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RESEARCH ARTICLE

NEWSBOY SINGLE PERIOD INVENTORY MODEL WITH FUZZY RANDOM VARIABLE DEMAND AND DETERIORATION

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ABSTRACT

This paper presents a single-period Newsboy inventory problem in an imprecise and uncertain mixed environment with deterioration. The aim of this paper is to introduce demand of Newsboy as a fuzzy random variable. To determine the optimal order quantity a new methodology is developed for this model in presence of fuzzy random variable demand where the optimum is achieved using a graded mean integration representation. To illustrate the model the classical newsboy problem is considered.

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INTRODUCTION

Several probabilistic inventory models have been studied in the literature, single-period inventory model (Hadley and Whiten, 1963; Buffa and Satin, 1987) is one of those elementary models in which only a single procurement is being made. There is a wide application of this model on production management systems, like stocking seasonal items (Christmas trees, woolen materials), perishable goods, spare parts, etc. To develop the methodology of this model, we consider the well-known newsboy problem, in which the decision-maker wants to know the optimal number of newspapers to be purchased daily to maximize his expected profit. In a real situation, the daily demand of the newspapers may vary day to day. Either due to lack of historical data or abundance of information it is worthwhile to consider a distribution for demand. Recently some researchers have considered the demand as a fuzzy number only (Kao and Hsu, 2002; Li *et al*., 2002). In (Ishii and Konno, 1998) the newsboy problem has been redefined considering shortage cost as fuzzy number and demand as random variable. However, no attempt is made to define the demand in mixed environment, where fuzziness and randomness both appears simultaneously. Thus, we consider the demand as a fuzzy random variable involving imprecise probabilities since the probability of a fuzzy event is a fuzzy number (Chakraborty, 2002). The concept of fuzzy random variable and its fuzzy expectation has been presented by H. Kwakernaak (Kwakernaak, 1978) and later by Purl and Ralescue (1986). Further, recently the notion of a fuzzy random variable has also been considered in (Kim and Ghil, 1997; Feng *et al*., 2001; Lopez-Diaz and. Gil, 1998). None of them do not considered the case of deteriorating or decaying items by different phenomena. In this regard we have considered that case in our model. The purpose of this study is to find the optimal order quantity i.e. the no of paper for the single-period inventory problem when the demands are fuzzy stochastic with deterioration. In Section 2, of this paper, the fuzzy random variable and its fuzzy expectation are defined and later a brief outline of graded mean integration representation of a triangular fuzzy number has been discussed. Next, in Section 3, a single period inventory model in presence of fuzzy random variable demand with deteriorating items is formulated and then the proposed methodology is being developed. A section 4, Deal with a numerical example and the conclusion has been made in Section 5.

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Preliminary Concepts

Fuzzy Random Variable and its Expectation

A fuzzy set $\tilde{u} : R \to [0,1]$ is called a fuzzy number if it satisfies,

$\tilde{u}$ is normal i.e., $\{x \in R / \tilde{u}(x) = 1\}$ is nonempty,
$\tilde{u}$ is fuzzy convex; i.e., $\tilde{u}(\alpha x + (1-\alpha)y) \geq \min(\tilde{u}(x), \tilde{u}(y))$ $for\ x, y \in R, \alpha \in [0,1]$,
$\tilde{u}$ is upper semi continuous,
The support of $\tilde{u}$ is compact, i.e. $\{x \in R / \tilde{u}(x) > 0\}$ is bounded.

Let F be the set of all fuzzy numbers. The α-cut $u_\alpha = \{x R / \tilde{u}(x) \geq \alpha\}$ of $\tilde{u} \in F$, is a closed interval for any $\alpha \in [0, 1]$. The addition and scalar multiplication on F are defined by the following:

$$\left.\begin{array}{l}[u+v]_\alpha = u_\alpha + v_\alpha \\ [\lambda u]_\alpha = \lambda u_\alpha\end{array}\right\} \tilde{u}, \tilde{v} \in F, \quad \lambda \in R, \qquad \alpha \in [0, 1]$$

A metric on F is defined by

$$d(\tilde{u}, \tilde{v}) = \frac{1}{2}\int_0^1 \left(\left| u_\alpha^- - v_\alpha^- \right|^2 + \left| u_\alpha^+ + v_\alpha^+ \right|^2 \right) d\alpha, \qquad \forall \tilde{u}, \tilde{v} \in F$$

