ISSN: 0975-833X

Available online at http://www.journalcra.com

International Journal of Current Research
Vol. 8, Issue, 09, pp.39334-39349, September, 2016

INTERNATIONAL JOURNAL OF CURRENT RESEARCH

RESEARCH ARTICLE

ASSESSMENT OF TEACHERS BELIEFS AND THEIR PROBLEM SOLVING VIEWS IN THE TEACHING AND LEARNING OF MATHEMATICS IN SECOND CYCLE PRIMARY SCHOOLS (GRADES 5-8): THE CASE OF OROMIA REGIONAL STATE, ETHIOPIA

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ARTICLE INFO

Article History:
Received 07th June, 2016
Received in revised form
23rd July, 2016
Accepted 17th August, 2016
Published online 30th September, 2016

Key words:

Mathematical belief, Problem solving, Discover questions, Modeling.

ABSTRACT

Teachers' views of mathematics, mathematics teaching and learning affects how it should be presented in the classroom. In this research 172 mathematics teachers of grades 5-8 were given a survey that examined their views on mathematics and problem solving approach in mathematics classroom. Questionnaires, observations and interviews were used to collect data. Findings revealed that majority of the teachers had an absolutist view of mathematics (61%) and their view on problem solving approach was traditional (64%) in that they considered problem solving as an end. Majority of the surveyed teachers used routine exercises (76%) in mathematics classroom. If instruction is to be transformed school districts, the Ministry of Education, Teachers Training Institutions should identify teachers' beliefs on mathematics and problem solving approach which are essential for teachers' professional development and implement through staff development and other measures what they perceive mathematics to be and how it should be taught via problem solving approach.

Citation: Dr. Charinet Tuge and Habtamu Garoma, 2016. "Assessment of teachers beliefs and their problem solving views in the teaching and learning of mathematics in second cycle primary schools (grades 5-8): the case of Oromia regional state, Ethiopia", *International Journal of Current Research*, 8, (09), 39334-39349.

INTRODUCTION

A review of the literature reveals that problem-solving forms a very important aspect of the teaching and learning of mathematics. As the National Council of Teachers of Mathematics (NCTM)indicated, "Problem-solving is central to inquiry and application and should be interwoven throughout the mathematics curriculum to provide a context for learning and applying mathematical ideas" (NCTM, 2000: 256). Moreover, according to the NCTM "Problem solving should be the central focus of the mathematics curriculum. As such, it is a primary goal of all mathematics instruction and an integral part of all mathematical activity" (NCTM, 1989: 7). Emphasis on problem-solving is a result of educators' growing understanding of how students learn and the demands of a rapidly changing world. A problem-solving approach in mathematics helps students to build conceptual understanding. In a problem-solving approach students demonstrate depth of understanding by translating mathematical concepts into their own words and applying those concepts to unfamiliar situations. Students develop a greater understanding of mathematics through problem-solving when they are actively engaged. They do not simply learn mathematics as a body of facts, but learn how to use mathematics and to do what mathematicians do. School teachers are, therefore, encouraged by the Ministry of Education of Ethiopia as well as by the local educational bureaus in the country to incorporate problem-solving in their classroom practice. According to Anderson and White (2004) and Anderson (2000) teachers agree that problem-solving is an important life-skill for students to develop, of which they need to develop a whole range.

They support the focus on problem-solving in syllabus documents. However, there is little evidence of the use of challenging, unfamiliar and open-ended problems in mathematics classrooms in Ethiopia. Given the amount of policy advice and resource development, there are concerns about the limited opportunities for Ethiopian students to engage in problems other than those of low procedural complexity. Much emphasis is placed on exercises, which typically entail practicing a procedure (*e.g.,* an algorithm) or rehearsing specific facts or concepts (*e.g*., multiplication, facts or definitions) to build proficiency and quickly obtain a correct answer. This suggests that teachers' beliefs about the importance of problem-solving are not being supported by their actions in the classrooms. It is unclear why problem-solving seems to have a less prominent place in mathematics classrooms than is expected.

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Statement of the problem

According to Ernest (inRuttkamp, 2002) most teachers of mathematics have little regard for the philosophy of mathematics, and justify this attitude in two ways: Firstly, teachers of mathematics are more often concerned with mathematics in a local, as opposed to a global sense, that is, teaching particular concepts, facts or algorithmic skills, and they are less concerned with issues such as the nature of mathematics and the nature of truth in mathematics. Secondly, when a teacher does consider global questions concerning the nature and purpose of mathematics and its teaching, the more useful answers are not to be found in the literature of the philosophy of mathematics, because the teacher is more likely to look at discussions of the nature and purpose of mathematics in mathematics education texts and journals. These educational sources do not give a well-rounded account of mathematics, but rather discuss a one-sided view from a fixed philosophical position (Ernest, in Ruttkamp, 2002). Despite these arguments, Ernest insists that a consideration of the views of the various schools of thought with regard to the philosophy of mathematics is of value for the teaching and learning of mathematics. To a large extent this argument holds true also in the educational circles of Ethiopia. Research conducted by Pepin (1999) indicates the relevance of the philosophy of mathematics for the teaching and learning of mathematics. She provides evidence that the teachers' instructional practices, especially in mathematics, reflect their conception of the subject matter. Pepin studied the conceptions and works of mathematics teachers in three countries, namely England, France and Germany. The study explored the issues concerning conceptions of mathematics, of mathematics teaching and learning, and the way in which mathematics teachers' classroom practices reflect their conception of mathematics and its teaching and learning. Pepin's findings suggest that teachers' conceptions are manifested in their practices and can be traced back to the educational trends of mathematics and mathematics education, as well as to personal constructions. The findings also suggest that teachers' pedagogical style are a personal response to a set of assumptions about the subject (mathematics) and its teaching and learning, to a set of educational and philosophical traditions, and of institutional and societal constraints. According to Ernest (1998) the philosophy of mathematics has a range of perspectives, of which one may be termed the ‘absolutist’ perspective. The absolutist perspective views mathematics as an objective, absolute, certain and incorrigible body of knowledge, which rests on the firm foundations of deductive logic. Many of the claims of the absolutist philosophy follow from the identification of mathematics as a rigid logical structure, introduced for epistemological purposes. An absolutist-like view may be communicated in school by giving students mainly unrelated routine mathematical tasks which involve the application of learnt procedures, and by stressing that every task has a unique, fixed and objectively correct answer, coupled with disapproval and criticism of any failure to achieve this answer.

Another promoted or more "fashionable and fruitful" (Phillip, 2000) conception of mathematics among teachers, is ‘fallibilism’, "…the image of mathematics, which is growing in popularity among mathematics educators" (Roulet, 1998). Even the reforms proposed by the NCTM (in Golafshani, 2002) are rooted in fallibilism and they support the transition of teachers' mathematics conceptions from the traditional absolutist view to a non-traditional fallibilist view (Roulet, 1998). According to the fallibilist view mathematics is a dynamic subject to be explored and investigated, termed a *contemporary* teaching approach. Classroom practices associated with this perspective usually involve group-work and the use of non-routine questions that promote mathematical thinking, and the development of problem-solving skills. This teaching approach may be accompanied by a belief that problem-solving is a *means* to learn mathematics.
It is relevant to identify beliefs as well as other factors or constraints which may prohibit teachers from implementing problem-solving techniques in their classrooms. Most of the identified constraints from previous studies can be grouped into four broad categories: those relating to the teachers themselves (*e.g.* Jaworski, 1991), to students (*e.g.* Thompson, 1992), to the school culture (*e.g.*, Hoyles, 1992), and to the requirements of the system (*e.g.* Clarke, 1993). According to Rickard (2005), clarifying what problem solving is and distinguishing between problems and exercises, is important in advancing the problem-solving approach in the teaching and learning of mathematics. He also mentions that if a teacher views problems as computation exercises and problem-solving as using memorized algorithms to determine the correct answer, then mathematics in such a classroom is typically taught and learnt as memorizing definitions and procedures that are disconnected. The latter (viewing problem solving as computation exercises) is rarely based on teachers’ and students’ understanding of the underlying concepts. According to Wasman (in Rickard, 2005) the way teachers shape the curriculum to meet the needs of their students is determined by their view of mathematics content, problem-solving, classroom discourse, as well as by other factors. From the above analysis of how mathematics teachers may view mathematics, mathematics teaching, mathematics learning and the impact of beliefs and conceptions on teachers’ classroom practices; the following research problems were formulated:

Research problems

A. Do mathematics teachers view mathematics as a fixed body of facts to be delivered by teachers and internalized by students, or as a dynamic subject to be explored and investigated?
B. How do teachers view the problem-solving approach in mathematics classrooms?
C. What factors prohibit teachers from implementing a problem-solving approach?
D. What support do teachers need for implementing the problem-solving approach in their classrooms?

