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RESEARCH ARTICLE

A MATHEMATICAL DISCUSSION ON A NEW OUTLOOK OF CONSUMER BEHAVIOUR THEORY: A CASE STUDY ON CES FUNCTIONAL FORM

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ABSTRACT

The consumer behaviour theory is not a new one. There are so many theories such as Marshallian, Hicks-Allen Indifference Curve and Samuelson's Revealed Preference theory and so on. But all these theories are based upon certain assumptions. The validity of these theories depends upon the rationality of these assumptions. The assumption of utility measurement i.e. whether utility is ordinal or cardinal is a debatable one. The Constant Elasticity of Substitution (CES) function is also not a new one. Recent theories developed Duality in Consumption, Linear Expenditure Function, application of Roy's identity, Shepherd's Lemma, Euler's theorem etc. Our paper concentrates much on these areas in the light of CES utility function to reach the consumer in an equilibrium position and tries to find out a relation between consumer theory and production theory. We tried to establish a single theory to analyse the consumer behaviour and the behaviour of the firm such that one is the dual of the other. Lastly, in this paper we tried to establish a valid relation between Marginal Utility of Money Income of a Consumer and Marginal Cost of a Firm.

Abbreviations: CES- Constant Elasticity of Substitution, MU-Marginal Utility, MRS- Marginal Rate of Substitution, LEF-Linear Expenditure Function, IC-Indifference Curve, MC-Marginal Cost.

INTRODUCTION

In economics the fundamental problem is to derive the consumer's demand functions on the basis of behaviouristic assumptions regarding utility maximization subject to the budget constraint and minimizing expenditure subject to the utility constraint. If the first problem is called the Primal, the second one is its corresponding Dual. These two problems are the core theme of this paper. A. Marshall, Hicks-Allen, Samuelson, Arrow, Chenery, Minhas and Solow developed many theories regarding this issue. Our analysis concentrates much on the Constant Elasticity of Substitution (CES) utility function to analyse consumer behaviour theory. Then try to discuss some basic properties, statement of some results and theorems and their proof in the light of CES functional form.

Definition of Indifference Curve

Suppose the consumer consumes two commodities, say, x & y. Then the total utility can be written as: U =f(x, y) where x, y > o, Ux, $Ux >$ o and Uxx, Uyy <o i.e. the utility curve is concave towards the origin. Now an indifference curve (IC) shows the pairs of locus combinations of two commodity bundles (x, y) which give the same level of satisfaction. Thus the equation of an indifference curve is $\bar{U}$ =f(x, y), x∈ S, y∈ S, $\overline{XY}$ ∈ S. A collection of IC is called indifference map shown in Figure-1.

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Properties of an IC

An IC has the following nice properties-

i) An IC is downward slopping
ii) Two IC cannot intersect or touch each other.
iii) An IC must be convex to the origin.
iv) A higher level of IC represents a higher level of satisfaction and a lower one a low level of satisfaction.

Assumptions of consumer preference theory

i) ***Preferences are complete-*** Suppose there are two commodity bundles, say A & B each containing both commodities x & y. The consumers can choose unambiguously at least one and only one in the following case:
A is preferred to B →APB
B is preferred to A →BPA
A is indifferent to B →AIB

ii) ***Preferences are Reflexive***- Suppose A & B are identical in all respect then their Preferences are indifferent:
if A = B i.e. if A= (x_1, y_1) & B=(x_2,y_2) and $x_1 = x_2$ & $y_1 = y_2$ → AIB & BIA.

iii) ***Preferences are transitive*** – This consistency assumption implies that if APB & BPC → APC and if AIB & BIC →AIC

iv) *Preferences are continuous-* If A is preferred to B and C is very close to B then A is preferred to C i.e. if APB and C tend to B→ APC. This assumption also indicates that the bundles are finely divisible and makes the IC continuous.