Where u_α^-, u_α^+ are the lower and upper end points of uα and (F, d) is a complete metric space.
Let (Ω, A, P) be a complete probability space. A fuzzy random variable (f.r.v.) is a Borel measurable function $\tilde{X} : (\Omega, A, P) \to (F, d)$

If $\tilde{X}$ is a f.r.v, then $[\tilde{X}]_\alpha = [X_\alpha^-, X_\alpha^+]$, $\alpha \in [0, 1]$ is a random dosed interval set, and X_α^-, X_α^+ are real valued random variables. The expectation of a f.r.v. $\tilde{X}$ is defined as a unique fuzzy number $\tilde{u} \in F$, whose α-cut us is given by

$$u_\alpha = [E\tilde{X}]_\alpha = E[X_\alpha] = [E(X_\alpha^-), E(X_\alpha^+)], \ \alpha \in [0, 1].$$

Definition

For a fuzzy random variable $\tilde{X} = \{(X_\alpha^-, X_\alpha^+) / 0 \leq \alpha \leq 1\}$ the expectation of $\tilde{X}$, is denoted by

$$E\tilde{X} = \int \tilde{X}\, dP = \left\{ \left(\int X_\alpha^-, \int X_\alpha^+ \right) \big/ 0 \leq \alpha \leq 1 \right\}$$

If $\tilde{X}$ is a discrete fuzzy random variable, such that $P(\tilde{X} = \tilde{x}_i) = \tilde{p}_i$, i = 1, 2, 3...., then its fuzzy expectation is given by $E\tilde{X} = \sum_{i=1}^{\infty} \tilde{x}_i\, \tilde{p}_i$ ………… (1)

It is also proved (11) that

$$E\tilde{X} \in F \ and \ [E\tilde{X}]_0 = \int_\Omega X_0\, dP = [EX_0^-, EX_0^+] \quad for\ \alpha = 0.$$

Defuzzification of a Fuzzy Number

Using its Graded Mean Integration Representation

For achieving computational efficiency, the method of defuzzification of a fuzzy number by its graded mean integration representation as proposed by Chen and Hsiesh (1999), is considered in this work. We now present below its salient features. Generally, a generalized fuzzy number $\tilde{A}$ is described as any fuzzy subset of the real line R, whose membership function $\mu_{\tilde{A}}(u)$ satisfies the following conditions,

$\mu_{\tilde{A}}(u)$ is a continuous mapping from R to the closed interval (0, 1),
$\mu_{\tilde{A}}(u) = 0, \ -\infty < u \le \underline{a}$,
$\mu_L(u) = L(u)$ is strictly increasing on $[\underline{a}, a]$,
$\mu_{\tilde{A}}(u) = w$, u=a
$\mu_R(u) = R(u)$ is strictly decreasing on $[a, \overline{a}]$,
$\mu_{\tilde{A}}(u) = 0, \ \overline{a} \le u < \infty$,

Where $0 < w \le 1$ and $\underline{a}, a$ and $\overline{a}$ are real numbers.

We call this type of generalized fuzzy number as triangular fuzzy number, and it is denoted by $\tilde{A} = (\underline{a},\ a,\ \overline{a}\ ;\ w)_{LR}$. When w=1, this type of generalized fuzzy number is called normal fuzzy number (Zimmermann, 1996) and is represented by $\tilde{A} = (\underline{a},\ a,\ \overline{a}\ )_{LR}$. Let L^{-1} and R^{-1} be the inverse function of functions L and R respectively, then the graded mean h-level value of $\tilde{A} = (\underline{a},\ a,\ \overline{a}\ )_{LR}$ is $h\left[L^{-1}(h) + R^{-1}(h)\right]/2$.

Therefore, the graded mean integration representation of generalized fuzzy number $\tilde{A}$ with grade w is

$$G(\tilde{A}) = \int_0^w h \left(\frac{L^{-1}(h) + R^{-1}(h)}{2} \right) dh \Big/ \int_0^w h \ dh$$

Where h lies between 0 and w, $0 < w \le 1$.