Definitions of terms

- **Belief:** A belief is the acceptance of the truth or actuality of anything without certain proof. It is a mental conviction, and beliefs are used to describe a wide range of affective responses to mathematics (McLeod, 1992)

Mathematics and Mathematics Education: Weisstein, (1999:1142) defines ‘mathematics’ as follows:

“Mathematics is the broad ranging field of study in which the properties andinteractions of idealized objects are examined. Whereas mathematics begins merely as a calculational tool for computation and tabulation of quantities, it has blossomed into extremely rich and diverse tools, terminologies and approaches which range from the purely abstract to the utilitarian.'' According to Cook (1995) people often equate mathematics with arithmetic. Arithmetic is concerned with numbers. When considering the mathematics curriculum, many teachers focus on computational skills and believe that they constitute the full set of competencies that students must have in mathematics. Traditionally, the major emphasis of the pre-kindergarten through grade 8 mathematics curriculum has been to teach children arithmetic, how to add, subtract, multiply and divide whole numbers, fractions, decimals, and percentages. Mathematics involves more than computation. Mathematics is the study of patterns and relationships; a science and a way of thinking; an art characterized by order and internal consistency; a language, using carefully defined terms and symbols; and a tool. Teachers and other educators working together to improve mathematics education must explore the broader scope of mathematics, as indicated above.

The Problem-solving Approach

In this study the problem-solving approach refers to teaching mathematical topics in problem-solving contexts and enquiry-oriented environments which are characterized by the teacher "…helping students construct a deep understanding of mathematical ideas and processes by engaging them in doing mathematics: creating, conjecturing, exploring, testing, and verifying." (Lester, F.K.Jr., Masingila, J.O., Mau, S.T., Lambdin, D.V., dos Santon, V.M. and Raymod, A.M.1994:154).

Structure of the Ethiopian School System

The current Ethiopian school structure consists of three phases: Phase one comprises grades 1–8, is called *Primary School* and the age level of the pupils is 7 to 14. This phase is further divided into two sub-phases termed first cycle primary school (grades 1 to 4) and second cycle primary school (grades 5 to 8). The second phase is called *General Secondary School* and includes grades 9 and 10. The age level of the pupils in these grades is 15 to 16. The third phase is called *Preparatory Secondary School,* grades11 to 12 with an age level of 17 to 18.

Research design

In this research a quantitative approach was employed and a questionnaire, interviews and classroom observations were used to gather data. The questionnaire consisted of five categories; these are, beliefs regarding problem solving, beliefs concerning mathematics, the use of different kinds of problems, teaching strategies, and open-ended problems. In beliefs regarding problem-solving, some of the constructs are consistent with progressive problem-solving views, and some are consistent with traditional problem-solving views. Likewise, in beliefs concerning mathematics some of the items are consistent with the absolutist view of mathematics, and others with the fallibilist view. In teaching strategies, parts of the items are traditional teaching, and others are progressive teaching strategies.

Research Instruments

A questionnaire entitled "Problem Solving in Mathematics Teaching: Teachers Views' and Practices", was compiled by the researcher, with reference to sources like Von Zoest, *et al.,* (1994) and Anderson, (1996). It includes fifty four questions that make use of a Likert-type scale and six open-ended questions. The Likert scale questions include items that relate to beliefs regarding problem-solving, mathematics, and the use of question-type and teaching strategies. Open-ended questions expected from the teachers to indicate question types recently used in the classroom, to explain why they prefer particular kinds of questions, and to describe the professional development needs of the staff members at their schools in relation to the implementation of problem-solving approaches. The respondents were also provided with the opportunity to reject problem-solving approaches to their teaching by responding to the following statement, "Give your opinion on the following statement: People who advocate problem-solving in mathematics obviously don't work in classrooms. It is a waste of time." Responses to this statement enabled the researcher to check for consistency with responses to earlier questions on the teachers' beliefs. To encourage teachers to respond as honestly as possible, questions on beliefs were presented as representing the views of two imaginary teachers, Naol and Ayantu. Naol's statement suggested that he uses problem-solving as an end in teaching mathematics. This perspective is based on the traditional approach, namely teaching mathematics as a body of knowledge to be internalized by students (Thompson, 1992). Ayantu's statement suggested that she uses problem-solving as a focus for learning mathematics. This perspective is based on a more contemporary view, namely teaching about problem-solving and via problem-solving; mathematics is considered to be a dynamic subject to be discovered by students (Clark, 1993). In order for teachers to more readily relate to each of these perspectives the statements were presented in the context of teaching the addition of rational numbers to grade five students. An example of a statement attributed to Naol was "Students must learn basic facts about rational numbers before theydo applications and unfamiliar problems", and an example of a statement attributed to Ayantu was "Mathematics lessons should focus on problems rather than on the practice of algorithms". Respondents were required to record their level of agreement with each of the statements on the following four-point scale: 'strongly agree', 'agree', 'disagree' and 'strongly disagree'. A neutral position of 'unsure' was not used, in an effort to encourage respondents to make definite decisions of either agreement or disagreement. An important consideration in the design of the instrument was to ensure that each item was interpreted in the same way by all the respondents, and that the meaning of the terms and expressions was clear. An earlier investigation highlighted the need to address the teachers' interpretation of the term 'problem' (Anderson, 1996). To overcome the diverse meanings attributed to this term, a set of question types and an example of each were presented at the beginning of the questionnaire as background information. The question type was chosen on the basis of typical questions or problems that are discussed in the literature (Clarke and McDonough, 1989) as well as mathematics textbooks for grades 5-8 in Ethiopia. It was indicated that the term 'exercise' was to be used for algorithms, and was not referred to as a 'problem' in this questionnaire. The problem types were 'application problems', 'open-ended problems' and 'unfamiliar problems'; the later was assumed to be a problem type the students haven't seen before.

The question types were those listed below (Table 1), and teachers were required to indicate the frequency of usage:

'hardly ever', 'sometimes', 'often' and 'almost always'. This was intended to gauge the type of questions teachers prefer to use; a subsequent question requested reasons for preferred choices. Most teachers probably use each of these types of questions at some stage in their teaching, so it is the frequency of use which provides an indication of the importance a teacher places on the role of problem-solving in learning/teaching mathematics.

Table 1. Examples of question types

Types of questions	Example
Exercise	3/5 + 5/2=________
Open ended problems	□ □ □ □ ________ 1 3 4
Application problems	If there are 23 female students in grade 5, 23 female students in grade 6 and 21 female students in grade 7 at a certain school, what percentage of the students are females in these three sections, where the total number of students is 200?
Unfamiliar problems	There are Donkeys and Dukes in grassland and altogether there are 23 heads and 68 legs. How many Donkeys and Dukes are there?

In another category of the questionnaire twenty items that relate to teaching strategies were listed. The aim was to explore teachers' practices when teaching mathematics, with most items specifically concentrating on problem-solving approaches. The items were chosen on Strategies included the grouping of students, kinds of problem-solving tasks, teacher explanations about problem-solving, and the use of concrete materials. Respondents were requested to rate the frequency of use of each of the strategies by making use of the comments, 'hardly ever', 'sometimes', 'often' and 'almost always'. It was anticipated that the frequency with which teachers use these strategies would provide an indication of their perceived importance and usefulness.

Participants

School principals from secondary schools in the Jimma zone of the Oromia Regional Government were invited to participate in the study. There are about 40 second cycle primary schools in the Jimma zone. Of the 29 principals contacted, four did not agree to participate, because of other priorities or participation in earlier studies. The remaining 25 principals invited the researcher to address the mathematics staff on the study, and agreed to distribute the questionnaires to the teachers. This initial distribution yielded a total of 132 completed questionnaires from the mathematics teachers in 25 schools. To increase the size of the sample, further groups were invited to participate. These included a group of first summer mathematics teachers participating in a teacher in-service course while completing a Bachelor of Education degree in mathematics. This yielded a total of 174 respondents. Many teachers felt uncomfortable and possibly even threatened by the idea of describing their teaching practices. In an effort to overcome the potentially threatening nature of completing the questionnaire, the researcher spent considerable time talking to teachers about the value of providing honest information. It was made clear to the respondents that all views were valuable and that participation was voluntary. Respondents were encouraged to request further information if required, and in addition, the researchers indicated that he would report back to the participating teachers when the results had been analyzed. It seemed that most teachers who responded had a particular interest in teaching mathematics or in issues surrounding the use of problem-solving approaches. Some of the teachers revealed that they had not heard of the 'problem-solving approach' as a philosophy of teaching mathematics and were interested in obtaining reference material to read and to develop a better understanding of how to apply a problem-solving approach in the classroom. The final group of respondents could be considered as a representative sample of second cycle primary school teachers in the Jimma zone since more than 50% of the mathematics teachers completed and returned the questionnaire. From a total of 205 mathematics teachers in the schools 174 participated in the research.

