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RESEARCH ARTICLE

ON ALMOST CONTRA- GENERALIZED #PRE-CONTINUOUS FUNCTIONS

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ABSTRACT

In this paper, we introduce a new classes of functions by using generalized#pre-closed sets and generalized #pre-open sets called strongly contrα- generalized #pre-continuous, strongly - generalized #pre-continuous function, contrα- generalized #pre-irresolute and almost contrα - generalized #pre-continuous function in topological spaces .Relationships between a new types of contrα - generalized #pre-continuous are established and we study some of basic properties.

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1. INTRODUCTION

Levine (1970) introduced the class of g-closed sets, Veera Kumar (2004) introduced generalized closed set namely $g^{\#}$-closed. The authors (2013) have already introduced $g^{\#}$p-closed sets and their properties, Subramanian in (2013) introduced $g^{\#}$p-continuous maps in topological spaces Ali in (2013) study contrα -$g^{\#}$p-continuous function in topological space. The notion of contrα-continuity was introduced by Donchev (1996). Jafari and Noiri (2002) introduced and investigated contrα pre-continuous function and contrα α-continuous function in topological space, Levine in (1960) studed strong continuity, almost contrα pre-continuous function was introduced by Ekici (2004). Throughout this paper (X,T) and (Y,T) (or simply X and Y) represents the non-empty topological space on which no separation axiom are assumed unless otherwise mentioned for a subset A of X ,cl(A) and int(A) represent the closure of A and interior of A respectively.

2.Preliminaries

In this section, we below list the definitions and results which are useful in the sequel.

Definition 2.1:

A subset A of a topological space (X,T) is called :

1- pre- open set (Mashhour *et al.*, 1982): if $A \subseteq \text{int}(cl(A))$ and pre-closed set $cl(\text{int}(A)) \subseteq A$.

2- a regular open set (Stone, 1970): if $A = \text{int}(cl(A))$

3- an α- open set (Njastad, 1965): if $A \subseteq \text{int}(cl(\text{int}(A)))$ and α-closed set $A \subseteq cl(\text{int}(cl(A)))$.

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Definition 2.2:

A subset A of a topological space (X,T) is called :

1- a generalized -closed (g-closed) set (Levine, 1970) if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open set in (X,T).
2- a generalized α-closed (gα-closed) set (Maki *et al.*, 1993) if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is α- open set in (X,T).
3- a generalized$^{\#}$-closed (g$^{\#}$-closed) set (Veera Kumar, 2004) if $cl(A) \subseteq U$ whenever $A \subseteq U$and U is αg- open set in (X,T), The complement of g$^{\#}$-closed set is g$^{\#}$- open.
4- generalized - preclosed (gp-closed) set (Maki *et al.*, 1996) if $pcl(A) \subseteq U$ whenever $A \subseteq U$ and U is open set in (X,T).
5- a generalized$^{\#}$- preclosed (g$^{\#}$ p-closed) set (Pious Missier *et al.*, 2013) if $pcl(A) \subseteq U$ whenever $A \subseteq U$ and U is g$^{\#}$- open set in (X,T).

Definition 2.3:

A function h: A → B is called :

1. Contrα - continuous (Dontchev, 1996) : if h^{-1} S is closed in A, ∀ open set S of B.
2. Contrα- precontinuous (Jafari and Noiri, 2002) : if h^{-1} S is preclosed in A, ∀ open set S of B.
3. Almost-continuous (Singal and Singal, 1968): if h^{-1} S is open in A, ∀ regular open set S of B.
4. Almost-contrα continuous (Noiri, 1989): h^{-1} S is closed in A, ∀ regular open set S of B.
5. Perfectly- continuous (Ekici, 2004): if h^{-1} S is clopen in A, ∀ open set S of B.
6. an R-map(Ekici,2008): if h^{-1} S is regular open in A, ∀ regular open set S of B.
7. g$^{\#}$p-continuous (Pious Missier *et al.*, 2013): if h^{-1} S is g$^{\#}$p-closed in A , ∀ closed set S of B.
8. g$^{\#}$p-irresolute (Pious Missier *et al.*, 2013): if h^{-1} S is g$^{\#}$p-closed in A , ∀ g$^{\#}$p-closed set S of B.
9. Strongly continuous (Levine, 1960): if h^{-1} S is clopen in A , ∀ subset S of B.