Here, $L(u) = w\left(\frac{u-\underline{a}}{a-\underline{a}}\right), \ \underline{a} \le u \le a$ and $R(u) = w\left(\frac{u-\overline{a}}{\overline{a}-a}\right), \ a \le u \le \overline{a}$

Then, $L^{-1}(h) = \underline{a} + (a-\underline{a})h / w, \quad 0 \le h \le w,$

$$R^{-1}(h) = \overline{a} + (\overline{a}-a)h / w, \quad 0 \le h \le w,$$

Now, using the formula (1) the graded mean integration representation if $\tilde{A}$ is

$$G(\tilde{A}) = \int_0^w h \left(\frac{L^{-1}(h) + R^{-1}(h)}{2} \right) dh \Big/ \int_0^w h \ dh = \frac{\underline{a} + 4a + \overline{a}}{6} \quad \text{………} (2)$$

Notation and Assumptions

We define the following symbols/notations and assumptions:

Consider a single-period inventory model,

$\hat{\tilde{X}}$:fuzzy random demand variable with the given set of data { $(\tilde{y}_1, \tilde{p}_1)$, $(\tilde{y}_2, \tilde{p}_2)$, ……., $(\tilde{y}_n, \tilde{p}_n)$ },

yi:i-item, to be procured, i=1 to n,

pi:Probability of occurrence of i-item,

S:Unit selling price of the item (S > C),

C:Unit price of an item at which it is procured (independent of number of Item procured),

Ch:holding cost per each item after the end of the period (Ch < S), (It can be Consider as unit selling price of the item after the end of the period),

Cd:deterioration cost per each item,

Cs:Unit shortages cost per item if there is a shortage,

ɵ:Rate of deterioration consider to be as constant,

$\tilde{P}\left(\tilde{y}_k, \hat{\tilde{X}}\right)$:Profit function of the model

Problem Formulation and Methodology

Single-Period Inventory Problem

Generally, the single-period inventory model of profit maximization with time independent costs can be represented as the classical newsboy problem where the newsboy must purchase an approximate number of newspapers for his corner newsstand such that he earns the maximum expected profit at the end of the day. Consider an item that can be procured at the beginning of a period and after the end of the period, it is either of no use or it is to be sold at a lower price than the cost at which it was procured. If $\tilde{y}_k$ items are procured at the beginning of the period, then the profit function $\tilde{P}$ is given by

$$\tilde{P}\left(\tilde{y}_k, \hat{\tilde{X}}\right) = \begin{cases} s\tilde{y}_i - c\tilde{y}_k - (C_h + C_d)(\tilde{y}_k - \tilde{y}_i), & \tilde{y}_i \le \tilde{y}_k \\ (s-c)\tilde{y}_k - {}^{-}C_d\tilde{y}_k - C_s(\tilde{y}_i - \tilde{y}_k), & \tilde{y}_i \ge \tilde{y}_k \end{cases} \quad \text{……..} \ (3)$$

for some i = 1 to n.

As the demand $\hat{\tilde{X}}$ is a fuzzy random variable so its profit function $\tilde{P}$ is also a fuzzy random variable and obviously its total expected value $E\tilde{P}$ becomes a unique fuzzy number.

Therefore the total expected profit is determined by

$$E\tilde{P} = E\tilde{P}\left(\tilde{y}_k, \hat{\tilde{X}}\right) = \sum_{i=1}^{k}[s\tilde{y}_i - c\tilde{y}_k - (C_h +^{-} C_d)(\tilde{y}_k - \tilde{y}_i)]\tilde{p}_i + \sum_{i=k+1}^{n}[(s-c)\tilde{y}_k -^{-} C_d\tilde{y}_k - C_s(\tilde{y}_i - \tilde{y}_k)]\tilde{p}_i \quad \text{…... (4)}$$

Mathematical Model with Fuzzy Random Variable Demand

We consider the above single-period inventory problem where the demand is considered as a fuzzy random variable. The data are imprecise with fuzzy probability, so for the sake of simplicity, we consider all the data set and its corresponding probabilities as triangular fuzzy number $\left(\underline{y}_i, y_i, \overline{y}_i\right)_{LR}$ and $\left(\underline{p}_i, p_i, \overline{p}_i\right)_{LR}$ for i = 1 to n, respectively.