Process

In the first stage of the research data were collected from 174 teachers in the Jimma zone of the Oromia Regional Government of Ethiopia, using a questionnaire designed by the researcher. The second stage of the researchers involved interviews and classroom observations in an effort to gain additional information about the constraints the teachers experience in their problem-solving approaches, as well as the opportunities that the approaches offer. A number of teachers, advocating a variety of beliefs, were selected for participation in the second stage of the study. These teachers were interviewed about their questionnaire responses in order to confirm the interpretation of items and to explore in more detail some of the issues that had been raised. From the initial interviewees, two teachers were selected to participate in classroom observations and further interviews. These teachers had highlighted a variety of important issues relating to possible constraints and opportunities that impacted on their efforts to implement problem-solving approaches. A combination of the information collected from the questionnaires, the interviews and the observations provided a detailed picture of the factors that influenced the beliefs and practices of these two teachers. The use of three data sources, namely the questionnaire, the interview and classroom observation, provided triangulation in this study, triangulation not merely to provide verification but as a means of highlighting the phenomenon from different angles and thus providing more and better evidence.

Scoring

The items in the survey that were positive to contemporary problem-solving views, namely teaching *about* problem-solving, teaching *through* problem-solving and problem-solving as a process were scored from 4 (strongly agree) to 1 (strongly disagree), and this was reversed for items supporting the traditional problem-solving views, namely teaching *for* problem-solving, and problem-solving as an end in itself. For example, the item "A vital task for the teacher is motivating children to solve their mathematical problems" was in agreement with contemporary problem-solving views, and hence it was scored from 4 (strongly agree) to 1 (strongly disagree). Similarly, items that are positive to the fallibilist philosophy of mathematics were scored from 4 (strongly agree) to 1 (strongly disagree), whereas items consistent with the absolutist philosophy of mathematics were scored from 1 (strongly agree) to 4 (strongly disagree). In the 'teaching strategies' section of the questionnaire items consistent with progressive teaching strategies were scored from 4 (strongly agree) to 1 (strongly disagree), and these scores were reversed for items that are true to traditional teaching strategies. In the 'use of different kinds of problems' section of the questionnaire, there were four items. These were the use of application problems, open-ended problems, unfamiliar problems and the use of exercises. The first three were scored from 4 (almost always) to 1 (hardly ever). These scores were reversed for the item 'use of exercises'. The scores were summarized for each section of the questionnaire. The respondents' score in each section of the questionnaire was the sum of the scored alternatives endorsed by the person. High scores on the section of the questionnaire 'Beliefs regarding problem-solving' reflected support for the view that problem-solving is a process, namely teaching via problem-solving and teaching about problem-solving.

The highest score any participant could obtain in this section of the questionnaire was 44 and the minimum score possible was 11. There were 11 questions. A high score on the section of the questionnaire 'Beliefs concerning mathematics', reflect approval of the fallibilist philosophy of mathematics as compared to the absolutist philosophy of mathematics. The highest score any participant could obtain in this section was 80 and the minimum score possible was 20. There were 20 questions. A high score on the section of the questionnaire 'The use of each of the question types', reflected approval of non-routine problems in the teaching and learning of mathematics as compared to the use of exercises. The highest score any participant could obtain in this section was 16 and the minimum score possible was 4. There were 4 questions. A high score on the section of the questionnaire 'Teaching strategies' reflect approval of progressive teaching strategies as compared to traditional teaching methods. The highest score any participant could obtain in this part was 80 and the minimum score possible was 20. There were 20 questions. Descriptive analyses were used to describe the participants' views regarding the constructs in each part of the questionnaire.

Data presentation, analysis and interpretation

The results presented in this chapter are inferred from the three types of data, namely those from the questionnaire, the interviews, and observations of selected teachers.

The results relating to each of the research questions are presented and discussed in turn. Furthermore, the results pertaining to each of the research questions will be compared in order to obtain an integrated picture of the teachers' beliefs regarding mathematics and their views of problem-solving and teaching techniques. The interview and observation data were also compared to ascertain to what extent the interviewees' reported beliefs are related to their observed practice and to their responses on the questionnaire survey. In the end, the results from the investigation were compared with other research findings.

Research question 1:

Beliefs regarding mathematics

Several of the items in the questionnaire survey (16 items) were on the subject of beliefs regarding mathematics. Each of the items in this category corresponds with either the fallibilist view or the absolutist view of mathematics. These items are intended to identify respondents' beliefs regarding mathematics. For each of the items the percentage of teachers who agreed or strongly agreed with the statement is indicated in Table 2. The data indicate that the majority of the teachers who took part in the survey view mathematics as absolute and certain (70%), and independent of human culture (67%), and they consider mathematics teachers and textbooks as authoritative sources of mathematical knowledge in the classroom (67%). The notion that "mathematics is a changing social product" was refuted by 67% of the respondents. Moreover, the data in Table 2. reveal that a significant number of the respondents believe that mathematical objects have their own independent existence (63%) and that mathematical objects are discovered (66%). The majority of the respondents also view it as important that a mathematics teacher has to understand how concepts and skills relate to one another, and they teach accordingly (64%). Likewise, the majority indicated that mathematics is a game played with symbols (64%), and they considered axioms to be the foundation of all mathematical knowledge (66%). In Table 2, items 3, 5,6,7,8,9,11 12, 14 and 16 relate to an absolutist view of mathematics whereas the remaining items reflect a fallibilist view of mathematics. Of all the items reflecting absolutism, in only the case of items 3 and 16 less than 63% of the teachers indicated that they agree or strongly agree. These two items reflect percentages of almost 50%. Likewise, three of the items that have a bearing on fallibilism indicate percentages of more than 50 (items 1, 2 and 15). These results indicate that some of the respondents have contrasting views. Indeed, a few of them appeared to support a section of the items that are consistent with the absolutist view, and at the same time support some other constructs that are indicators of the fallibilist view. For example, the item "Mathematical objects have their own realm of existence" attracted 63% of the responses, while the item reflecting a fallibilst view "Most students forget mathematics procedures and so it is best to leave them to work out their own methods first" attracted 62% of the responses. In order to categorize the respondents' views as either fallibilist or absolutist, the 'agree' or 'strongly agree' responses were combined, and similarly the 'disagree' or 'strongly disagree' ones. Moreover, the items in the survey of 'Beliefs regarding mathematics' that relate to the fallibilist view were scored from 4 (strongly agree) to 1 (strongly disagree), while the scores were reversed for items supporting the absolutist view. There were 16 items in the survey and the maximum score a respondent could gain was 64 and the minimum 16. Those respondents who scored less than the mean value ((64+16)/2=40) are supposed to have an absolutist view of mathematics, and respondents with scores greater or equal to 40 are included in the fallibilist view category. Table 3 summarizes the respondents' beliefs regarding mathematics. The data in Table 3 indicate that the majority of the teachers who were surveyed could be labelled as teachers who hold an absolutist view of mathematics. In conclusion the survey on the beliefs regarding mathematics revealed that the majority of the teachers who took part in the survey uphold an absolutist view of mathematics. Indeed, these teachers view mathematics as a fixed body of facts to be delivered by teachers; mathematical objects have their own realm of existence independent from human beings. Mathematical objects were discovered and the history of mathematics is not related to the teaching and learning thereof. They also indicated that mathematics teachers and textbooks are reliable sources of knowledge and that mathematics is a game played with mathematical symbols, such as numbers. Ernest (1989b) outlined three views of the nature of mathematics, namely the platonic view, which views mathematics as a static but unified body of knowledge; mathematics is a monolith immutable product, it is discovered, not created. The instrumentalist view views mathematics as a bag of tools made up of an accumulation of facts, rules and skills to be skilfully used by trained artists in the pursuance of some external end; mathematics is a set of unrelated but utilitarian rules and facts. The problem-solving view views mathematics as a dynamic, problem driven, continually expanding field of human creation and invention; mathematics is viewed as a process of enquiry and coming to know, not a finished product, for its results remain open to revision. Comparing Ernest's (1989b) view of mathematics with the results from Tables 2 and 3, it is evident that 61% (Table 3) of the respondents hold either a platonist or instrumentalist view of mathematics, which can be called an absolutist view, whereas the remaining 39% of the respondents hold the problem-solving view of mathematics, which is consistent with the fallibilist view.

