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RESEARCH ARTICLE

MRT TRANSFORM FOR 1-D SIGNALS

***AswathiSanal and AnuRani Philip**

Department of ECE, Viswajyothi College of Engineering & Technology, Vazhakulam, Kerala, India, Pin 686670

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ABSTRACT

A new real transform, named M-dimensional real transform is developed, which can help to represent signals using real additions without complex arithmetic, and which offers a different way of signal analysis. The MRT exhibits redundancy. All the implementation is build in FPGA using verilog.

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INTRODUCTION

A digital signal is a sequence of numbers, real or complex, that represents an information-bearing quantity over discrete co-ordinates of time, space or another variable. Processing of digital signal play a vital role in almost all fields of technology. It involves various methods of dealing with digital signals, e.g., analysis and modeling of signals, their coding and transmission, compression, restoration etc. In the process of analyzing signals for further processing, signal processing theory makes use of a fundamental class of operators called signal transforms. Transforms convert the original signal into a form which enables relatively simpler analysis than in the original form. The exact effect of the transform on the signal is unique for each transform. Hence, different signal processing applications may use different transforms, of which many kinds exist. Transforms can thus be considered to be the fundamental components in processing of digital signal, where a majority of operations are performed in the transform-domain. Continuous transforms like Fourier and Laplace transforms were the earliest transforms used in analog signal manipulation. Signal processing operations utilizing these analog transforms were implemented using low pass, high pass, and band pass filters and spectrum analyzers. With the advent of digital computers, signal processing became digital in nature, and digital signal processing has been an area of heavy research activity. Signal transforms are mathematical tool used to arrive at the underlying phenomena in a signal. Frequency domain analysis can be done using variety of transforms. The DFT in the form of its fast version, FFT is widely used signal transform in various application. A transform, M-Dimensional Real Transform (MRT) (Rajesh Cherian Roy and Gopikakumari, 2009) for one-dimensional signals is proposed and this transform helps to do the frequency domain analysis of one-dimensional signals without any complex operations but in terms of real additions. The MRT exhibits redundancy. Removal of redundancy could result in a simpler 1-D transform. The 1-D MRT could be useful tool for 1-D signal processing application and the 1-D MRT shares the property of the Haar transform (Rajesh Cherian Roy and Gopikakumari, 2011) in that it is a real transforms.

****Corresponding author:*** **AswathiSanal,**
Department of ECE, Viswajyothi College of Engineering & Technology, Vazhakulam, Kerala, India, Pin 686670

Forward 1-D MRT

The M-dimensional Real Transform (MRT) is obtained by exploiting the symmetries in the exponential kernel of the Discrete Fourier Transform (DFT).Let x_n , 0≤n≤N-1,be a 1-D sequence and let Y_k,0≤k≤N-1 be its DFT. The DFT is given by (Rajesh Cherian Roy and Gopikakumari, 2009)

$$Y_k = \sum_{n=0}^{N-1} x_n W_N^{nk} \quad ,0\leq k\leq N-1 \qquad (1)$$

where $W_N^q = e^{-j2\pi q/N}$

Using the periodicity of the twiddle factor W_N,(1) can be rewritten as

$$Y_k = \sum_{n=0}^{N-1} x_n W_N^{((nk))N} \qquad (2)$$

The exponent $((nk))_N$ can have a value p, 0≤p≤N-1. For a given value of k, by grouping the data n that share the same value p for the exponent $((nk))_N$ and also using $W_N^{p+\left(\frac{N}{2}\right)} = -W_N^p$, (2) can be expressed as

$$Y_k = \sum_{n=0}^{M-1} Y_k^p W_N^p \quad (3)$$

where Y_k^p ,defined as

$$Y_k^{(p)} = \sum_{\forall n|((nk))_N=p} x_n - \sum_{\forall n|((nk))_N=p+M} x_n \quad (4)$$

k=0,1,2,……..N-1, p=0,1,2,………M-1 and M=N/2.is called the 1-D MRT of x.

