}\right]^{\frac{1}{2}} \tag{7}$$

Substituting equation (7) in (4) , we get,

$$JTC(n) = 2\left(\frac{S_V R}{n}+S_B R\right)^{\frac{1}{2}}\left(\frac{rH_V}{2}\left[n\left(1-\frac{R}{P}\right)-1+\frac{2R}{P}\right]+\sim n+\;^{-}\; n+\frac{rH_B}{2}\right)^{\frac{1}{2}} \tag{8}$$

In equation (8), squaring on both sides, we get

$$\begin{aligned}[JTC(n)]^2 = {} & 2RS_B rH_V n - \frac{4R^2 S_B rH_V n}{P} - 2RS_B rH_V + \frac{2R^2 S_B rH_V}{P} + 2RS_B rH_B + 4RS_B\sim n + \\ & 4RS_B\;^{-}n + 2RS_V rH_V - \frac{2R^2 S_V rH_V}{P} - \frac{2RS_V rH_V}{n} + \frac{4R^2 S_V rH_V}{nP} + \frac{2RS_V rH_B}{n} + \\ & 4RS_V\sim \;+\; 4RS_V\;^{-}\end{aligned} \tag{9}$$

Differentiating equation (8) partially with respect to ‘n’, we have,

$$\frac{\partial JTC(n)}{\partial n} = RS_B rH_V - \frac{2R^2 S_B rH_V}{P} + 2RS_B\sim \;+ 2RS_B\;^{-}\; + \frac{RS_V rH_V}{n^2} - \frac{2R^2 S_V rH_V}{n^2 P} - \frac{RS_V rH_B}{n^2} \tag{10}$$

Again, differentiate equation (10) partially with respect to ‘n’, we have,

$$\frac{\partial^2 JTC(n)}{\partial n^2} = \frac{2RS_V r}{n^3}\left[H_B - H_V\right] + \frac{4R^2 S_V rH_V}{n^3 P} > 0 \tag{11}$$

Now, set the equation (10) to zero and we compute for n, then,

$$n = \left\{\frac{S_V r\left[H_B - H_V\left(1-\frac{2R}{P}\right)\right]}{S_B\left[rH_V\left(1-\frac{R}{P}\right)+2\sim \;+2\;^{-}\right]}\right\}^{1/2} \tag{12}$$

The optimal value of n = n* is get when,

$$JTC(n^*-1) \le JTC(n^*) \quad \text{and} \quad JTC(n^*) \le JTC(n^*+1) \tag{13}$$

From (12) and (13), we get,

$$n^*(n^*-1) \le \left\{ \frac{S_V r\left[H_B - H_V\left(1-\frac{2R}{P}\right)\right]}{S_B\left[rH_V\left(1-\frac{R}{P}\right) + 2\sim \; + 2 \;^{-}\right]} \right\}^{1/2} \le n^*(n^*+1) \tag{14}$$

3.2 Proposed Joint Integrated Inventory Model in Fuzzy Sense

We consider the joint integrated inventory model in fuzzy environment. Here, holding cost of the vendor (HV), holding cost of the buyer (HB), production cost of the vendor (SV) and purchasing cost of the buyer (SB) are taken as a triangular fuzzy numbers.

Suppose $\tilde{H}_V = \left(H_{V_1}, H_{V_2}, H_{V_3}\right), \tilde{H}_B = \left(H_{B_1}, H_{B_2}, H_{B_3}\right), \tilde{S}_V = \left(S_{V_1}, S_{V_2}, S_{V_3}\right) and\ \tilde{S}_B = \left(S_{B_1}, S_{B_2}, S_{B_3}\right)$ are fuzzy triangular numbers, in LR form and

$H_{V_1}, H_{V_2}, H_{V_3}, H_{B_1}, H_{B_2}, H_{B_3}, S_{V_1}, S_{V_2}, S_{V_3}, S_{B_1}, S_{B_2}\ and\ S_{B_3}$ are known real positive numbers.

Now, we fuzzify the total annual inventory cost of the vendor, total annual inventory cost of the buyer and joint integrated total annual inventory cost of the vendor and the buyer.

Apply graded mean integration representation method to defuzzify the fuzzy total annual cost of the vendor, fuzzy total annual cost of the buyer, joint total annual cost of the vendor and the buyer, total number of shipments, optimal order quantity for both the vendor and the buyer.