Research question 2:

Beliefs concerning problem-solving

Ten items of the questionnaire were directed at obtaining an indication of the teachers' views on problem-solving. Some of the items (items 1, 2, 3, 4, 7 and 10) underscore the traditional problem-solving view, where a problem-solving approach is considered as an end. The others (items 5, 6, 8 and 9) emphasize the contemporary problem-solving view where a problem-solving approach is considered a process. Table 4. indicates the percentage of teachers who agree or strongly agree with each of the statements. The data suggest that a majority of the surveyed teachers, at least 65% of them, agreed with the various items reflecting the traditional view. Respectively 90% and 83% of the teachers entertain the view that 'number facts and some basic mathematics' and 'examples by the teacher' are needed before students can tackle problems. Also, many teachers favoured the notion that problems can motivate students (86%) and that students can benefit from discussing their solution strategies with one another (86%). In addition, many of the teachers agreed that application and unfamiliar problems are best left to the end of the topic in mathematics (67%).Encouraging students to solve their own mathematical problems as being a vital task of a mathematics teacher was rejected by a great number of the respondents (86%). Given the overwhelming support for the item on basic number facts before students do application and unfamiliar problems, it is clear that most mathematics teachers in the survey believed that students should learn basic facts such as axioms, definitions and theorems before any example is given by the teacher, and after some examples are given, the students have to practice exercises from the textbooks. This is not surprising as almost all the textbooks for grades 5-8 are compiled in such a way that for any new topic definitions and other basic facts are given first, after which exercises based on the examples follow. This could be one of the reasons for the majority of teachers favouring the traditional view of the problem-solving approach as demonstrated in Table 4. Sixty-five percent of the respondents were of the opinion that "Solving mathematics problems is a procedure to be memorized, practiced, and habituated" is important. This result indicates that there is support for a more traditional practice of focusing on students' memorizing of procedures, algorithms, and of practicing the use of these procedures to become familiar with them. The great support for the item about language difficulties indicates that most teachers believe that problems may be more difficult because of the language that is used. This is not astonishing, as placing mathematical problems in real-life contexts usually requires the use of much more language than is often required in mathematical exercises. Interestingly enough, many of the teachers favoured items 6 and 8 in Table 4. These items reflect contemporary views of the problem-solving approach, namely either teaching *through* problem-solving or teaching *about* problem-solving. The responses indicate that teachers recognized the fact that students can learn from one another by discussing their solution strategies, and also acknowledged that problems have some motivational value for students' learning. The items in this section were separated according to whether they have a bearing on either of the views of problem-solving by the following mechanism: The items in the survey that reflect on the contemporary problem-solving view (Teaching *about* problem-solving and teaching *through* problem-solving; problem-solving as a process) were scored from 4 (strongly agree) to 1 (strongly disagree), and this was reversed for items supporting the traditional problem-solving view (Teaching *for* problem-solving; problem-solving as an end in itself). Since there were 10 items in the survey, the maximum score a respondent could attain was 40 and the minimum 10. Those respondents who scored less than the mean value ((10+40)/2=25) were considered to hold a traditional problem-solving view and respondents with scores higher or equal to 25 were included in the progressive problem-solving view category. For the purpose of analysis, the responses of either 'agree' or 'strongly agree' were combined, as were those for 'disagree' or 'strongly disagree'. Table 5 gives a summary of this analysis. The data in Table 5 indicate that the majority of the teachers who were surveyed could be labelled as teachers who hold a traditional problem-solving view.

In conclusion, the survey on the problem-solving views of the teachers revealed that the majority of the surveyed teachers believe that students should learn basic facts before they do application and unfamiliar problems, examples should be given by the teacher before students are made to solve problems, and application and unfamiliar problems should be left to the end of the topic. Moreover, a teacher has to provide students with clear and concise solution methods. The goals of problem-solving for the majority of the respondents were to teach the concepts so that students will later be able to apply the knowledge to problem-solving situations. This approach has been commonly seen in the textbooks of grades 5-8 in Ethiopia, where a page of skill practice is followed by a story problem that applies the same concept. The practice of the majority of teachers is consistent with what Schroeder and Lester (1989) call teaching *for* problem-solving, or with Nunokawa's (2005:328) view of "…emphasizing the application of the mathematical knowledge the student has", both of which are consistent with a traditional problem-solving view. This result implies that only 36% of the respondents either have a teaching *through* problem-solving view or a teaching *about* problem-solving view (Schroeder and Lester 1989), or a view of emphasizing management solving processes themselves (Nunokawa 2005), which are all consistent with a contemporary problem-solving view.

The use of each of the question types

The items that explored how often teachers use different types of questions also revealed a range of responses from teachers.

Table 4.5 lists the four question types and the frequency of use expressed as percentages. Overall, open-ended problems and unfamiliar problems were less frequently presented to students than application problems and exercises. These results suggest that while the majority of teachers use more exercises in mathematics lessons, a rather small percentage of the teachers appear to use open-ended and unfamiliar problems on a regular basis. These teachers (teachers who used open-ended and unfamiliar problems) view problem-solving as a *means* rather than as an *end* in the teaching of mathematics. To categorize the respondents as either 'exercise users' or 'problem users' based on their responses, the alternatives on the use of exercises were scored from 1 (almost always) to 4 (hardly ever). Those question types called 'problems' (open-ended, application and unfamiliar problems) were scored from 4 (almost always) to 1 (hardly ever). The maximum score a respondent could attain was 16 and the minimum was 4. Those respondents who scored less than the mean (10 = (4+16)/ 2) were considered to be teachers who are more intent on using exercises while respondents with scoreshigher than or equal to 10 were considered to be 'problems users'. The outcome of this classification is included in Table 7. From Table 7 it can be inferred that a greater number of the respondent teachers are inclined to use exercises as opposed to problems. In the following table an analysis is given of a comparison between respondents' problem-solving views and their actual use of the various problem types. The comparisons revealed that the majority of the respondents who sustain a traditional problem-solving view (Table 5) favoured the use of exercises.

The data from Table 8 indicate that 72 % of the respondents who hold a traditional problem-solving view favour the use of exercises, whereas 57% of the respondents who favour a contemporary problem-solving view supported the use of problems (open-ended or unfamiliar problems). This implies that the majority of respondents believe that the goal of problem-solving is to teach the concepts so that students will later be able to apply the knowledge by doing exercises (story problems) that apply the same concepts as the given examples. For this group problem-solving comprises giving examples and then making students practise the examples by means of exercises that have a step-by-step solution strategy and one correct answer. From Table 8 it can also be inferred that 31 (about 27%) of the respondents who court a traditional problem-solving view reported that they use open-ended or unfamiliar problems in their classrooms, whereas 27 (about 47%) of the respondents who hold a contemporary problem-solving view reported that they regularly use exercises in their classrooms. This result gives the impression that some of the respondents either have difficulty in identifying their problem-solving strategies, or their views and their classroom practices are not in agreement.

Respondents' teaching strategies

Seventeen items in the questionnaire concern the issue of teaching strategies. Each of the items in this category corresponds with either the traditional or the contemporary teaching strategy of mathematics. The traditional teaching strategy employs a problem-solving approach as an *end,* and the contemporary teaching strategy as a *process*. These items are intended to ascertain respondents' teaching strategies. For each of the items the percentage of teachers who often or almost always made use of the strategies is indicated in Table 9. The data from Table 9 indicate that most of the teachers explain in detail what the students have to do (74%); they believe that for students to be effective in learning mathematics they have to listen carefully to what the teacher is explaining (69%); they accept the notion that teaching mathematics is to demonstrate procedures (74%), and to give the students examples and assignments to practice the procedures that were demonstrated (74%). Items 12, 15, 16, and 17, which were supported by at least 50% of the respondents, refer to traditional teaching strategies; item 17 specifically, refers to a teaching strategy that is called 'content-focused with an emphasis on performance' (Van Zoest *et al.*, 1994). The other three items (2, 15 and 16) support a platonist view of the nature of mathematics (Ernest, 1989b) More than 50% of the respondents entertain beliefs consistent with the latter view, or an instrumentalist view of the nature of mathematics, which is consistent with traditional teaching strategies. The results also indicate that the majority of the teachers abandoned the use of open-ended and unfamiliar problems in the classroom, and this implies that the teachers mostly used exercises in their classes. This result confirms that the majority of teachers supported the traditional teaching strategies. It is also evident from Table 9 that more than 50% of the teachers did not give the students many opportunities to clarify, interpret and attempt to construct solutions of their own and they did not teach certain aspects of problem-solving like drawing a diagram, making a list, etc. The majority of teachers agreed that they do not encourage students to make generalizations about rules and concepts using a problem-solving approach. It is also evident that the teachers did not permit the students to present their own problems and to discuss problem-solving strategies. The teachers did not seem interested in how students think, and did not produce alternative ideas to what the students thought. They supported the notion that students have to grapple with their problems, and accepted their responses in a non-evaluative way. All the items in Table 9, with the exception of items 2, 15, 16 and 17, reflect a problem-solving approach or contemporary teaching strategies (Van Zoest, *et al.,* 1994; Cobb *et al.,* 1991; Lester *et al.,* 1994). The results from the empirical investigation indicate that the majority of teachers felt that they are responsible to clarify, interpret and construct different solution strategies for the students in the classroom. It seems that the majority of teachers did not directly teach certain aspects of the problem-solving approach whereas, according to Evan and Lappin (1994), one of the methods of creating a good problem-solving approach is to directly teach some features of a problem-solving approach such as guessing, systematic listing, drawing graphs or looking for related easier problems. Another characteristic of the problem-solving approach is that it can be used to encourage students to make generalizations about rules and concepts, a process which is central to mathematics (Evan and Lappin, 1994) Only 49 % of the teachers who took part in the survey acknowledged the use of a problem-solving approach to make generalizations about rules and concepts, implying that the majority of teachers believe that they are responsible to present generalizations of rules and concepts, which is in contrast to a progressive teaching strategy. In order to categorize the respondents' teaching strategies as either traditional or progressive, the 'almost always' and 'often' responses were combined, as well as the 'sometimes' and 'hardly ever' ones. Moreover, the items that relate to progressive teaching strategies were scored from 4 (almost always) to 1 (hardly ever) while the scores were reversed for items supporting the traditional teaching strategies. From the 17 items in the survey, the maximum score a respondent could obtain was 68 and the minimum was 17.