In (4),k is the frequency index and p is the phase index .The computation of the DFT coefficients involves complex multiplication. The 1-D MRT maps an array of length N into M arrays, each of length N. Since the relation $W_N^{p+\left(\frac{N}{2}\right)} = -W_N^p$ is exploited,the 1-D MRT is valid only for even values of N.In contrast to the DFT, the MRT involves computation of MN coefficients in terms of real additions only.

The 1-D MRT can be also expressed as (Rajesh Cherian Roy and Gopikakumari, 2009),

$$Y_k^{(p)} = \sum_{n=0}^{N-1} A_{k,p,n}\, x_n \qquad 0\le k\le N-1,\ 0\le p\le M-1 \quad (5)$$

$$A_{k,p,n} = \begin{cases} 1, & ((nk))_N = p \\ -1, & ((nk))_N = p+M \\ 0, & Otherwise \end{cases} \quad (6)$$

Thus, the kernel $A_{k,p,n}$ maps the data x_n into the 1-D MRT $Y_k^{(p)}$.

Direct 1-D MRT Computation

The 1-D MRT $Y_k^{(p)}$,k=0,1,2,……N-1, and p=0,1,2,…..M-1, of the given sequence x_n , 0≤n≤N-1 ,is computed as follows, using real addition only.

1. For a given k&p ,initialize $Y_k^{(p)} = 0$.
2. For each value of n, compute $z=((nk))_N$
   If z=p, $Y_k^{(p)} = Y_k^{(p)} + x_n$,
   else if z=p+M, , $Y_k^{(p)} = Y_k^{(p)} - x_n$,
   else go to next value of n.
3. For each value of k and p, repeat steps 1-2.

Example1 :

Let x=(95 23 61 49 89 76 46 2) N=8,
then ,$Y_k^{(p)}$ the corresponding MRT of x, is

$Y_k^{(0)}$ =(441 6 77 6 141 6 77 6)
$Y_k^{(1)}$ =(0 -53 0 47 0 53 0 -47)
$Y_k^{(2)}$ =(0 15 48 -15 0 15 -48 -15)
$Y_k^{(3)}$ =(0 47 0 -53 0 -47 0 53)

The direct method requires N computation of z and its logical checking for every MRT coefficient. Computation of all MRT coefficients corresponding to one frequency involves addition of N data. Thus, the total number of additions involved in the MRT computation is N(N-1).

Examples

The relation between 1-D MRT coefficients and corresponding data elements, for N=4 and N=8 are given below

1-D MRT for N=4

The relation between data elements and corresponding MRT coefficients, for N=4 are

1. $Y_0^{(0)} = x_0+x_1+x_2+x_3$
2. $Y_1^{(0)} = x_0-x_2$
3. $Y_1^{(1)} = x_0-x_3$
4. $Y_2^{(0)} = x_0-x_1+x_2-x_3$
5. $Y_3^{(0)} = -x_0-x_2$
6. $Y_3^{(1)} = -x_1+x_3$

1-D MRT for N=8

The relation between data elements and corresponding MRT coefficients, for N=8 are

1. $Y_0^{(0)} = x_0+x_1+x_2+x_3+x_4+x_5+x_6+x_7$
2. $Y_1^{(0)} = x_0-x_4$
3. $Y_1^{(1)} = x_0-x_5$
4. $Y_1^{(2)} = x_2-x_6$
5. $Y_1^{(3)} = x_3-x_7$
6. $Y_2^{(0)} = x_0-x_2+x_4-x_6$
7. $Y_2^{(2)} = x_1-x_3+x_5-x_7$
8. $Y_3^{(0)} = x_0-x_4$
9. $Y_3^{(1)} = x_3-x_7$
10. $Y_3^{(2)} = -x_2+x_6$
11. $Y_3^{(3)} = x_1-x_5$
12. $Y_4^{(0)} = x_0-x_1+x_2-x_3+x_4-x_5+x_6-x_7$
13. $Y_5^{(0)} = x_0-x_4$
14. $Y_5^{(1)} = -x_1+x_5$
15. $Y_5^{(2)} = x_2-x_6$
16. $Y_5^{(3)} = -x_3+x_7$
17. $Y_6^{(0)} = x_0-x_2+x_4-x_6$
18. $Y_6^{(2)} = -x_1+x_3-x_5-x_7$
19. $Y_7^{(0)} = x_0-x_4$
20. $Y_7^{(1)} = -x_3+x_7$
21. $Y_7^{(2)} = -x_2+x_6$
22. $Y_7^{(3)} = -x_1+x_5$