Now, the fuzzy expected profit function $E\tilde{P} = \left(\underline{EP}, EP, \overline{EP}\right)_{LR}$ is given by

$$E\tilde{P} = \sum_{i=1}^{k}[s\tilde{y}_i - c\tilde{y}_k - (C_h +^{-} C_d)(\tilde{y}_k - \tilde{y}_i)]\tilde{p}_i + \sum_{i=k+1}^{n}[(s-c)\tilde{y}_k -^{-} C_d\tilde{y}_k - C_s(\tilde{y}_i - \tilde{y}_k)]\tilde{p}_i$$

$$= \sum_{i=1}^{k}[(s + C_h +^{-} C_d)\tilde{y}_i - (c + C_h +^{-} C_d)\tilde{y}_k]\tilde{p}_i + \sum_{i=k+1}^{n}[(s - c -^{-} C_d + C_s)\tilde{y}_k - C_s\tilde{y}_i]\tilde{p}_i \quad \text{….. (5)}$$

Where,

$$EP = E\left[\tilde{P}_{\sim=1}\right] = \sum_{i=1}^{k}\left[\left(s + C_h + {}^{-}C_d\right) y_i\, p_i - \left(c + C_h + {}^{-}C_d\right) y_k\, p_i\right] + \sum_{i=k+1}^{n}\left[\left(s - c - {}^{-}C_d + C_s\right) y_k\, p_i - C_s y_i\, p_i\right] \quad \text{........ (6)}$$

$$\underline{EP} = E\left[\tilde{P}^{-}_{\sim=0}\right] = \sum_{i=1}^{k}\left[\left(s + C_h + {}^{-}C_d\right)\underline{y}_i\,\underline{p}_i - \left(c + C_h + {}^{-}C_d\right)\overline{y}_k\,\overline{p}_i\right] + \sum_{i=k+1}^{n}\left[\left(s - c - {}^{-}C_d + C_s\right)\underline{y}_k\,\underline{p}_i - C_s\,\overline{y}_i\,\overline{p}_i\right] \text{ and, } \overline{EP} = E\left[\tilde{P}^{+}_{\sim=0}\right] \quad \text{.............. (7)}$$

$$= \sum_{i=1}^{k}\left[\left(s + C_h + {}^{-}C_d\right)\overline{y}_i\,\overline{p}_i - \left(c + C_h + {}^{-}C_d\right)\underline{y}_k\,\underline{p}_i\right] + \sum_{i=k+1}^{n}\left[\left(s - c - {}^{-}C_d + C_s\right)\overline{y}_k\,\overline{p}_i - C_s\,\underline{y}_i\,\underline{p}_i\right] \quad \text{............. (8)}$$

Here the α-level set of the fuzzy number $E\tilde{P}$ are considered $\left[E\tilde{P}\right]_{\sim} = E\left[\tilde{P}_{\sim}\right] = \left[E\left(P_{\sim}^{-}\right), E\left(P_{\sim}^{+}\right)\right], \quad 0 \le \sim \le 1$ and we get different α -cut intervals for the fuzzy number $E\tilde{P}$ for different α between 0 and 1.

The membership function of this unique fuzzy number $E\tilde{P}$ is given by

$$«_{E\tilde{P}}(u) = \begin{cases} L(u), & \underline{EP} \le u \le EP \\ R(u), & EP \le u \le \overline{EP} \\ 0, & otherwise \end{cases}$$

Where L(u) is the left continuous function from $\left[\underline{EP},\ EP\right]$ to (0, 1) and R(u) is the right continuous function from $\left[EP,\ \overline{EP}\right]$ to (0, 1).