Table 2. Percentage of teachers who agree or strongly agree with statements on 'Beliefs regarding mathematics'

No	Item	% teachers who agree or strongly agree
1	Most students forget mathematics procedures and so it is best to leave them to work out their own methods first.	62%
2	All mathematics problems should challenge students, based on their previous experience.	52%
3	Only a few naturally able (gifted) students can learn mathematics successfully.	47%
4	The history of mathematics is relevant to the teaching and learning of mathematics	43%
5	Mathematics is a body of absolute and certain knowledge.	70%
6	Mathematics is independent of human culture.	67%
7	Mathematical objects have their own realm of existence.	63%
8	Mathematical objects are discovered.	66%
9	Mathematics teachers and mathematics textbooks are authoritative sources of knowledge in the classroom.	67%
10	Mathematics is a social construct.	36%
11	Mathematics is computation.	63%
12	It is important for teachers to understand the structured way in which mathematics concepts and skills relate to one another.	64%
13	Mathematics is a changing social product.	33%
14	Mathematics is a game played with mathematical symbols, such as numbers, variables ,etc.	64%
15	Axioms provided the basis for all mathematical knowledge.	66%
16	Students are considered to be receivers of knowledge in the teaching and learning of mathematics.	49%

Table 3. Beliefs regarding mathematics

Beliefs regarding mathematics	Number of respondents	Percentage
Fallibilist	68	39%
Absolutist	106	61%

Table 4. Percentage of teachers who agree or strongly agree with statements on 'problem-solving views'

No	Item	% of teachers who agree or strongly agree
1	Students should learn basic number facts before they do application and unfamiliar problems.	90%
2	Students cannot solve problems until examples are given to them by the teacher.	83%
3	Some students have trouble solving problems unless they know how to do the problem before they begin.	83%
4	Application and unfamiliar problems are best left to the end of the topic in mathematics.	67%
5	Some students find problem-solving difficult because of the language involved in the problem.	68%
6	Problems help students to become motivated to learn basic facts and algorithms because they can see a reason in the problem.	86%
7	Solving mathematics problems is a procedure to be memorized, practiced, and habituated.	65%
8	Children always benefit by discussing their solutions to mathematical problems with one another.	86%
9	An important task for the teacher is to motivate students to solve their own mathematical problems.	14%
10	It is the teacher's responsibility to provide students with clear and concise methods to solve mathematical problems.	72%

Table 5. Views of teachers on problem-solving

Problem Solving Views	Number of respondents	Percentage
Traditional	111	64%
Contemporary	63	36%

Table 6. Frequency of use of each of the question types (expressed as a percentage)

	Type	Almost always	Often	Sometimes	Hardly ever
1	Do you use exercises?	51%	20%	22%	6%
2	Do you use open-ended problems?	6%	8%	63%	24%
3	Do you use application problems?	20%	23%	40%	17%
4	Do you use unfamiliar problems	5%	7%	36%	51%

Table 7. Frequency of use of each of the question types

Question type	Number of respondents	Percentage
Exercises	132	76
Problems	42	24

Table 8. Comparison between problem-solving views and use of problem types

Problem solving view (number of respondents)	Use of question type(number of respondents)	Number and percentage of respondents who hold a traditional problem-solving view and favoured the use of exercises	Number and percentage of respondents who hold a contemporary problem-solving view and favoured the use of problems
Traditional (111)	Exercises (132)	N =81	N = 36
Contemporary(63)	Problems (open-ended or unfamiliar problems) (42)	72.3% (81/111)	57.1% (36/63)

Table 9. Percentage of teachers who often or almost always made use of the teaching strategies

No	Item	% of teachers who often or almost always used the strategies
1	You present unfamiliar and open-ended problems to the class with little indication of how to solve them.	27%
2	You explain in detail what the students have to do to solve the problems.	74%
3	You provide just enough information to establish the background of a problem and let the students clarify, interpret and attempt to construct one or more solutions to a problem by themselves.	49%
4	You teach some aspects of problem-solving , *e.g.* make a list, draw a diagram, guess, work backwards, etc.	47%
5	You accept correct/wrong answers in a non-evaluative way in a problem-solving approach in the classroom.	51%
6	You use a problem-solving approach to encourage students to make generalizations about rules and concepts.	49%
7	You present problems to students with little guidance of appropriate procedures.	51%
8	You allow students to spend a considerable time on one problem.	44%
9	You encourage students to identify their own problems and solve them themselves.	60%
10	You encourage students to struggle with problems.	65%
11	You present complex and thought-provoking problems.	32%
12	You let the class discuss the various methods of solution.	35%
13	You encourage students to record their own procedures and methods of solving problems.	28%
14	You provide students with interesting problems to investigate in small groups.	43%
15	Listening carefully to the teacher explaining a mathematics lesson is the most effective way to learn mathematics.	69%
16	If a student's explanation of a mathematical solution does not make sense to the teacher it is best to ignore it.	52%
17	Teaching mathematics entails to demonstrate procedures, to show a few examples and to give students assignments for exercise and to practice.	74%

Table 10. Teaching strategies of teachers

Teaching strategy	Number of respondents	Percentage of respondents
Traditional	104	60%
Progressive	70	40%

Table 11. Conventional and activist teachers

Belief regarding mathematics (number of respondents)	Problem -solving view (number of respondents)	Use of problem type (number of respondents)	Teaching strategy (number of respondents)	Teacher type: conventional/ activist (number of respondents)
Absolutist (106)	Traditional(111)	Exercises(132)	Traditional (104)	Conventional (77)
Fallibilist(68)	Contemporary(63)	Problems(42)	Progressive(70)	Activist (40)

Table 12. Relationship of beliefs regarding Mathematics, Mathematics Teaching and Learning

Beliefs regarding mathematics (Ernest, 1989b)	Beliefs about mathematics Teaching (Van Zoest *et al.*, 1994)	Beliefs about mathematics learning (Ernest, 1989b)
Instrumentalist	Content-focused with an emphasis on performance.	Skill mastery, passive reception of knowledge
Platonist	Content-focused with an emphasis on understanding.	Active construction of understanding.
Problem-solving	Learner-focused	Autonomous exploration of own interests

Those respondents who scored less than the mean value (42.5) were considered to use a traditional teaching strategy and those who scored higher than or equal to 42.5, as using progressive teaching strategies. Table 10 summarizes the number and percentage of respondents on either of the teaching strategies. The data in Table 10 indicate that the majority of the mathematics teachers who took part in the survey use traditional teaching strategies. In conclusion, as is revealed in Table 10, the majority of the teachers do not make use of open-ended or challenging problems. The students are consequently missing many advantages that can be gained from making use of such problems. For example, Mayor, Cai and Grampp (1997) describe the importance of using open-ended and challenging problems as it accommodates diverse learning styles, and Hiebert, *et al.* (1997) state that traditional problem-solving approaches and traditional teaching strategies often do not meet the learning needs of many students. Hiebert, *et al.* (1997) continue by saying that learning by means of open-ended problem-solving helps students to develop an understanding that can be adjusted or transferred to new situations. The respondents' problem-solving views, beliefs regarding mathematics, use of different kinds of problems and their teaching strategies, were compared (Table 11). An overwhelming majority of the respondents with traditional problem-solving views confirmed that they employed traditional teaching strategies. Referring to Table 11, from the 111 respondents advocating a traditional-problem solving view, 104 confirmed that they use traditional teaching strategies. Table 4.10 summarizes the alliance among 'beliefs regarding mathematics', 'problem-solving views', 'the use of problem type' and 'teaching strategies', as stated by the respondent teachers. The teachers were grouped into two categories, namely 'conventional teachers' and 'activist teachers'. 'Conventional teachers' are those who have an absolutist view of mathematics, hold a traditional problem-solving view, use exercises in the classroom and appear to apply traditional teaching strategies, whereas 'activist teachers' are those who have a fallibilist view of mathematics, hold a contemporary problem-solving view, make use of problems (open-ended and unfamiliar problems) and progressive teaching strategies in the classroom.