Observations

The following observations can be made about the relationships between MRT coefficients and the data, as laid out in sections 2.2.1 – 2.2.2 for various values of *N.*

1) For all values of *N,* MRT coefficient is made up of the simple addition of all the elements of the data.
2) For all values of *N,* there is one and only one MRT coefficient which is made up of alternate additions and subtractions among all the elements of the data. Such coefficients are seen in coefficient no. 4 corresponding to *N* = 4, coefficient and coefficient no. 12 corresponding to *N= 8.*
3) For all values of *N,* some MRT coefficients are made up of only a subtraction among two elements of the data. Examples are in coefficient nos. 2, 3 & 5 corresponding to *N* = 4.
4) Some MRT coefficients are made up of only one addition among two elements of the data. Some MRT

coefficients are made up of a negated addition among two elements of the data.

5) There exist MRT coefficients that are exact negations of other MRT coefficients. If the sign of such an MRT coefficient is inverted, another MRT coefficient of the same order *N* is obtained.
6) There exist MRT coefficients that are exactly equal to other MRT coefficients. More than one MRT coefficients share the same value for some values of order *N*.
7) For any *N*, $Y_0^{(0)}$ and $Y_M^{(0)}$are the only MRT coefficients that involve all the elements of the data.
8) The 1-D MRT shares the property of the Haar transform in the real transform and involves real addition only.

Analysis of 1D MRT

From the preliminary observations made above, a detailed analysis of the 1-D MRT coefficients is necessary. Data elements form positive and negative groups. The phase index of an MRT coefficient has particular significance. The existence of a 1-D MRT coefficient can be explained on the basis of number theoretic principles. The conditions of existence relate the phase and frequency indices. The index of a data element in positive and negative groups can be found using different methods. It is possible to re-write the forward 1-D MRT in the form of an arithmetic series. The MRT has physical significance.

Data Elements in an MRT Coefficients

(a) Positive Data Group:
The group of data elements whose indices satisfy the congruence relation *((nk))N= p* is defined as the positive data group of the 1-D MRT coefficient $Y_k^{(p)}$.
(b) Negative Data Group:
The group of data elements whose indices satisfy the congruence relation *((nk))N= p + M* is defined as the negative data group of the 1-D MRT coefficient $Y_k^{(p)}$.

Phase Index in MRT

An MRT coefficient has two indices, the frequency index and the phase index. By formal definition of the MRT, the phase index has values in the range (0, *M* - 1).

(a) Valid Phase Index

Although the MRT definition is such that the phase index has values in the range (0, *M* - 1), the nature of the linear congruence equations involved makes it theoretically possible for the value of phase index *p* to have values in the range (0, *N* - 1). Given an MRT coefficient $Y_k^{(p)}$, a value for the phase index pin the range (0, *N* - 1) is defined to be a valid phase index for a given frequency index*k*|*p*.

Example :

For *N* = 6, if *k*= 2, then *p* = 0, 2, & 4 satisfy *k*|*p,* and hence these are valid phase indices for this value of *k.*

(b) Allowable Phase Index

A phase index *p* is defined to be an allowable phase index if *p* <*M.* The allowable phase index actually is the phase index that is referred to in the formal definition of MRT.