Remark 2.4:

A space (X,T) is called a:

(1) $T_P^{\#}$ - space (Pious Missier *et al.*, 2013) if every g$^{\#}$p-closed set is closed.
(2) Every preclosed set (resp.α-closed, gα-closed and closed set) (Pious Missier *et al.*, 2013) is g$^{\#}$p-closed set .
(3) The intersection of an open set and g$^{\#}$p-open sets is a g$^{\#}$p-open set (Pious Missier *et al.*, 2013).
(4) The union of any family of g$^{\#}$p-open sets is a g$^{\#}$p-open set (Pious Missier *et al.*, 2013).

3.On Contrα-g$^{\#}$p-continuous functions

In this section we introduce the following definitions:

Definition 3.1:

A function h: A → B is called

1. Contrα- g$^{\#}$ PRE-continuous (contrα - g$^{\#}$p-continuous) (Alli, 2013) if h^{-1} S is g$^{\#}$p-closed set in A, ∀ open set S of B.
2. Strongly- g$^{\#}$ PRE-continuous (strongly- g$^{\#}$p-continuous) if h^{-1} S is open set in A, ∀ g$^{\#}$p- open set S of B.
3. Contrα Strongly-g$^{\#}$ PRE-continuous(contrα strongly-g$^{\#}$p-continuous) if h^{-1} S is closed set in A , ∀ g$^{\#}$p- open set S of B.
4. Contrα - g$^{\#}$ PRE-irresolute (contrα - g$^{\#}$p-irresolute) if h^{-1} S is g$^{\#}$p-closed set in A, ∀ g$^{\#}$p- open set S of B.

Example 3.2

1. Let A=B={1,2,3} with topologies T={A,Ø,{3}} and σ={B,Ø, {1,2}}, Let h : A → B defined by h(1)=1, h(2)=2, h(3) =3 , Since h^{-1} 1,2 = {1,2} is g$^{\#}$p- closed in A. Hence h is contrα -g$^{\#}$p-continuous.
2. Let A=B={1,2,3} with topologies T={A,Ø,{1,2}} and σ={B,Ø, {3}}, let h : A → B defined by h(1)=1, h(2) =2,h(3) =3, Since h^{-1} 3 = {3} is closed in A. Hence h is contrα strongly-g$^{\#}$p-continuous.
3. Let A=B={1,2,3} with topologies T={A,Ø,{1}} and σ={B,Ø,{1}}, let h : A → B defined by h(1) = 1, h(2) = 2, h(3) = 3, Since h^{-1} 1 = {1} is open in A. Hence h is strongly-g$^{\#}$p-continuous
4. Let A=B={1,2,3} with topologies T={A,Ø,{3,2}} and σ={B,Ø,{1},{2},{1,2}} . Afunction h : A → B defined by h(1)= h(2) =h(3)=1, Clearly h is contrα -g$^{\#}$p-irresolute.

Theorem 3.3:

Every contrα –continuous function is contrα -g$^{\#}$p-continuous

Proof: Let B contain any open set say S , let the function h: A → B be contrα - continuous, then $h^{-1}(S)$ is closed in A , since every closed set is $g^{\#}$p-closed, then $h^{-1}(S)$ is $g^{\#}$p-closed in A. Therefore h is contrα -$g^{\#}$p-continuous. Not be true the converse of above theorem , as shown in the following example:

Example 3.4:

Let A=B={1,2,3} with topologies T={A,Ø,{1},{1,2}} and σ={B,Ø,{2}} let h : A → B defined by h(1) =1, h(2) =2, h(3) =3. Hence h is contrα -$g^{\#}$p-continuous, but f is not contrα-continuous, since $h^{-1}(2)$ = {2} is not closed in A.

Theorem 3.5

If a function h : A → B is contrα-$g^{\#}$p-continuous and A is $T_P^{\#}$- space then h is contrα continuous.

Proof: Let B contain any open set say S , Since h is contrα -$g^{\#}$p-continuous, Then $h^{-1}(S)$ is $g^{\#}$p- closed in A ,Since A is $T_P^{\#}$-space, Then $h^{-1}(S)$ is closed in A, Therefore h is contrα -continuous.