Table 11 indicates that 117, all of the 'conventional' and 'activist' teachers, or 67% of them, confirmed that their beliefs regarding mathematics, their problem-solving views, and their use of problem types were consistent with their teaching strategies or classroom practice. Indeed, 77 (44%) of the respondents can be labeled as 'conventional teachers' seeing that they hold an absolutist view of mathematics, as well as a traditional problem-solving view, make use of exercises, and practice traditional teaching strategies in the mathematics classroom. And, 40 (23%) of the respondents can be labeled as 'activist teachers', because of their fallibilist view of mathematics, contemporary problem-solving views, their use of open-ended and unfamiliar problems in the classroom and their practice of different progressive teaching strategies. The remaining 57 (174-117=33%) respondents have contrasting views or were not able to identify their beliefs, as it is recognized that individuals may hold beliefs that they do not articulate and, in some cases, are not even aware of (Buzeika, 1996). It is assumed that teachers who hold an absolutist view of mathematics and a traditional problem-solving view, created a teacher-centred instructional environment to teach mathematics as rules to be memorized, and to describe mathematics as an infallible discipline. Such teachers tend to present mathematics to students in a way that suggests that mathematics is a linear subject, namely facts and skills related to numbers, which generally feature a paper and pencil activity. Since these teachers' main objective is the students' mastery of mathematical skills, it follows that a clear presentation of a step-by-step mathematical procedure with the emphasis on correct or wrong answers, and by making use of the simple exercises which are found in their textbooks, are practised. Teachers' holding a fallibilist view of mathematics and progressive problem-solving views appeared to adopt a teacher-student interaction mode of instruction by allowing students to explore and investigate while teachers act as facilitators. Problem-solving and the use of open-ended or unfamiliar problems are central to their teaching. For teachers advocating this view purposeful activity stems from problem situations that require reasoning and creative thinking. Consequently, the classroom takes on a progressive teaching atmosphere. Research has also shown that teachers with an absolutist conception of mathematics describe the subject as a vast collection of fixed and infallible concepts and skills (Romberg, 1992) and a useful but unrelated collection of facts and rules (Ernest, in Golafshani, 2002). For them mathematical knowledge becomes certain and absolute truths. It represents the unique realm of certain knowledge (Ernest, 1991). Ernest (1996:2) summarizes teachers' absolutist views about mathematics by saying: "Absolutist views of mathematics are not concerned to 'describe' mathematics or mathematical knowledge…thus mathematical knowledge is timeless…it is superhuman…it is pure isolated which happens to be useful because of its universal validity; it is value-free and culture-free, for the same reason." Research related to beliefs regarding mathematics and its teaching and learning is summarized in Table 12. Beliefs on the same row are regarded as theoretically consistent with one another, and those in the same column are regarded by some researchers as a continuum (Van Zoest *et al.,* 1994).

The results in Table 12 are consistent with the findings in this research in respect of the fact that the instrumentalist and platonist views of mathematics are grouped under the absolutist view, and the problem-solving view under the fallibilist view of mathematics. A teaching strategy that focuses on the content of mathematics and passive student learning that is based on the students' mastery of skills, rules and procedures characterizes the traditional teaching strategy. The results in Table 4.11 indicate that teachers advocating an absolutist view of mathematics tend to make their classrooms teacher-centred and are thus called 'conventional teachers'. At the same time, teachers with a student-focused teaching strategy and who help students to explore and construct their own knowledge, tend to make their classrooms student-centred and are called 'activist teachers', making the result consistent with the findings in Table 11. Other research findings have pointed out that a teacher's understanding of the nature of mathematics predicates his/her view of how teaching should take place in the classroom (Hersh, in Golafshani, 2002). It has also been indicated that mathematics teachers' views on the subject matter, as well as the teaching and learning thereof, influence their actions in the classroom (Dougherty, 1990). In spite of some literature suggesting that teachers' beliefs of mathematics and its teaching and learning are not related in a simple cause and effect way to their instructional practices (Pepin, 1999), and even though there exists some disparities between teachers' conceptions of the subject and their actual practice due to many constraints (*e.g*., fixed curricula, time pressure and many other external factors) (Raymond,(1993) many researchers and research findings indicate that there exists considerable agreement that beliefs influence action (Abelson, in Golafshani, 2002). Contextual constraints also exert significant influence on the enactment of beliefs (Sullivan & Mousley, 2001).They describe beliefs as influencing contextual constraints as well as being influenced by them, and argue that beliefs not only impact on practice, but practice also influences beliefs. Beswick (2003b) concludes that consistency cannot be expected when the contexts in which the teachers' beliefs are considered and their practices observed are not closely matched and according to Pajares, (1992) such context-matching must include correspondence between the degree of generality of the beliefs and practice being considered. That is, it is unreasonable to expect consistency between broad collections of beliefs that are not closely linked with a specific context, and practice that is not described in equally broad, contextually independent terms.

Research questions 3 and 4:

Factors prohibiting teachers from implementing a problem-solving approach, and the support they need in implementing this approach

Two of the open-ended questions on the survey demanded from the respondents to complete the following questions:

1. Why is there little evidence of problem-solving in the grades 5-8 mathematics classrooms?
2. What do you see as the professional development needs of teachers at your school?

The 174 survey responses from the teachers were analyzed and the comments grouped into three broad areas:

A. Teachers

Thirty five comments (20% of the responses) were made on teachers' knowledge and beliefs, or their confidence and competence. Of these, 18 referred to teachers' beliefs about mathematics or problem-solving and the role of problem-solving in learning mathematics. It was suggested by some respondents that teachers need to rethink their approaches to teaching mathematics and to move from more formal approaches to innovative, creative methods. However, others indicated that teachers need to be more aware of the benefits of problem-solving in the mathematics curriculum, since in their view, problem-solving is not seen as a legitimate mathematical activity. Problem-solving is seen by many teachers as not serious mathematics. Another group of the replies of teachers deals with teachers' confidence and competence, suggesting that teachers need support and encouragement. One teacher described the lack of teachers' control of students' learning in 'problem-solving' classrooms as follows:

"Many (teachers) are scared to do it because they are not sure themselves of whatthey should be telling the children to do. They feel they are losing a 'grasp' on exactly whatthe children are doing."

This notion of loss of control associated with a lack of confidence to allow students to explore and investigate freely is an important factor prohibiting teachers to adopt problem-solving approaches, particularly if problem-solving is considered to be a process of inquiry (Schwan Smith 2000). In addition, a teacher's competence in relation to problem-solving was considered to be a real issue for one respondent, who suggested:

"Teachers themselves need to develop competence in problem-solving processes and then through staff in-service training and interaction, devise appropriate problem solving strategies for their students."

The comments about teachers also suggest that it is necessary to consider teachers' beliefs and knowledge about problem-solving, and that teachers need to be encouraged and supported in their efforts, so that they can overcome the lack of confidence.

B. Practical Teaching Advice

Seventy-six percent of the respondents commented on the need for practical teaching advice relating to the following: modelling problem-solving approaches, increasing school resources, and providing information about incorporating problem-solving approaches into the teaching of mathematics through appropriate activities, relevant planning, and the use of a range of problems. Some respondents mentioned the need to have problem-solving approaches demonstrated to staff members by expert teachers. The modelling could take several forms, including peer mentoring, workshops by experts, or instructional videos. These comments indicate that while teachers are aware of recommendations to teach problem-solving, it may be difficult for them to know what it actually comprises, how to manage a class while students are doing problem-solving assignments, and what the role of the teacher is. This supports Lester *et. al*.'s (1994) suggestions that advice to teachers needs to include a clear description of the teacher's role as well as an indication of what problem-solving entails.

Another professional development requirement for many respondents was the availability and accessibility of good resource materials. Twenty-one teachers mentioned books, posters, commercial products, software, and concrete materials. Some suggested a demonstration of materials while others indicated the need for more equipment in classrooms. Although suitable material is available to support problem-solving approaches, yet, according to Clarke (1997), this is not sufficient to encourage the adoption of these approaches. Nineteen teachers commented on support to incorporate problem-solving into daily mathematics lessons and into the curriculum. Concerns were raised by several of these respondents in respect of adding problem-solving to an overcrowded curriculum. It was apparent from some of these comments that problem-solving is viewed as an added topic or an additional focus for teachers to incorporate into the curriculum. Several teachers indicated that time to implement new ideas is an issue and that schools need to recognize this; this includes time to plan and implement, as well as time to reflect on the success, or otherwise, of potential changes. Using problem-solving approaches would take up valuable time that needs to be spent on other sections of the syllabus. Respondents mentioned that the curriculum is overcrowded. Associated with this view was the accountability of the teachers to cover the syllabus to prepare students for examinations, in particular the national examination in grade 8. This view was captured in a teacher's comment:

"The emphasis on accountability with the external exam such as grade 8 national examinations and the amount of work required by the syllabuses is such that teachers push through the content rather than 'losing' time on enrichment areas such as problem-solving."