Then, the joint total annual cost is given by,

$J\tilde{T}C(Q,n)$ = ordering cost + set up cost + holding cost + screening cost + reworking cost

In vendor's inventory model, the total annual cost of the vendor is given by,

$T\tilde{C}_V(Q,n)$ = set up cost + holding cost + screening cost + reworking cost

In buyer's inventory model, the total annual cost for the buyer is given by,

$T\tilde{C}_B(Q)$ = ordering cost + holding cost

The average inventory for vendor IV is determined as follows ,

$$I_V = \frac{\frac{nQ^2}{2P} + Q^2\left(\frac{1}{R}-\frac{1}{P}\right) + 2Q^2\left(\frac{1}{R}-\frac{1}{P}\right) + 3Q^2\left(\frac{1}{R}-\frac{1}{P}\right) + \dots + nQ^2\left(\frac{1}{R}-\frac{1}{P}\right)}{nQ/R}$$

$$= \frac{R}{nQ}\left[\frac{nQ^2}{2P} + \frac{n(n-1)Q^2}{2}\left(\frac{1}{R}-\frac{1}{P}\right)\right]$$

$$I_V = \frac{Q}{2}\left[n\left(1-\frac{R}{P}\right) - 1 + \frac{2R}{P}\right] \tag{15}$$

Vendor's total annual cost is,

$$T\tilde{C}_V(Q,n) = \frac{\tilde{S}_V R}{nQ} + \frac{rQ\tilde{H}_V}{2}\left[n\left(1-\frac{R}{P}\right) - 1 + \frac{2R}{P}\right] + \sim nQ + \;^{-}\; nQ \tag{16}$$

Buyer's total annual cost is,

$$\tilde{TC}_B(Q) = \frac{\tilde{S}_B R}{Q} + \frac{rQ\tilde{H}_B}{2} \tag{17}$$

Then, the joint total annual cost is given by,

$$\tilde{JTC}(Q,n) = \text{Vendor's total annual cost + Buyer's total annual cost}$$

$$= \tilde{TC}_V(Q,N) + \tilde{TC}_B(Q)$$

$$\tilde{JTC}(Q,n) = \frac{\tilde{S}_V R}{nQ} + \frac{rQ\tilde{H}_V}{2}\left[n\left(1-\frac{R}{P}\right)-1+\frac{2R}{P}\right] + \sim nQ + \,^{-}nQ + \frac{\tilde{S}_B R}{Q} + \frac{rQ\tilde{H}_B}{2} \tag{18}$$

Partially differentiate equation (18) with respect to Q, we get

$$\frac{\partial \tilde{JTC}(Q,n)}{\partial Q} = -\frac{\tilde{S}_V R}{nQ^2} + \frac{r\tilde{H}_V}{2}\left[n\left(1-\frac{R}{P}\right)-1+\frac{2R}{P}\right] + \sim n + \,^{-}n - \frac{\tilde{S}_B R}{Q^2} + \frac{r\tilde{H}_B}{2} \tag{19}$$

Again, differentiate equation (19) with respect to Q, we get,

$$\frac{\partial^2 \tilde{JTC}(Q,n)}{\partial^2 Q} = \frac{2\tilde{S}_V R}{nQ^3} + \frac{2\tilde{S}_B R}{Q^3} > 0 \tag{20}$$

Now, set the equation (19) to zero and we compute for Q, then,

$$Q^* = \left[\frac{2R\left(\frac{\tilde{S}_V}{n} + \tilde{S}_B\right)}{r\tilde{H}_B + r\tilde{H}_V\left[n\left(1-\frac{R}{P}\right)-1+\frac{2R}{P}\right] + 2\sim n + 2\,^{-}n}\right]^{\frac{1}{2}} \tag{21}$$

Substituting equation (21) in (18) , we get,

$$\tilde{JTC}(n) = 2\left(\frac{\tilde{S}_V R}{n} + \tilde{S}_B R\right)^{\frac{1}{2}} \left(\frac{r\tilde{H}_V}{2}\left[n\left(1-\frac{R}{P}\right)-1+\frac{2R}{P}\right] + \sim n + \,^{-}n + \frac{r\tilde{H}_B}{2}\right)^{\frac{1}{2}} \tag{22}$$