v) *Preferences are strongly monotonic-* This assumption implies, more is always better than less i.e. the consumer prefers more of any commodity and is not over-supplied of any commodity (non-satiation and this assumption is also called the law of dominance). Suppose that A= (x_1, y_1) & B=(x_2, y_2) and $x_1 > x_2$ & $y_1 > y_2$ →APB.

vi) *The law of diminishing marginal rate of substitution* (MRS) operates and hence there is a trade-off with one good to get more of another. Which implies the IC is smooth downward slopping and convex towards the origin.

vii) *Reflexivity-* This assumption states that A must be indifferent to itself i.e. AIA & BIB.

viii) *Symmetry-* This assumption states that if A is indifferent to B then B is indifferent to A i.e. if AIB → BIA

ix) *Goods are available in all quantities and are finely divisible infinitely small.*

Budget Line

A budget constraint represents the combination of goods and services that a consumer can purchase with given prices and income. The budget constraint equation can be written as:

$$P_x X + P_y Y = M$$

where P_x = the price of specific good X
P_y = the price of specific good Y
X = the quantity of specific good X
Y = the quantity of specific good Y
M = money income of the consumer (after saving and borrowing)

The equation can be re-written as:

$Y = \frac{M}{Py} - (\frac{Px}{Py})$ x where $-(\frac{Px}{Py}) < 0$ as $Px, Py > 0$ is the slope of the budget line which is downward.

In Figure-2, PQ is the budget line. OP $= \frac{M}{Py}$ and OQ $= \frac{M}{Px}$. The Δ OPQ is closed and convex set. Any point in this set is feasible and affordable to the consumer and any point like Z is not attainable to him with his given income M. If he wants to spend his entire income to purchase these two commodities he must lie on this line. When M changes the budget line shift parallel and when the relative price i.e. $\frac{Px}{Py}$ changes the slope of the budget line changes i.e. it will steeper or flatter.

Substitutes and Complements

Two goods are substitutes if an increase in the price of one good increases the demand for other (e.g. tea and coffee) and as complements if an increase in the price of one good decreases the demand for other (e.g. tea and sugar). Thus for substitutegoods $\frac{\partial y_j}{\partial pi} > 0$

and for complementary goods $\frac{\partial y_j}{\partial pi} < 0$

Perfect Substitutes

Suppose two goods X &Y are perfect substitutes e.g. Coke and Pepsi are perfect substitutes to each other and they can exchange at the rate1:1. In this absolute case the IC will be a straight line with negative slope and the consumer will be switch between them at a fixed ratio and the MRS will be constant. In this 'monomania' situation the consumer spends his all income in one commodity and corner point will be the equilibrium. This is shown in Figure-3.

Suppose the utility function is U =f(x, y) = p.x +q.y
Then the Ux =p and Uy =q and the slope of the IC is $-\frac{q}{p}<0$ and the IC is straight line downward slopping.

Perfect Complements

In this situation the consumer preferred two goods equally and MRS is either zero or infinite e.g. Left and Right shoes. The consumer will not be better off having several right shoes iff he has only one left shoe because addition of right shoes have zero MU without more left shoes i.e. he needs a pair of shoes. In this situation the IC will be L-shaped as in fig.-4
In this case IC will be U =f(x, y) = min. {p x, q y}.

Derivation of the Equilibrium Condition

The consumer faces the problem of maximization of utility subject to his budget constraint i.e. Max. U =f(x, y)

Sub. To Px.X + Py.Y =M

For maximization we construct the Lagrangian function as follows:

L = U =f(x, y) +μ [M- Px. X - Py. Y], where μ is called Lagrange multiplier.

For maximization the First Order Condition requires

$$\frac{\partial U}{\partial x} = Ux - \mu\, Px = 0 \quad \text{......... (i)}$$
$$\frac{\partial U}{\partial y} = Uy - \mu\, Py = 0 \quad \text{.......... (ii)}$$
$$\frac{\partial U}{\partial \mu} = M - Px.X - Py.Y = 0 \quad \text{........... (iii)}$$

Combining two equations (i) & (ii) we get

$$\frac{Ux}{Uy} = \frac{Px}{Py} = \mu$$

→ the slope of the IC = the slope of the Budget Line.