Now, using the method of representation of generalized fuzzy number based on the integral value of graded mean h-level, we find a defuzzified representative of the unique fuzzy number $E\tilde{P}$ as

$$G\left(E\tilde{P}\right) = \int_0^1 h \left(\frac{L^{-1}(h) + R^{-1}(h)}{2}\right) dh \Big/ \int_0^1 h \ dh$$

Where,

$$L(u) = w\left(\frac{u - \underline{EP}}{EP - \underline{EP}}\right), \quad \underline{EP} \le u \le EP$$

And,

$$R(u) = w\left(\frac{u - \overline{EP}}{\overline{EP} - EP}\right), \quad EP \le u \le \overline{EP}$$

And using it we get

$$G\left(E\tilde{P}\right) = \frac{\underline{EP} + 4EP + \overline{EP}}{6} \quad \text{.......... (9)}$$

Further for the optimal order quantity $\tilde{y}_k$, we must have

$$G\left(E\tilde{P}\left(\tilde{y}_k, \hat{\tilde{X}}\right)\right) > G\left(E\tilde{P}\left(\tilde{y}_{k-1}, \hat{\tilde{X}}\right)\right) \quad \text{.......... (10)}$$

And

$$G\left(E\tilde{P}\left(\tilde{y}_k, \hat{\tilde{X}}\right)\right) > G\left(E\tilde{P}\left(\tilde{y}_{k+1}, \hat{\tilde{X}}\right)\right) \quad \text{.......... (11)}$$

The result of (10) and (11), gives us the optimal order quantity $\tilde{y}_k$, which satisfies the following condition

$$\text{Lower limit} < \frac{s - c - {}^{-}C_d + C_s}{2s - c + C_h + C_s} < \text{upper limit} \quad \text{............ (12)}$$

Where,

Lower limit

$$= \frac{4(y_k - y_{k-1})\sum_{i=1}^{k-1} p_i + (\underline{y}_k - \underline{y}_{k-1})\sum_{i=1}^{k-1} \underline{p}_i + (\overline{y}_k - \overline{y}_{k-1})\sum_{i=1}^{k-1} \overline{p}_i}{4(y_k - y_{k-1})\sum_{i=1}^{n} p_i + (\underline{y}_k - \underline{y}_{k-1})\sum_{i=1}^{n} \underline{p}_i + (\overline{y}_k - \overline{y}_{k-1})\sum_{i=1}^{n} \overline{p}_i}$$

Upper limit

$$= \frac{4(y_{k+1} - y_k)\sum_{i=1}^{k} p_i + (\underline{y}_{k+1} - \underline{y}_k)\sum_{i=1}^{k} \underline{p}_i + (\overline{y}_{k+1} - \overline{y}_k)\sum_{i=1}^{k} \overline{p}_i}{4(y_{k+1} - y_k)\sum_{i=1}^{n} p_i + (\underline{y}_{k+1} - \underline{y}_k)\sum_{i=1}^{n} \underline{p}_i + (\overline{y}_{k+1} - \overline{y}_k)\sum_{i=1}^{n} \overline{p}_i}$$

Thus, we conclude that the optimal order quantity can be found for a single-period inventory model when the demand is prescribed in mixed imprecise and uncertain environment, which is more realistic for making an inventory. The calculation part of equation (10) and (11) is given as an appendix. Next we give a numerical example how to achieve the optimal order quantity when the data set of the demand is imprecisely given with fuzzy probability.

Numerical Example

To illustrate this model, suppose the newsboy has no opportunity to repurchase during the day if he needs more papers. Let the purchase cost per item be c = 4, the selling cost s = 6, the holding cost Ch = 2, the deterioration cost Cd =1, the shortage cost Cs = 3 and rate of deterioration is ɵ=0.2. The Daily demand for the papers is unknown; however, based on his past experience and sales data, he can assign probabilities to various levels of demand. The demand data and associated probabilities are given in Table1. Now, using our proposed methodology we can find out the optimal order quantity $\tilde{y}_k$ (say) from Table 2.

Table 1. Demand associated with probabilities

$(\underline{y}_i, y_i, \overline{y}_i)_{LR}$	$(\underline{p}_i, p_i, \overline{p}_i)_{LR}$
$(41, 43, 45)_{LR}$	$(.045, .05, .055)_{LR}$
$(46, 48, 50)_{LR}$	$(.143, .15, .157)_{LR}$
$(51, 53, 55)_{LR}$	$(.292, .30, .308)_{LR}$
$(56, 58, 60)_{LR}$	$(.192, .20, .208)_{LR}$
$(61, 63, 65)_{LR}$	$(.092, .10, .108)_{LR}$
$(66, 68, 70)_{LR}$	$(.093, .10, .107)_{LR}$
$(71, 73, 75)_{LR}$	$(.094, .10, .106)_{LR}$