In equation (22), squaring on both sides, we get

$$\begin{aligned}\left[\tilde{JTC}(n)\right]^2 = {} & 2R\tilde{S}_B r\tilde{H}_V n - \frac{4R^2\tilde{S}_B r\tilde{H}_V n}{P} - 2R\tilde{S}_B r\tilde{H}_V + \frac{2R^2\tilde{S}_B r\tilde{H}_V}{P} + 2R\tilde{S}_B r\tilde{H}_B + 4R\tilde{S}_B \sim n + \\ & 4R\tilde{S}_B \,^{-}n + 2R\tilde{S}_V r\tilde{H}_V - \frac{2R^2\tilde{S}_V r\tilde{H}_V}{P} - \frac{2R\tilde{S}_V r\tilde{H}_V}{n} + \frac{4R^2\tilde{S}_V r\tilde{H}_V}{nP} + \frac{2R\tilde{S}_V r\tilde{H}_B}{n} + \\ & 4R\tilde{S}_V \sim + 4R\tilde{S}_V \,^{-}\end{aligned} \tag{23}$$

Partially differentiate equation (22) with respect to 'n', we have,

$$\frac{\partial J\tilde{T}C(n)}{\partial n} = R\tilde{S}_B r\tilde{H}_V - \frac{2R^2\tilde{S}_B r\tilde{H}_V}{P} + 2R\tilde{S}_B \sim + 2R\tilde{S}_B \ - + \frac{R\tilde{S}_V r\tilde{H}_V}{n^2} - \frac{2R^2\tilde{S}_V r\tilde{H}_V}{n^2 P} - \frac{R\tilde{S}_V r\tilde{H}_B}{n^2} \qquad (24)$$

Again, partially differentiate equation (24) with respect to ‘n’, we have,

$$\frac{\partial^2 J\tilde{T}C(n)}{\partial n^2} = \frac{2R\tilde{S}_V r}{n^3}\left[\tilde{H}_B - \tilde{H}_V\right] + \frac{4R^2\tilde{S}_V r\tilde{H}_V}{n^3 P} > 0 \qquad (25)$$

Now, set the equation (24) to zero and we compute for n, then,

$$n = \left\{ \frac{\tilde{S}_V r\left[\tilde{H}_B - \tilde{H}_V\left(1 - \frac{2R}{P}\right)\right]}{\tilde{S}_B\left[r\tilde{H}_V\left(1 - \frac{R}{P}\right) + 2\sim + 2 \ -\right]} \right\}^{1/2} \qquad (26)$$

The optimal value of n = n* is get when,

$$J\tilde{T}C(n^* - 1) \le J\tilde{T}C\left(n^*\right) \quad \text{and} \quad J\tilde{T}C(n^*) \le J\tilde{T}C\left(n^* + 1\right) \qquad (27)$$

From (26) and (27), we get,

$$n^*(n^* - 1) \le \left\{ \frac{\tilde{S}_V r\left[\tilde{H}_B - \tilde{H}_V\left(1 - \frac{2R}{P}\right)\right]}{\tilde{S}_B\left[r\tilde{H}_V\left(1 - \frac{R}{P}\right) + 2\sim + 2 \ -\right]} \right\}^{1/2} \le n^*(n^* + 1) \qquad (28)$$

4. Numerical Example

4.1 Numerical Example in Crisp Sense

The annual demand of an item is Rs. 800 unit / year and the production rate is Rs.3200 unit / year. Annual inventory holding cost is Rs. 2 per unit. The holding cost for the vendor is Rs.10 / unit, the holding cost for the buyer is Rs.16 / unit, the production cost for the vendor is Rs. 400 / set up and the production cost for the buyer is Rs. 25 / order. If there is 5 % defective items then the duplicate cost for the defective items is Rs. 2 / unit and the screening cost is Rs. 3 / unit.

Total number of shipments, order quantity, vendor’s order quantity, total annual cost of the vendor, buyer’s order quantity, total annual cost of the buyer, joint total annual cost for both the vendor and the buyer are determined.