In fig.-5 E is the equilibrium point where IC is tangent to the Budget Line and the optimum commodity bundle purchased by the consumer is $(\overline{ox}, \overline{oy})$.

The second –order (or sufficient condition) condition requires the relevant Bordered Hessian Determinant be positive i.e.

$$|\bar{H}| = \begin{vmatrix} o & -Px & -Py \\ -Px & Uxx & Uxy \\ -Py & Uyx & Uyy \end{vmatrix} > 0$$

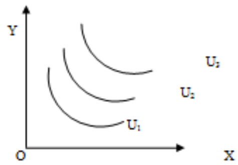
RESULTS AND DISCUSSION

This section deals in the use of CES production function in the Consumer Behaviour Theory. Moreover it deals in establishing different lemmas, statements and identity.

Combining (i) & (ii) we get

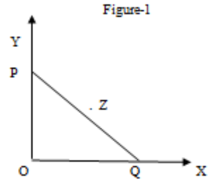
$$\frac{A\frac{1}{\rho}[ax^{\rho}+by^{\rho}]^{\frac{1}{\rho}-1}.a\rho x^{\rho-1}}{A\frac{1}{\rho}[ax^{\rho}+by^{\rho}]^{\frac{1}{\rho}-1}.b\rho y^{\rho-1}} = \frac{Px}{Py} \therefore y = \left(\frac{b}{a}\frac{Px}{Py}\right)^{\frac{1}{1-\rho}}.x$$

Putting this value into the budget equation we get



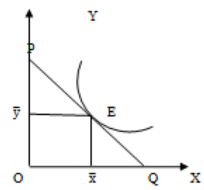
INDIFFERENCE MAP

Figure-1



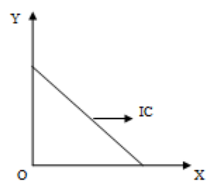
BUDGET – LINE

Figure-2



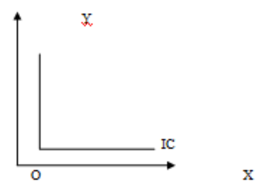
CONSUMER EQUILIBRIUM

Figure-5



PERFECT SUBSTITUTE

Figure-3



PERFECT COMPLEMENT

Figure-4

Mathematical Derivation

Let the utility function be $U = A[ax^{\rho} + by^{\rho}]^{\frac{1}{\rho}}$, $\rho \leq 1$, the co-efficient a & b are share parameters of two goods x & y respectively, A (autonomous parameter) > 0 and x,y ≥ 0 (non-negativity restrictions). The consumer faces the problem of maximizing utility subject to his budget constraint. In mathematical notation-

Max. $U = A[ax^{\rho} + by^{\rho}]^{\frac{1}{\rho}}$
Sub. To Px.X + Py.Y =M

For maximization we construct the Lagrange function as follows:

$L = A[ax^{\rho} + by^{\rho}]^{\frac{1}{\rho}} + \mu[M - Px.X - Py.Y]$; where μ is called Lagrange multiplier.

Now setting First Order Condition (FOC) we require

$$\frac{\partial L}{\partial x} = o \therefore A\frac{1}{\rho}[ax^{\rho} + by^{\rho}]^{\frac{1}{\rho}-1}.a\rho x^{\rho-1} - \mu Px = 0 \quad \text{............. (i)}$$

$$\frac{\partial L}{\partial y} = o \therefore A\frac{1}{\rho}[ax^{\rho} + by^{\rho}]^{\frac{1}{\rho}-1}.b\rho y^{\rho-1} - \mu Py = 0 \quad \text{............. (ii)}$$

$$\frac{\partial L}{\partial \mu} = o \quad \therefore \quad M - Px.X - Py.Y = 0 \quad \text{........... (iii)}$$