Theorem 3.6:

1. Every strongly-$g^{\#}$p-continuous is continuous.
2. Every contrα strongly-$g^{\#}$p-continuous is contrα -continuous.
3. Every contrα strongly-$g^{\#}$p-continuous is contrα - $g^{\#}$p-continuous.
4. Every contrα strongly-$g^{\#}$p-continuous is contrα - $g^{\#}$p-irresolute.

Proof:

1) Let B contain any open set say S, Since every open set is $g^{\#}$p- open, Then S is $g^{\#}$p- open set in B, Since h is strongly-$g^{\#}$p-continuous, hence $h^{-1}(S)$ open in A, Therefore h is continuous.
2) Let B contain any open set say S, Since every open set is $g^{\#}$p- open, then S is $g^{\#}$p- open set in B, since h is contrα strongly-$g^{\#}$p-continuous, hence $h^{-1}(S)$ closed in A. Therefore h is contrα -continuous.
3) The proof by theorem (3.3) is obvious.
4) By the same proof of (3) using the fact that (every closed is $g^{\#}$p-closed)

Not be true the converse of above theorem in general.

Theorem 3.7:

A function h: A → B is

1. Strongly-$g^{\#}$p-continuous iff for every $g^{\#}$p-closed set in B the inverse image is closed in A.
2. Contrα Strongly-$g^{\#}$p-continuous iff for each $g^{\#}$p-closed set in B the inverse image is open in A.
3. Contrα -$g^{\#}$p-irresolute iff for each $g^{\#}$p-closed set in B the inverse image is $g^{\#}$p- open in A.
4. Contrα -$g^{\#}$p-irresolute iff for each closed set in B the inverse image is $g^{\#}$p- open in A.

Proof:(1) → Let S be any $g^{\#}$p-closed set in B, Then B-S is $g^{\#}$p-open set in A Since h is strongly - $g^{\#}$p - continuous, Then $h^{-1}(B-S)$ is open in A, Therefore $h^{-1}(S)$ is closed in A. ← Let B contain any open set say S, Then B-S is closed set in B, since every closed is $g^{\#}$p-closed, hence B-S is $g^{\#}$p-closed in B, but $h^{-1}(B-S) = A - h^{-1}(S)$ is closed in A, therefore $h^{-1}(S)$ is open in A. Hence h is strongly -$g^{\#}$p-continuous. By the same way of (1)we can prove (2),(3)&(4).

Theorem 3. 8:

Let h : A→B is $g^{\#}$p-continuous and Z: B→C is strongly-$g^{\#}$p-continuous Z∘h : A→C is $g^{\#}$p-irresolute.
Proof: Let S be a $g^{\#}$p-closed set in C, since Z is strongly-$g^{\#}$p-continuous function, then $Z^{-1}(S)$ is closed set in B, $h^{-1}(Z^{-1}(S))$ is $g^{\#}$p-closed in A, but $h^{-1}(Z^{-1}(S))=(Z∘h)^{-1}(S)$ is $g^{\#}$p-closed set in A, Therefore Z∘h is $g^{\#}$p-irresolute.

Theorem 3. 9:

Let h : A→B is contrα strongly-$g^{\#}$p-continuous and Z: B→C is $g^{\#}$p-continuous Z∘h : A→C is contrα continuous.

Proof: Let C contain any open set say S, Since Z is $g^{\#}$p-continuous function, Then $Z^{-1}(S)$ is $g^{\#}$p-open set in B, Therefore $h^{-1}(Z^{-1}(S))$ is closed in A, Since h is contrα strongly-$g^{\#}$p-continuous, Therefore $h^{-1}(Z^{-1}(S))=(Z∘h)^{-1}(S)$ is closed set in A. Hence Z∘h is contrα continuous.

Theorem 3. 10:

Let h : A→B is contrα -$g^{\#}$p-continuous and Z:B→C is strongly-$g^{\#}$p-continuous Z∘h : A→C is contrα -$g^{\#}$p-irresolute.