Sol:

R = Rs. 800 unit / year
P = Rs. 3200 unit / year
r = Rs. 2 / unit
HV = Rs. 10 / unit
HB = Rs. 16 / unit
SV = Rs. 400 / setup
SB = Rs. 25 / order
− = 5 %
~ = Rs. 3 / unit
= Rs. 2 / unit

4.1.1 Total Number of Shipments:

$$n^* = \left\{ \frac{S_V r \left[H_B - H_V \left(1 - \frac{2R}{P} \right) \right]}{S_B \left[rH_V \left(1 - \frac{R}{P} \right) + 2\sim \; + 2 \; - \right]} \right\}^{1/2}$$

n^* = 4

4.1.2 Order Quantity:

$$Q^* = \left[\frac{2R\left(\frac{S_V}{n} + S_B \right)}{rH_B \; + \; rH_V \left[n\left(1 - \frac{R}{P} \right) - 1 + \frac{2R}{P} \right] + 2\sim n + 2 \; - \; n} \right]^{\frac{1}{2}}$$

Q^* = 43.27

4.1.3 Vendor’s Order Quantity:

$$Q_V = \left[\frac{2R\left(\frac{S_V}{n} \right)}{rH_V \left[n\left(1 - \frac{R}{P} \right) - 1 + \frac{2R}{P} \right] + 2\sim n + 2 \; - \; n} \right]^{\frac{1}{2}}$$

Q_V = 46.25

4.1.4 Total Annual Cost of the Vendor

$$TC_V\,(Q,n) = \frac{S_V R}{nQ} + \frac{rQH_V}{2}\left[n\left(1 - \frac{R}{P} \right) - 1 + \frac{2R}{P} \right] + \sim nQ + \; - \; nQ$$

$TC_V\,(Q,n)$ = Rs. 3459.48

4.1.5 Buyer’s Order Quantity

$$Q = \left\{ \frac{2S_B R}{rH_B} \right\}^{1/2}$$

Q = 35.36

4.1.6 Total Annual Cost of the Buyer

$$TC_B\,(Q) = \frac{S_B\, R}{Q} + \frac{rQH_B}{2}$$

$TC_B\,(Q)$ = Rs. 1,131.37

4.1.7 Joint Total Annual Cost of the Vendor and the Buyer

$$JTC\,(Q,n) = \frac{S_V R}{nQ} + \frac{rQH_V}{2}\left[n\left(1 - \frac{R}{P} \right) - 1 + \frac{2R}{P} \right] + \sim nQ + \; - \; nQ + \frac{S_B\, R}{Q} + \frac{rQH_B}{2}$$

$JTC\,(Q,n)$ = Rs. 4590.85

4.2 Numerical Example in Fuzzy Sense

Let R = Rs. 800 unit / year

P = Rs. 3200 unit / year

r = Rs. 2 / unit

$\tilde{H}_V$ = (Rs.8, Rs.10, Rs.12) unit/year

$\tilde{H}_B$ = (Rs.12, Rs.16, Rs.20) unit/ year

$\tilde{S}_V$ = (Rs.200, Rs.400, Rs.600) / setup

$\tilde{S}_B$ = (Rs.20, Rs.25, Rs.30) / order

$-$ = 5 %

$\sim$ = Rs. 3 / unit

= Rs. 2 / unit

Sol:

4.2.1 Total Number of Shipments

$$n^* = \left\{ \frac{\tilde{S}_V r \left[\tilde{H}_B - \tilde{H}_V \left(1 - \frac{2R}{P} \right) \right]}{\tilde{S}_B \left[r\tilde{H}_V \left(1 - \frac{R}{P} \right) + 2\sim + 2 \; - \right]} \right\}^{1/2}$$

n^* = (1.818, 4.0748, 7.2628)

Using Graded integration method to defuzzify the above value,

$$p(n^*) = \frac{a_1 + 4a_2 + a_3}{6} = \frac{1.818 + 4 \times 4.0748 + 7.2628}{6} = 4.23 = 4(\text{app})$$

4.2.2 Fuzzy Order Quantity

$$Q^* = \left[\frac{2R\left(\frac{\tilde{S}_V}{n} + \tilde{S}_B \right)}{r\tilde{H}_B + r\tilde{H}_V \left[n\left(1 - \frac{R}{P} \right) - 1 + \frac{2R}{P} \right] + 2\sim n + 2 \; - \; n} \right]^{\frac{1}{2}}$$

Q^* = (29.957, 43.274, 56.9495)

Using Graded integration method to defuzzify the above value,

$$p(Q^*) = \frac{a_1 + 4a_2 + a_3}{6} = \frac{29.957 + 4 \times 43.274 + 56.9495}{6} = 43.33$$