By simple algebraic manipulation we get

$$x = \frac{M}{Px}.\frac{Py^{\frac{\rho}{1-\rho}}}{\left(Py^{\frac{\rho}{1-\rho}} + Px^{\frac{\rho}{1-\rho}} + \left(\left(\frac{b}{a}\right)^{1+r}.Py^{\frac{\rho}{1-\rho}}\right)\right)}$$

Let $r = \frac{\rho}{1-\rho}$, t= $\frac{b}{a}$ = the ratio of the co-efficient of share parameters of two goods x & y respectively & p= $\frac{Px}{Py}$ = the relative price of the two commodities (i.e. the slope of the budget line).

Then **x** $= \frac{M}{Px}.\frac{Py^{r}}{(Py^{r}) + Px^{r} + t^{1+r}(Py^{r})} = \frac{M}{Px}\left(\frac{1}{1+Px^{r}+t^{1+r}}\right)$

Therefore, $x^{*} = \phi(Px, Py, M)$

Now putting this value in the budget equation and by some algebraic manipulation lly, lly we can get $y = \frac{M}{Py}\left(\frac{Px^{r}+t^{1+r}}{1+Px^{r}+t^{1+r}}\right)$

Therefore, $y^{*} = \theta(Px, Py, M)$
Now $k^{o}x^{*} = \phi(kPx, kPy, kM)$ and $k^{o}y^{*} = \theta(kPx, kPy, kM)$- which show that these demand functions are homogeneous of degree zero in prices and income. These demand functions have both price and income elasticity.

Lemma

In each demand function the sum of three partial elasticities will be equal to zero.

Proof

Let $x^*=\phi(Px, Py, M)$. Since the function is homogenous of degree zero. Applying Euler's theorem we get

$$\frac{\partial\phi}{\partial x}.Px + \frac{\partial\phi}{\partial Py}.Py + \frac{\partial\phi}{\partial M}.M=0$$

Dividing both sides by x we get

$$\frac{\partial\phi}{\partial x}.\frac{Px}{x} + \frac{\partial\phi}{\partial Py}.\frac{Py}{x} + \frac{\partial\phi}{\partial M}.\frac{M}{x}=0$$

The elasticity of ϕ w.r.to $Px +$ elasticity of ϕ w.r.to $Py +$ elasticity of ϕ w.r.to M=0

Thus $E_{Px} + E_{Py} + E_M = 0$

lly, lly the elasticity of θ w.r.to $Px +$ elasticity of θ w.r.to $Py +$ elasticity of θ w.r.to M=0

Thus $E_{Px} + E_{Py} + E_M = 0$

Hence the Lemma.

Adding- up Theorem

If each good is multiplied by its MU then total utility will be exhausted. The theorem is simple because the utility function is linearly homogenous.

Proof

$x.\,MU_x + y.\,MU_y = x.\frac{1}{\rho}A[ax^{\rho}+by^{\rho}]^{\frac{1}{\rho}-1}.a\rho x^{\rho-1} + y.A\frac{1}{\rho}[ax^{\rho}+by^{\rho}]^{\frac{1}{\rho}-1}.b\rho y^{\rho-1}$

$=A[ax^{\rho}+by^{\rho}]^{\frac{1}{\rho}-1}(ax^{\rho}+by^{\rho}) = A[ax^{\rho}+by^{\rho}]^{\frac{1}{\rho}}$
=U Q.E.D.