Proof: Let S be a $g^{\#}$p-open set in C, Since Z is strongly-$g^{\#}$p-continuous function, Then Z^{-1} S is open set in B, Therefore $h^{-1}(Z^{-1}(S))$ is $g^{\#}$p-closed in A, Since h is contrα -$g^{\#}$p-continuous ,Hence $h^{-1}(Z^{-1}(S))=(Z\circ h)^{-1}(S)$ is $g^{\#}$p-closed set in A. Therefore $Z\circ h$ is contrα $g^{\#}$p-irresolute.

Theorem 3. 11:

Let h: $A\rightarrow B$ and $Z:B\rightarrow C$ be a function

1. If Z is $g^{\#}$p-continuous and h is contrα $g^{\#}$p-irresolute then is $Z\circ h$ is contrα $g^{\#}$p-continuous.
2. If Z is $g^{\#}$p-irresolute and h is contrα $g^{\#}$p-irresolute then is $Z\circ h$ is contrα $g^{\#}$p-irresolute.
3. If Z is contrα $g^{\#}$p-irresolute and h is $g^{\#}$p-irresolute then is $Z\circ h$ is contrα $g^{\#}$p-irresolute.
4. If Z is continuous and h is contrα $g^{\#}$p-continuous then is $Z\circ h$ is contrα $g^{\#}$p-continuous.
5. If Z is contrα - continuous and h is $g^{\#}$p-irresolute then is $Z\circ h$ is contrα $g^{\#}$p-continuous.

Proof:

(1) Let C contain any open set say S, Since Z is $g^{\#}$p-continuous function, Then Z^{-1} S is $g^{\#}$p-open set in B,Since h is contrα $g^{\#}$p-irresolute, then $h^{-1}(Z^{-1}(S))$ is $g^{\#}$p- closed in A, Therefore $h^{-1}(Z^{-1}(S))=(Z\circ h)^{-1}(S)$ is $g^{\#}$p-closed set in A. Hence $Z\circ h$ is contrα $g^{\#}$p-continuous.

(2) Let S be a $g^{\#}$p-open set in C, Since Z is $g^{\#}$p-irresolute function, Then Z^{-1} S is $g^{\#}$p-open set in B, Since h is contrα -$g^{\#}$p-irresolute, Therefore $h^{-1}(Z^{-1}(S))=(Z\circ h)^{-1}(S)$ is $g^{\#}$p-closed set in A. Hence $Z\circ h$ is contrα $g^{\#}$p-irresolute.

(3) Let S be a $g^{\#}$p-open set in C, Since Z is contrα - $g^{\#}$p-irresolute function, then Z^{-1} S is $g^{\#}$p-closed set in B, Since h is $g^{\#}$p-irresolute, Therefore $h^{-1}(Z^{-1}(S))=(Z\circ h)^{-1}(S)$ is $g^{\#}$p-closed set in A. Hence $Z\circ h$ is contrα -$g^{\#}$p-irresolute.

(4) Let C contain any open set say S, Since Z is continuous function, Then Z^{-1} S is open set in B, Since h is contrα -$g^{\#}$p-continuous, Then $h^{-1}(Z^{-1}(S))=(Z\circ h)^{-1}(S)$ is $g^{\#}$p-closed set in A. Hence $Z\circ h$ is contrα- $g^{\#}$p-continuous.

(5) Let C contain any open set say S, then S is $g^{\#}$p-open . Since Z is $g^{\#}$p-irresolute function, then Z^{-1} S is $g^{\#}$p-open set in B,By theorem (3.3) h is contrα -$g^{\#}$p-continuous ,therefore $h^{-1}(Z^{-1}(S))=(Z\circ h)^{-1}(S)$ is $g^{\#}$p-closed set in A. Hence $Z\circ h$ is contrα $g^{\#}$p-continuous.

4-Almost contrα- $g^{\#}$p- continuous functions

In this section, we introduce and study basic properties of a new continuity called almost contrα- $g^{\#}$p- continuous.

Definition 4.1:

A function $h: A\rightarrow B$ is called Almost contrα - $g^{\#}$ PRE-continuous (almost contrα - $g^{\#}$p-continuous) if h^{-1} S is $g^{\#}$p-closed set in A for each S of B where S be regular open set.