Table 2. Optimum order quantity

$\tilde{y}_k$						
$4(y_{k+1} - y_k)\sum_{i=1}^{k} p_i$	1	4	10	14	16	18
$(\underline{y}_{k+1} - \underline{y}_k)\sum_{i=1}^{k} \underline{p}_i$	0.225	0.94	2.4	3.36	3.82	4.285
$(\overline{y}_{k+1} - \overline{y}_k)\sum_{i=1}^{k} \overline{p}_i$	0.275	1.06	2.6	3.64	4.18	4.715
	1.5	6	15	21	24	27
$4(y_{k+1} - y_k)\sum_{i=1}^{n} p_i$	20	20	20	20	20	20
$(\underline{y}_{k+1} - \underline{y}_k)\sum_{i=1}^{n} \underline{p}_i$	4.755	4.755	4.755	4.755	4.755	4.755
$(\overline{y}_{k+1} - \overline{y}_k)\sum_{i=1}^{n} \overline{p}_i$	5.245	5.245	5.245	5.245	5.245	5.245
	30	30	30	30	30	30
$\frac{s-c-C_d+C_s}{2s-c+C_h+C_s}$	0.3692	0.3692	0.3692	0.3692	0.3692	0.3692
$\frac{A}{B}$	0.05	0.2	0.5	0.7	0.8	0.9

Here, we take A and B as the numerator and denominator of the upper limit of (12), respectively. Hence, the optimal order quantity $\tilde{y}_k$ is given as $(51, 53, 55)_{LR}$, i.e., it should be better for the newsboy to buy around 53 papers for maximizing his daily expected profit.

Conclusion

It is important to note that, as in the case of development of stochastic inventory models, the implementation of fuzzy random variable as demand gives more realistic information where the variable values are imprecise. Here, a single-period inventory model has been discussed in the presence of imprecision and uncertainty demand with deterioration. This is a nice model to help the Newsboy to take the necessary steps to buy how many papers in a day that maximize his daily expected profit. This amalgamation may be extended for other inventory models also deterioration may be imprecision and uncertainty.

Appendix

The optimality condition of the optimal order quantity may be defined as

$$G\left(E\tilde{P}\left(\tilde{y}_k,\hat{\hat{X}}\right)\right) > G\left(E\tilde{P}\left(\tilde{y}_{k-1},\hat{\hat{X}}\right)\right)$$

$$\Rightarrow G\left(E\tilde{P}\left(\tilde{y}_k,\hat{\hat{X}}\right)\right) - G\left(E\tilde{P}\left(\tilde{y}_{k-1},\hat{\hat{X}}\right)\right) > 0 \qquad \text{……….. (13)}$$

The left hand side of above inequality in the extended form reads like

$$= \frac{1}{6}\left[\begin{array}{l} 4\sum_{i=1}^{k}\{(s+C_h+\bar{C}_d)y_i p_i-(c+C_h+\bar{C}_d)y_k p_i\}+\sum_{i=k+1}^{n}[\{(s-c-\bar{C}_d+C_s)y_k p_i\}-C_s y_i p_i] \\ \quad +\sum_{i=1}^{k}\{(s+C_h+\bar{C}_d)\underline{y}_i\underline{p}_i-(c+C_h+\bar{C}_d)\overline{y}_k\overline{p}_i\}+\sum_{i=k+1}^{n}[\{(s-c-\bar{C}_d+C_s)\overline{y}_k\overline{p}_i\}-C_s\underline{y}_i\underline{p}_i] \\ \quad +\sum_{i=1}^{k}\{(s+C_h+\bar{C}_d)\overline{y}_i\overline{p}_i-(c+C_h+\bar{C}_d)\underline{y}_k\underline{p}_i\}+\sum_{i=k+1}^{n}[\{(s-c-\bar{C}_d+C_s)\overline{y}_k\overline{p}_i\}-C_s\underline{y}_i\underline{p}_i] \end{array}\right]$$