Elasticity of Substitution

Let the utility function be $U = A[ax^{\rho}+by^{\rho}]^{\frac{1}{\rho}}$.
Or $logU = \log[A(ax^{\rho}+by^{\rho})^{\frac{1}{\rho}}]$
Or $logU = logA + \frac{1}{\rho}log[ax^{\rho}+by^{\rho}]$

$$\text{MRS} = -\frac{MUx}{MUy} = -\frac{A[ax^{\rho}+by^{\rho}]^{\frac{1}{\rho}-1}.a\rho x^{\rho-1}}{A[ax^{\rho}+by^{\rho}]^{\frac{1}{\rho}-1}.b\rho y^{\rho-1}}$$
$$= -\left(\frac{a}{b}\right)\left(\frac{x}{y}\right)^{\rho-1}$$

Now log MRS $= -\log\left(\frac{a}{b}\right) - log\left(\frac{x}{y}\right)^{\rho-1} = -\log\left(\frac{a}{b}\right) - (\rho-1)log\left(\frac{x}{y}\right)$

Thus dlog MRS$= -(\rho-1)dlog\left(\frac{x}{y}\right) = (1-\rho)dlog\left(\frac{x}{y}\right)$

or $\frac{dlog\left(\frac{x}{x}\right)}{\text{dlog MRS}} = \frac{1}{1-\rho}$ or $\sigma = \frac{1}{1-\rho}$.
Now if $\rho = 0$ then σ =1. But it should be noted that in this case U is undefined. Since, $\rho \to 0$ then $\sigma \to 1$ and the function would be linear.Let $\sigma = \frac{1}{1-\rho} \to \rho = \frac{\sigma-1}{\sigma}$. Then the function becomes
$U = A\left[ax^{\sigma-1/\sigma} + by^{\sigma-1/\sigma}\right]^{(\sigma-1)/\sigma}$.

Therefore the goods x & y are perfect substitute if $\sigma \to$ infinity (as $\rho \to 1$) and perfect complements when $\sigma \to zero$ (as $\rho \to$ infinity).

Indirect Utility Function

The utility function be $U = A[ax^{\rho}+by^{\rho}]^{\frac{1}{\rho}}$ is called the direct utility function. Now putting the values of x* & y* in the utility function we get the Indirect Utility Function as:
$U^* = A[ax^{*\rho}+by^{*\rho}]^{\frac{1}{\rho}}$

$$U^* = \text{A.M}\left(\frac{1}{1+P^r+t^{1+r}}\right)\left(\frac{a}{Px^{\rho}} + \frac{b}{Py^{\rho}}(p^r+t^{1+r})^{\rho}\right)^{\frac{1}{\rho}}$$

$U^* = U(Px, Py, M)$
$k^0U^* = U(kPx, kPy, kM)$- which is homogeneous of degree zero.

Roy's identity

This identity enables us to find the ordinary demand curve from the indirect utility function.

$$U^* = \text{A.M}\left(\frac{1}{1+P^r+t^{1+r}}\right)\left(\frac{a}{Px^{\rho}} + \frac{b}{Py^{\rho}}(p^r+t^{1+r})^{\rho}\right)^{\frac{1}{\rho}}$$

Now $-\frac{\frac{\partial U*}{\partial px}}{\frac{\partial U*}{\partial M}} = \frac{M}{Px(1+P^r+t^{1+r})}$ =x

lly, lly $-\frac{\frac{\partial U*}{\partial py}}{\frac{\partial U*}{\partial M}} = \frac{M}{Py(1+P^r+t^{1+r})}$ =y. Hence the identity.

Duality of Utility and Expenditure

The equilibrium of the consumer is analysed in terms of duality.
The Primal problem is: Max. Utility
S.T. Budget Constraint
The corresponding Dual problem is:
Min. Expenditure
S.T. Utility Constraint
Let the total expenditure of the consumer be E. Then the Dual problem is:

Min. E= Px.X + Py.Y

S. T. $U^0 = A[ax^{\rho}+by^{\rho}]^{\frac{1}{\rho}}$
For minimization we construct the Lagrange function as follows:

$$\text{L} = Px.\text{X} + Py.\text{Y} + \lambda\left[U^0 - A[ax^{\rho}+by^{\rho}]^{\frac{1}{\rho}}\right]$$