Existence of 1-D MRT Coefficient

An MRT coefficient exists $Y_k^{(p)}$ exists for data of order N if either of the following two conditions is satisfied (in the following , g(a,b) signifies the greatest common divisor of integers a and b, and a|b indicates that integer a divides integer b):

Condition l: g(k. N)|p
Condition 2: g(k,N)|(p+ M)

If condition 1 holds, there are elements in the positive data group of the MRT coefficient. If condition 2 holds, there are elements in the negative data group of the MRT coefficient.

Redundancy

1-D MRT is that the coefficients are sometimes exact negations of other coefficients. i.e a number of 1D MRT coefficients can be obtained by reversing the sign of an MRT coefficient. Also- MRT coefficients are sometimes exactly equal to other MRT coefficients. i.e. more than one MRT coefficients share the same values. In general. a group of MRT coefficients having different frequency and phase indices have the same magnitude. The sign of the coefficients may or may not be the same. The presence of the same value at different frequency/phase indices indicate an element of redundancy in the transform. If this repetition can be accurately predicted, then the simpler transform structure can be evolved by removing the redundant MRT coefficients to yield a transform that has no redundancy.

Theorem1 :

Given $Y_k^{(p)}$, for all h such that g(h,N)=1,

$$Y_{((hp))_N}^{((hp))_N} = Y_k^{(p)} \qquad for\ ((hp))_N < M \tag{9}$$

and

$$Y_{((hp))_N}^{((hp))_N\ -M} = -Y_k^{(p)} \quad for\ \left((hp)\right)_N \geq M \tag{10}$$

Theorem1shows that the redundancy can be predicted using the co-primes of N and values of the frequency and phase indices. Given a pair of frequency and phase index (k,p) ,the frequency and phase index pairs of all other MRT coefficient that are redundant with respect to the MRT coefficient with frequency and phase index pair (k,p). Also it states that the condition for redundancy is that the multiplication factor that relates the frequency indices of two redundant MRT coefficients is co-prime to N.

Example:

Let N=8,coefficients nos.2,3,4&5 corresponding to k=1 and from coefficients nos.8,9,10&11 corresponding to k=3 that MRT coefficients corresponding to k=3 that MRT coefficients corresponding to k=3 are redundant with those corresponding to k=1.Relations of redundancy exist also between MRT coefficients with k=2 and k=6.The integer h, other than 1 in (0,7), that satisfies g(h,8)=1 are h=3,5&7.Since $3=((3*1))_8$ and $6=((3*2))_8$ this explains redundancy between k=1&k=3 and k=2&k=6 respectively.

Theorem 2:

MRT coefficients with frequency indices that have common gcd with respect to N redundant to each other.

Example:

Let N=8, the following relations can be verified:

$Y_1^{(0)} = Y_3^{(0)} = Y_5^{(0)} = Y_7^{(0)}$ and $Y_1^{(1)} = Y_3^{(3)} = -Y_5^{(1)} = -Y_7^{(3)}$ and $Y_1^{(2)} = Y_3^{(3)} = -Y_5^{(2)} = -Y_7^{(2)}$ and $Y_1^{(3)} = Y_3^{(1)} = -Y_5^{(3)} =- Y_7^{(1)}$
Since k=1,3,5&7 share the same gcd of 1 w.r.t N there is redundancy amoung MRT coefficients of these frequencies. Similarly, since g(2,8)=g(6,8)=2.Thus,all frequency indices of 1-D MRT can be classified on the basis of their gcd w.r.t N.

Conclusion

The MRT is a new transform is to process data on the basis of associated kernel values while avoiding the multiplication with the kernel, and to utilize the symmetry and periodicity properties of the exponential transform kernel of the DFT, while maintaining the capability for frequency-domain analysis. The MRT expresses the data in terms of simple additions among various data elements. If required, the DFT can be obtained from the MRT. Hence, the MRT is a new and simple way of expressing and analyzing 1-D and implemented in FPGA using verilog.

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