The need was expressed for ideas on using particular problem types, and their role in promoting and developing students' problem-solving skills. The comments on particular problem types were influenced by the survey which listed problem types for teachers. These included application problems, open-ended problems and unfamiliar problems. In particular the teachers would like to learn more about the use of open-ended and unfamiliar problems and to be provided with more examples of these kinds of problems.

C. Students

Nineteen respondents recorded comments on professional development issues that relate to the diverse needs of students of grades 5 to 8. Most of these comments relate to students' abilities and in particular to the need to develop the problem-solving abilities of students not achieving well. It was also recognized that textbooks frequently determine how teachers handle students. This implies that if problem-solving is to be promoted, textbooks would need to change. One respondent suggested that many mathematics teachers underwent more traditional practices when they were at school, and consequently they cannot understand why students do not respond to them. The teachers mentioned student behavior as a constraining factor, and most of them mentioned the level of understanding of the students. It is clear that for many teachers, a sound basic knowledge of mathematics is required before students are able to engage in investigations and problem-solving in mathematics.

In conclusion, factors prohibiting teachers from implementing a problem-solving approach in their classrooms can be summarized as follows:

- The teachers' lack of awareness of the benefits of problem-solving in the mathematics curriculum, since in their view problem-solving is not seen as a legitimate mathematical activity. Many teachers do not consider problem-solving as serious mathematics. They view it as additional work and not as a process that can be woven into every part of mathematics.
- The teachers' concern about covering the content of the curriculum. They believe that problem-solving may take up too much time, time that could rather be used to prepare students for the national examinations.
- The lack of confidence and competence in applying a problem-solving approach in the mathematics classrooms.
- The lack of practical teaching advice related to three key areas: modelling problem-solving approaches, increasing school resources, and providing information about incorporating problem-solving approaches into the teaching of mathematics through appropriate activities, relevant planning, and the use of a range of problems.
- Mathematics textbooks present few non-routine (open-ended and unfamiliar) problems. The teachers indicated that textbooks do not provide a variety of problems to choose from.
- Many teachers are not comfortable deviating from the scope and sequence of the textbook. They are not confident to search for and develop material to supplement their textbooks.

On the support teachers need in implementing a problem-solving approach in mathematics, teachers identified issues that should be considered. Opportunities for teachers to plan and learn together in collaborative environments would support the development of knowledge and confidence in teaching problem-solving. Teachers need to be convinced that providing opportunities for students to engage in rich learning experiences will eliminate poor behaviour and a lack of enthusiasm.

Interview data

To gain additional information on the relationship between teachers' problem-solving beliefs and practices, data from two teachers, each a representative of the traditional and the contemporary convictions were presented. Each of these teachers was interviewed with a view of encouraging them to elaborate on their questionnaire responses. Chala (a pseudonym) has been teaching for 13 years and was teaching mathematics for grades 6 and 7. Overall, his questionnaire responses placed him in the *traditional* category. During the interviews, Chala's reported beliefs from the questionnaire were confirmed. Basic skills were viewed as necessary knowledge before students could do problem-solving. In Chala's classes students are given short tasks with guidelines as to the process required to complete the task. He believes that a structured approach is desirable in mathematics. He stated,

"It seems to me that students who aren't particularly capable in math will only become amenable to sitting down … to solve a problem if they know some way of doing that."

Chala's decisions and planning seemed to centre on the needs of the students in his class. He reported that he seldom uses problem-solving approaches in his teaching since the students need practice on basic skills, and generally find problem-solving difficult.

Chala views problem-solving as an added extra to the curriculum and not as a process of inquiry, particularly suitable for more able students as an additional activity, thus in contrast to the view that problem-solving can be a teaching approach suitable for all students. In fact, Chala rejected this view and was highly critical of this approach during the interview.

Bachu (a pseudonym) has been teaching for four years and her responses placed her in the *contemporary* category. At the time of completing the questionnaire she was teaching mathematics for grades 5 and 6. Bachu indicated that she often uses open-ended and application problems, sometimes unfamiliar problems, and rarely gives her students exercises. Her response to the question why she prefers to make use of problems was that

"…open-ended problems allow children to bring their own knowledge and strategies to thetask as well as to respond at their own level. My children write their own problems which are often in the form of application problems."

During the interview Bachu indicated that she could now probably qualify some of the statements she had rejected or supported in the questionnaire. While she still agrees that "…problem-solving can actually teach you some of the basic number facts", she now feels that this was not the case for all students. To support her change of mind she said, "I think I was going too far into problem-solving and not giving them enough of the traditional stuff." However, she still supported the use of problem-solving approaches, since she stated " I like the idea of using problem-solving because many of the problems you use relate math tothe real world and Iknow that every time I mention mathematics in the real world with my class they immediately attend much better than they normally would ... it does motivate them because they like a challenge. Any child likes a challenge. But the problem is the textbook prepared for this grade level by the Ministry of Education has almost no open-ended or unfamiliar problems like those of your survey examples."

It appears that her initial enthusiasm in using problem-solving approaches in the school was influenced by her experiences in the pre-service education course on problem-solving. She embraced much of the advice from the problem-solving literature.

Observation data

The researcher observed Chala and Bachu while they were teaching in the classroom to see the extent to which they practise what had been discussed in the interview. The researcher got the impression that both of them were very careful to present their lesson in accordance with their responses in the survey and the interview. Chala started his lesson presentation for grade 7 by revising what they had discussed the day before, while the students listened to him for about three minutes. Thereafter he defined 'linear equations'. He gave examples pertaining to the definition and three examples on solving linear equations. He asked the students if they had any questions or if there was anything they did not understand. This took 25 minutes. Next he gave the students class-work of five tasks. All were exercises that could be solved following the same procedure as the examples. He copied them from the students' textbook. He moved around in the classroom while the students were doing the exercises. The researcher counted that he checked three students' class-work. Then he went to the blackboard and did all five exercises. He finally gave the students homework exercises from their textbook. It is evident that Chala's class is completely teacher-led and is based on the transmissional approach. The students were passive recipients of knowledge. He used the textbook as his source of knowledge. The transmissional approach to teaching describes instructional methods as a set of procedures that a teacher would precisely follow to produce student learning. The researcher observed that much of the instruction provided in the textbook and the teachers' manual is of this kind. The teachers' manual provides directions on how to conduct the lesson, includes the questions to ask students, and makes suggestions for student assignments and assessments. Teachers like Chala who follows the transmissional approach rely extensively on textbook teaching to select instructional methods and material for their students. Unfortunately, by relying so much on the textbook and using these instructional approaches teachers often fail to meet the unique needs of their students. What they do achieve are rote learning skills and static textbook information. The transmissional view also perceives the teacher as a kind of inert conduit for the flow of information from teacher to student. Transmissional teaching minimizes the needs, interests, and motivations of both teachers and students. It is based on the assumption that following textbook classroom procedures ensures that all students will learn at a consistent and predetermined level. This is in contrast to the problem-solving approach where open-ended problems can also be used to address the learning styles of different students in the classroom. Bachu's class was noticeably different from Chala's. She started her class by questioning the students on what they had discussed the day before. The majority of the students in the grade 5 class raised their hands to respond to her question. She gave three students the chance and they explained what they had discussed the last time; the teacher seemed satisfied. Thereafter she introduced the objective of the lesson. She discussed word-problems on the division and multiplication of rational numbers. Then, without saying anything other than introducing the objective, she posed an open-ended problem as follows, "With two rational numbers 406 and 6, write down a question that canbe solved using division." The researcher observed that almost all the students were very active, constructing problems, while she went around in the classroom observing the students. One student wrote a question as follows, "If a textile factory produces 406 meters of cloth in 6 days, how many meterscan it produce per day?"She came to the student upon his requestand said, "This is fine, but how do you solve the problem?" The student then started to try and find the answer.