$$-\frac{1}{6}\left[\begin{array}{l} 4\sum_{i=1}^{k-1}\{(s+C_h+\bar{C}_d)y_i p_i-(c+C_h+\bar{C}_d)y_k p_i\}+\sum_{i=k}^{n}[\{(s-c-\bar{C}_d+C_s)y_k p_i\}-C_s y_i p_i] \\ \quad +\sum_{i=1}^{k-1}\{(s+C_h+\bar{C}_d)\underline{y}_i\underline{p}_i-(c+C_h+\bar{C}_d)\overline{y}_k\overline{p}_i\}+\sum_{i=k}^{n}[\{(s-c-\bar{C}_d+C_s)\overline{y}_k\overline{p}_i\}-C_s\underline{y}_i\underline{p}_i] \\ \quad +\sum_{i=1}^{k-1}\{(s+C_h+\bar{C}_d)\overline{y}_i\overline{p}_i-(c+C_h+\bar{C}_d)\underline{y}_k\underline{p}_i\}+\sum_{i=k}^{n}[\{(s-c-\bar{C}_d+C_s)\overline{y}_k\overline{p}_i\}-C_s\underline{y}_i\underline{p}_i] \end{array}\right]$$

$$= \frac{1}{6}\left[\begin{array}{l} \sum_{i=1}^{k}\{A(4y_i p_i+\underline{y}_i\underline{p}_i+\overline{y}_i\overline{p}_i)-B(4y_k p_i+\underline{y}_k\underline{p}_i+\overline{y}_k\overline{p}_i)\} \\ \quad +\sum_{i=k+1}^{n}\{C(4y_k p_i+\underline{y}_k\underline{p}_i+\overline{y}_k\overline{p}_i)-D(4y_i p_i+\underline{y}_i\underline{p}_i+\overline{y}_i\overline{p}_i)\} \end{array}\right]$$

$$-\frac{1}{6}\left[\begin{array}{l} \sum_{i=1}^{k-1}\{A(4y_i p_i+\underline{y}_i\underline{p}_i+\overline{y}_i\overline{p}_i)-B(4y_{k-1}p_i+\underline{y}_{k-1}\underline{p}_i+\overline{y}_{k-1}\overline{p}_i)\} \\ \quad +\sum_{i=k}^{n}\{C(4y_{k-1}p_i+\underline{y}_{k-1}\underline{p}_i+\overline{y}_{k-1}\overline{p}_i)-D(4y_i p_i+\underline{y}_i\underline{p}_i+\overline{y}_i\overline{p}_i)\} \end{array}\right]$$

Where, $A=s+C_h+\bar{C}_d$, $B=c+C_h+\bar{C}_d$, $C=s-c-\bar{C}_d+C_s$, $D=C_s$

$$=\frac{1}{6}\left[\begin{array}{l}(-B)\left\{4(y_k-y_{k-1})\sum_{i=1}^{k-1}p_i+\left(\underline{y}_k-\underline{y}_{k-1}\right)\sum_{i=1}^{k-1}\underline{p}_i+\left(\overline{y}_k-\overline{y}_{k-1}\right)\sum_{i=1}^{k-1}\overline{p}_i\right\}\\ \quad +C\left\{4(y_k-y_{k-1})\sum_{i=k+1}^{n}p_i+\left(\underline{y}_k-\underline{y}_{k-1}\right)\sum_{i=k+1}^{n}\underline{p}_i+\left(\overline{y}_k-\overline{y}_{k-1}\right)\sum_{i=k+1}^{n}\overline{p}_i\right\}\\ \quad +(A-B+D)\left(4y_kp_k+\underline{y}_k\underline{p}_k+\overline{y}_k\overline{p}_k\right)-C\left(4y_{k-1}p_k+\underline{y}_{k-1}\underline{p}_k+\overline{y}_{k-1}\overline{p}_k\right)\end{array}\right]$$

$$=\frac{1}{6}\left[\begin{array}{l}(-B)\left\{4(y_k-y_{k-1})\sum_{i=1}^{k-1}p_i+\left(\underline{y}_k-\underline{y}_{k-1}\right)\sum_{i=1}^{k-1}\underline{p}_i+\left(\overline{y}_k-\overline{y}_{k-1}\right)\sum_{i=1}^{k-1}\overline{p}_i\right\}\\ \quad +C\left\{4(y_k-y_{k-1})\sum_{i=k+1}^{n}p_i+\left(\underline{y}_k-\underline{y}_{k-1}\right)\sum_{i=k+1}^{n}\underline{p}_i+\left(\overline{y}_k-\overline{y}_{k-1}\right)\sum_{i=k+1}^{n}\overline{p}_i\right\}\\ \quad +C\left(4y_kp_k+\underline{y}_k\underline{p}_k+\overline{y}_k\overline{p}_k\right)-C\left(4y_{k-1}p_k+\underline{y}_{k-1}\underline{p}_k+\overline{y}_{k-1}\overline{p}_k\right)\end{array}\right]$$, A-B+D=C