4.2.3 Vendor's Fuzzy Order Quantity

$$Q_V = \left[\frac{2R\left(\frac{\tilde{S}_V}{n} \right)}{r\tilde{H}_V \left[n\left(1 - \frac{R}{P} \right) - 1 + \frac{2R}{P} \right] + 2\sim n + 2 \; - \; n} \right]^{\frac{1}{2}}$$

Q_V = (30.7148, 46.2497, 60.858)

Using Graded integration method to defuzzify the above value,

$$p(Q_V) = \frac{a_1 + 4a_2 + a_3}{6} = \frac{30.7148 + 4 \times 46.2497 + 60.858}{6} = 46.10$$

4.2.4 Total Fuzzy Annual Cost of the Vendor

$$T\tilde{C}_V\,(Q,n) = \frac{\tilde{S}_V R}{nQ} + \frac{rQ\tilde{H}_V}{2}\left[n\left(1 - \frac{R}{P}\right) - 1 + \frac{2R}{P}\right] + \sim nQ + \ ^{-}\ nQ$$

$T\tilde{C}_V\,(Q,n)$ = (2327.04, 3466.838, 4606.636)

Using Graded integration method the defuzzified value is

$$p(T\tilde{C}_V\,(Q,n)) = \frac{a_1 + 4a_2 + a_3}{6} = \frac{2327.04 + 4 \times 3466.838 + 4606.636}{6} = 3466.838$$

4.2.5 Buyer's Fuzzy Order Quantity

$$Q = \left\{\frac{2\tilde{S}_B R}{r\tilde{H}_B}\right\}^{1/2}$$

Q = (28.2848, 35.355, 44.721)

Using Graded integration method to defuzzify the above value,

$$p(Q) = \frac{a_1 + 4a_2 + a_3}{6} = \frac{28.2848 + 4 \times 35.355 + 44.721}{6} = 35.74$$

4.2.6 Fuzzy Total Annual Cost of the Buyer

$$T\tilde{C}_B\,(Q) = \frac{\tilde{S}_B\,R}{Q} + \frac{rQ\tilde{H}_B}{2}$$

$T\tilde{C}_B\,(Q)$ = (876.563, 1131.434, 1386.304)

Using Graded integration method to defuzzify the above value,

$$p(T\tilde{C}_B\,(Q)) = \frac{a_1 + 4a_2 + a_3}{6} = \frac{876.563 + 4 \times 1131.434 + 1386.304}{6} = \text{Rs. } 1131.434$$

4.2.7 Joint Fuzzy Total Annual Cost of the Vendor and the Buyer:

$$J\tilde{T}C\,(Q,n) = \frac{\tilde{S}_V R}{nQ} + \frac{rQ\tilde{H}_V}{2}\left[n\left(1 - \frac{R}{P}\right) - 1 + \frac{2R}{P}\right] + \sim nQ + \ ^{-}\ nQ + \frac{\tilde{S}_B\,R}{Q} + \frac{rQ\tilde{H}_B}{2}$$

$J\tilde{T}C\,(Q,n)$ = (3203.603, 4598.272, 5992.94)

Using Graded integration method to defuzzify the above value,

$$p(J\tilde{T}C\,(Q,n)) = \frac{a_1 + 4a_2 + a_3}{6} = \frac{3203.603 + 4 \times 4598.272 + 5992.94}{6} = \text{Rs. } 4598.272$$

5. Sensitivity Analysis

	QV	QB	Q*	TCV(Q, n)	TCB(Q)	JTC(Q, n)
n = 1	239.05	35.36	125.46	2,677.31	1,131.37	3,808.68
n = 2	99.08	35.36	74.77	3,219.94	1,131.37	4,351.31
n = 3	63.09	35.36	54.40	3,381.53	1,131.37	4,512.90
n = 4	46.25	35.36	43.27	3,459.48	1,131.37	4,590.85
n = 5	36.51	35.36	36.23	3,505.43	1,131.37	4,636.80
n = 6	30.17	35.36	31.35	3,535.72	1,131.37	4,667.09

6. Conclusion

In this model, we derived a Joint integrated total annual inventory cost of the vendor and the buyer in the crisp sense as well as in fuzzy sense. Holding cost and purchasing cost of the vendor and buyer are taken as a fuzzy numbers. When the number of shipments (n) is increasing, the joint total integrated cost is increasing. It implies that the trader will order more number of quantities to reduce the annual inventory cost. This model is solved analytically by minimizing the total inventory cost.

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