She did the same with many students in the class and finally asked a student to read what she had written. She repeated this process with three more students, and then asked all of the students to solve the problems they wrote. Many different questions were stated by the students, but all with the same answer. She finally gave them two problems, both of them open-ended, for homework. The researcher paged through a textbook of one of the students to check if there were any open-ended problems in it, but he could not find any. This brought him to the conclusion that Bachu prepared the problems herself. It is also probable that she recognized the advantage of including open-ended problems for student learning. In contrast to Chala, Bachu was an a*ctivist* type of teacher. In her view, learners construct their own knowledge by searching for meaning through experience of the world. Although a teacher can and should provide experience and guidance for student learning, all new knowledge is synthesized by the learner to a unique understanding based on previous knowledge (Brooks and Brooks, 1993). According to his view, the professional judgment of the teacher is critical as he or she makes decisions in the classroom to design experiences that encourage student learning. Learning cannot be coerced from students but must be skilfully elicited. This type of teaching requires a skilful, intelligent and sensitive teacher, like Bachu. It requires from a teacher to be reflective about the events in the classroom and carefully plan lessons based on what students know and what they need to learn to bring them to the next level in their development. Dewey (1933) noted that reflective thinking frees teachers from blindly following classroom routines (textbook instruction), like that of Chala. In conclusion, the interview and observation data indicate that there was a dichotomy of views among the teachers taking part in the survey. The beliefs of the two teachers who were observed in their classes and in the interviews were in harmony. Moreover, in the survey Chala was categorized as a conventional teacher. Chala's interview and classroom observation supported the fact that he was a *conventional* type, while Bachu was an *activist* type of teacher.

Summary of the findings

Firstly, with regard to the beliefs of the teachers who took part in the survey, it seemed that even though the majority of teachers hold an absolutist view of mathematics there were also respondents with a fallibilist view. It can consequently be assumed that the majority of teachers advocate the view that mathematics is a body of absolute and certain knowledge, all mathematical objects are discovered, mathematical objects have their own realm of existence independent of human culture, and mathematics is a game played with mathematical symbols and variables. Secondly, regarding the teachers' problem-solving views, it appeared that the majority of teachers advocate a traditional problem-solving view. They believe that basic facts and principles should be learned before students are confronted with open-ended and unfamiliar problems, and examples should be given before students begin to do the assignments. Thus, it is the teachers' responsibility to provide the students with clear and concise methods of solving mathematical problems. The research also suggested that the majority of teachers made use of exercises, as compared to open-ended and unfamiliar problems. There also existed a strong relationship between the teachers' problem-solving views and the use of different kinds of problems. Indeed, teachers with a traditional problem-solving view reported that they prefer the use of exercises to problems in the classroom, whereas a significant majority of teachers with a contemporary problem-solving view reported that they preferred 'problems' to 'exercises'. Thirdly, the survey on the teaching strategies of teachers established that more than 50% of the teachers reported that they make use of different traditional teaching strategies, such as explaining in detail what the students have to do to solve problems, requiring students to carefully listen to what the teacher is explaining, with the presupposition that the heart of teaching is the students' mastery of mathematical skills and procedures.

This research also revealed that there exists a strong relationship among mathematics beliefs, problem-solving views, the use of different kinds of problems and teaching strategies. Indeed, the majority of respondents with an absolutist view of mathematics, who hold a traditional problem-solving view and who make use of exercises, were found to employ traditional teaching strategies. Alternatively, the majority of respondents with a fallibilist view of mathematics, who hold a contemporary problem-solving view and who make use of problems, used different progressive teaching strategies in the classroom.

The research also identified some of the problems that restrained teachers from implementing aproblem-solving approach in their mathematics classrooms. These problems are:

- The belief that teachers have about mathematics, about the problem-solving approach and about the use of different kinds of problems.
- Teachers' lack of awareness regarding the benefits of implementing a problem-solving approach in mathematics, since in their view problem-solving is not seen as a legitimate mathematical activity.
- The lack of confidence and competence in applying a problem-solving approach in mathematics classrooms.
- The lack of practical teaching advice.
- Mathematics textbooks that contain only a few non-routine (open-ended and unfamiliar) problems.
- Many teachers are not comfortable deviating from the scope and sequence that the textbook provides.

Conclusion

This study revealed that the teachers' beliefs about mathematics and their problem-solving views strongly influence what happens in the classroom. This, in turn, implies that the more a mathematics teacher is aware of adopting a particular belief of the nature of mathematics or a particular view on problem-solving the more he/she would probably be inclined to cling to that particular belief. That is, if one is aware of adopting a particular belief, then he/she may strive to change his/her belief and accompanying practice. Moreover, it is evident that some teachers regard problem-solving as an extra load in the curriculum and that students are only able to solve problems after they have acquired the basic skills and procedures. The possibility of teaching *through* problem-solving and teaching *about* problem-solving is nominal and the majority of teachers may yet be teaching mathematics by means of 'outdated' traditional methods. The research also indicated that to a large extent school mathematics is seen as a set of computational skills and the learning of mathematics as acquiring those skills by imitating demonstrations by the teachers and textbooks. Teachers and textbooks were considered to be a reliable source of knowledge. Consequently, the majority of teachers regard their role as that of transmitting information to students, with the students the passive recipients of knowledge. It also seems that open-ended problems and unfamiliar problems were not commonly used in mathematics classes. The mathematics textbooks for grades 5-8 do not provide enough open-ended and unfamiliar problems for the teachers to choose from. Furthermore, it appears that some of the teachers who took part in the survey do not adequately understand what the problem-solving approach is. This implies that Teacher Training Institutes and Universities training mathematics teachers for grades 5-8 in Ethiopia do not include problem-solving approaches in their mathematics teaching courses.

Recommendations

Professional Development Needs

Providing teachers with support for the implementation of problem-solving approaches in classrooms is vital if classroom practices are to change. Finding the most appropriate professional development program can be challenging, given the constraints of time, cost and personnel. Involving teachers in decisions about their professional development needs is crucial if they are to embrace such approaches. In general, considerable support is necessary, particularly in the form of time and resources so that teachers can share teaching ideas, reflect on practices, and develop new understandings about these approaches. Given the evidence that students in grades 5-8 mathematics classes experience insufficient complex problem-solving, there needs to be a greater emphasis on explicit mathematical reasoning and higher-order thinking. One way to support the teachers' efforts in this regard is to provide professional development that challenges commonly held beliefs, but at the same time, provides the resources for teachers to investigate ways to implement new approaches. Therefore, professional development programs for teachers must include an introduction to the nature of mathematics, since teachers must be made aware that the teaching of mathematics is closely related to their views of what the nature of mathematics comprises.

Challenges in adopting a problem-solving approach that is related to a particular philosophy of teaching mathematics and which should be addressed in professional development programs include the following:

- Mathematics teachers must be convinced that mathematics is an active, creative endeavor involving inquiry and discovery, not simply involving correct answers and infallible procedures consisting of arithmetic operations, algebraic procedures, and geometric terms and theorems.
- Mathematics teachers must be convinced to view their roles as facilitators, challenging students to think and to question their findings and assumptions, not merely as presenters of mathematical concepts, procedures, facts, and theorems with a focus on student practice and memorization where the meaning and context associated with many of these theorems and procedures may be relegated to the fringes.
- Mathematics teachers have to accept the fact that teaching non-routine problem-solving is difficult. True problem-solving is as demanding to the teacher as it is to the students. The art of teaching mathematical problem-solving is best mastered over a long period of time. Teachers must acquire the mathematical expertiseto understand the different approaches that students may have to solve a problem and how promising those approaches will be.
- Teachers' teaching strategies, their use of different problem types and their problem-solving views need to be analyzed and understood in terms of the larger

cultural context, because a lack of such understanding is likely to slow down the process of change at all levels.

- Teachers must be convinced that their problem solving performance is enhanced by teaching students to use a variety of strategies; problem-solving practice without direct instruction on strategies does not elicit improvement. Students can learn how and when to use problem-solving strategies to successfully solve problems when provided with explicit instruction on the strategies.
- Furthermore, the professional development programs to empower teachers to implement problem-solving approaches in mathematics classrooms should also address the following:
- Allow mathematics to be problematic. This means posing problems that are just within the student's reach, allowing them to struggle to find solutions, and making them examine the methods they have used. Allowing mathematics to be problematic requires a different mindset about what mathematics is, how students learn mathematics with understanding, what role the teacher should play, and the need for students to struggle with (challenging) problems if they are to truly learn mathematics. This notion is in a direct contrast to what many mathematics teachers feel their main goal is, namely to step in and to remove the struggle and the challenge.

Research

Further research into the teachers' role in problem-solving is necessary. Ways in which teachers can build learning communities within their schools to research their practice should be explored by using outside experts or academic partners.

Student Textbooks

Textbooks need to include many more open-ended problems. In addition, in Ethiopian schools there is only one textbook for a grade. This has to change, and a greater number of textbooks should be made available for each grade based on the standards of the country. This will give teachers the chance to select a 'good' textbook that can be used to implement a problem-solving approach.

The Role of the Stakeholders

All teachers, so to speak, need to be on the same page as far as mathematical teaching is concerned, if they are to advance technologically. Colleges and universities have to offer pre-service and in-service courses that include methods which involve problem-solving approaches. All stakeholders have to make sure that teachers receive the training in mathematics that advocates a problem-solving approach that will equip them to implement it in mathematics teaching in their classes.

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