Remark 4.2:

Every contrα - $g^{\#}$p-continuous is almost contrα - $g^{\#}$p-continuous (Since every regular open set is open)

Not be true the converse of remark above as shown in the following example:

Example 4.3:

Let A=B={1,2,3} with topologies T={A,Ø,{1},{1,2},{1,3}} and σ={B,Ø , {1},{1,2}}, let $h : A\rightarrow B$ defined by h(1) =1, h(2) =2, h(3) =3,Clearly h is almost contrα -$g^{\#}$p-continuous ,But h is not contrα - $g^{\#}$p -continuous.

Definition 4.4:

A space (A,T) is called locally $g^{\#}$p- indiscrete if every $g^{\#}$p-closed set is open.

Theorem 4.5:

If a function $h : A\rightarrow B$ is almost contrα -$g^{\#}$p- continuous and (A,T) is locally $g^{\#}$p- indiscrete then h is almost -continuous.

Proof: Let B contain regular open set say S, Since h is almost contrα -$g^{\#}$p- continuous, Then h^{-1} S is $g^{\#}$p- closed set in A, Since A is $g^{\#}$p-locally indiscrete, Then h^{-1} S is open set in A. Therefore h is almost-continuous.

Theorem 4. 6:

If a function $h : A\rightarrow B$ is an almost contrα -$g^{\#}$p-continuous, Then h^{-1} S is $g^{\#}$p-open set in A, ∀ regular closed set S in B.

Proof: Let B contain regular closed set say S, Then B-S is regular open, Since h is almost contrα -$g^{\#}$p-continuous, then h^{-1} B − S) = A − h^{-1}(S is $g^{\#}$p-closed set in A. So h^{-1} S is $g^{\#}$p-open set in A.

Theorem 4. 7:

If a function h : A → B is an almost contrα -$g^{\#}$p-continuous function and C subset of A,C is an open set, Then the restriction $h \backslash_C$:C→B is also almost contrα - $g^{\#}$p-continuous.

Proof: Let S be a regular closed set in B, Since h is almost contrα - $g^{\#}$p-continuous function, hence h^{-1} S is $g^{\#}$p-open set in A, since C is open, By remark (2,4(3)) hence $(h \backslash_C)^{-1}$ (S) = C ∩ h^{-1}(S) is h^{-1} S is $g^{\#}$p-open set in C. Therefore $h\backslash_C$ is an almost contrα-$g^{\#}$p-continuous.

Theorem 4. 8:

Let h : A→B is almost contrα -$g^{\#}$p-continuous and g:B→C is almost-continuous then Z∘h : A→C is almost contrα -$g^{\#}$p-continuous.

Proof: Let C contain regular open set say S, Since Z is almost-continuous function, Hence Z^{-1} S is open set in B, Since h almost contrα -$g^{\#}$p-continuous $h^{-1}(Z^{-1}(S))=(Z\circ h)^{-1}(S)$ is $g^{\#}$p-closed set in A. Therefore Z∘h is almost contrα -$g^{\#}$p-continuous.

Theorem 4. 9:

Let h : A→B is almost contrα -$g^{\#}$p-continuous and Z:B→C is perfectly continuous, then Z∘h : A→C is contrα -$g^{\#}$p-continuous.

Proof: Let C contain open set say S, Since Z is perfectly continuous function, Then Z^{-1} S is clopen (open and closed) set in B, Since h almost contrα -$g^{\#}$p-continuous $h^{-1}(Z^{-1}(S))=(Z\circ h)^{-1}(S)$ is $g^{\#}$p-closed set in A. Therefore Z∘h is contrα -$g^{\#}$p-continuous.

Theorem 4. 10:

Let h : A→B is almost contrα -$g^{\#}$p-continuous and Z:B→C is an R-map then Z∘h : A→C is almost contrα -$g^{\#}$p-continuous.

Proof: Let C contain regular open set say S , Since Z is an R-map, then Z^{-1} S is regular open set in B, Since h almost contrα -$g^{\#}$p-continuous function, Hence $h^{-1}(Z^{-1}(S))=(Z\circ h)^{-1}(S)$ is $g^{\#}$p-closed set in A. Therefore Z∘h is almost contrα -$g^{\#}$p-continuous.

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