$$=\frac{1}{6}\left[\begin{array}{l}(-B)\left\{4(y_k-y_{k-1})\sum_{i=1}^{k-1}p_i+\left(\underline{y}_k-\underline{y}_{k-1}\right)\sum_{i=1}^{k-1}\underline{p}_i+\left(\overline{y}_k-\overline{y}_{k-1}\right)\sum_{i=1}^{k-1}\overline{p}_i\right\}\\ \quad +C\left\{4(y_k-y_{k-1})\sum_{i=k}^{n}p_i+\left(\underline{y}_k-\underline{y}_{k-1}\right)\sum_{i=k}^{n}\underline{p}_i+\left(\overline{y}_k-\overline{y}_{k-1}\right)\sum_{i=k}^{n}\overline{p}_i\right\}\end{array}\right]$$

$$=\frac{1}{6}\left[\begin{array}{l}(-B+C)\left\{4(y_k-y_{k-1})\sum_{i=1}^{k-1}p_i+\left(\underline{y}_k-\underline{y}_{k-1}\right)\sum_{i=1}^{k-1}\underline{p}_i+\left(\overline{y}_k-\overline{y}_{k-1}\right)\sum_{i=1}^{k-1}\overline{p}_i\right\}\\ \quad +C\left\{4(y_k-y_{k-1})\sum_{i=k}^{n}p_i+\left(\underline{y}_k-\underline{y}_{k-1}\right)\sum_{i=k}^{n}\underline{p}_i+\left(\overline{y}_k-\overline{y}_{k-1}\right)\sum_{i=k}^{n}\overline{p}_i\right\}\end{array}\right]$$

Now, from (13) we have

$$\frac{4\,(y_k-y_{k-1})\sum_{i=1}^{k-1}p_i+\left(\underline{y}_k-\underline{y}_{k-1}\right)\sum_{i=1}^{k-1}\underline{p}_i+\left(\overline{y}_k-\overline{y}_{k-1}\right)\sum_{i=1}^{k-1}\overline{p}_i}{4\,(y_k-y_{k-1})\sum_{i=1}^{n}p_i+\left(\underline{y}_k-\underline{y}_{k-1}\right)\sum_{i=1}^{n}\underline{p}_i+\left(\overline{y}_k-\overline{y}_{k-1}\right)\sum_{i=1}^{n}\overline{p}_i}<\frac{C}{B+C}=\frac{s-c-{}^{-}C_d+C_s}{2s-c+C_h+C_s}$$

……………..………..… (14)

Similarly, from (11) we have

$$\frac{s-c-{}^{-}C_d+C_s}{2s-c+C_h+C_s}<\frac{4\,(y_{k+1}-y_k)\sum_{i=1}^{k}p_i+\left(\underline{y}_{k+1}-\underline{y}_k\right)\sum_{i=1}^{k}\underline{p}_i+\left(\overline{y}_{k+1}-\overline{y}_k\right)\sum_{i=1}^{k}\overline{p}_i}{4\,(y_{k+1}-y_k)\sum_{i=1}^{n}p_i+\left(\underline{y}_{k+1}-\underline{y}_k\right)\sum_{i=1}^{n}\underline{p}_i+\left(\overline{y}_{k+1}-\overline{y}_k\right)\sum_{i=1}^{n}\overline{p}_i}$$

…………………….…. (15)

From, (14) and (15) we have

Lower limit < $\frac{s-c-{}^{-}C_d+C_s}{2s-c+C_h+C_s}$ < Upper limit

Hence, from equation (10) and (11), we get the optimality condition of the optimal order quantity $y_k=\left(\underline{y}_k,y_k,\overline{y}_k\right)_{LR}$.

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