For minimization the F.O.C. requires

$$\frac{\partial U^0}{\partial x} = 0 \; or \; Px - \lambda A\frac{1}{\rho}[ax^{\rho}+by^{\rho}]^{\frac{1}{\rho}-1}.a\rho x^{\rho-1} = 0 \quad \ldots \text{(iv)}$$
$$\frac{\partial U^0}{\partial y} = 0 \; or \; Py - \lambda A\frac{1}{\rho}[ax^{\rho}+by^{\rho}]^{\frac{1}{\rho}-1}.a\rho y^{\rho-1} = 0 \ldots.. \quad \text{(v)}$$
$$\frac{\partial U^0}{\partial \lambda} = 0 \; or \; U^0 - A[ax^{\rho}+by^{\rho}]^{\frac{1}{\rho}} =0 \quad \ldots\ldots.\text{(vi)}$$

Combining (iv) & (v) we get $y=\left(\frac{Px}{Py}\right)^{\frac{1}{1-\rho}}\left(\frac{b}{a}\right).x$Putting this value into the utility constraint we get

$$x = \frac{U^0.a.}{\left[a^{1+\rho}+Py^{\frac{\rho}{1-\rho}}+b^{1+\rho}+Px^{\frac{\rho}{1-\rho}}\right]}Py^{\frac{1}{1-\rho}}$$

or, $x^*=x(Px, Py, U^0)$

lly, lly $x=\left(\frac{Py}{Px}\right)^{\frac{1}{1-\rho}}\frac{b}{a}.y$

Putting this value into the utility constraint we get y= $\frac{U^0.b.}{\left[a^{1+\rho}.Py^{\frac{\rho}{1-\rho}}+b^{1+\rho}.Px^{\frac{\rho}{1-\rho}}\right]} Px^{\frac{1}{1-\rho}}$

or, y* =y $(Px, Py,\ U^0)$

Thus we get x*=x$(Px, Py,\ U^0)$ & y*=y $(Px, Py,\ U^0)$
It would be noted that functions are homogenous of degree zero in prices and utility. These two demand functions have only price elasticities.

Now substituting x*& y*we get

E= $Px.x^* +\ Py.y^*$

$=Px.\frac{U^0.a.}{\left[a^{1+\rho}+Py^{\frac{\rho}{1-\rho}}+b^{1+\rho}+Px^{\frac{\rho}{1-\rho}}\right]} Py^{\frac{1}{1-\rho}}$

$+ Py.\frac{U^0.b.}{\left[a^{1+\rho}.Py^{\frac{\rho}{1-\rho}}+b^{1+\rho}.Px^{\frac{\rho}{1-\rho}}\right]} Px^{\frac{1}{1-\rho}}$

$$E^* = U^0\left(Px.\frac{a}{\left[a^{1+\rho}+Py^{\frac{\rho}{1-\rho}}+b^{1+\rho}+Px^{\frac{\rho}{1-\rho}}\right]} Py^{\frac{1}{1-\rho}} + Py.\frac{b}{\left[a^{1+\rho}.Py^{\frac{\rho}{1-\rho}}+b^{1+\rho}.Px^{\frac{\rho}{1-\rho}}\right]} Px^{\frac{1}{1-\rho}}\right)$$

Thus E*= E*$(U^0, Px.Py)$ – is the Linear Expenditure Function. This function is strictly monotonically increasing function of $U^0 i.e.$ higher utility can be achieved only if the consumer increases his expenditure. Moreover, this function is linearly homogenous function of prices.

The second –order (or sufficient condition) condition requires the relevant Bordered Hessian Determinant be negative i.e.

$$|\bar{H}| = \begin{vmatrix} o & -\text{Ux} & -\text{Uy} \\ -\text{Ux} & \text{Uxx} & \text{Uxy} \\ -\text{Uy} & \text{Uyx} & \text{Uyy} \end{vmatrix} < 0$$

Shepherd's Lemma

The partial derivative of the Linear Expenditure Function w.r.to price of the i th commodity gives the compensated demand function for the i th commodity.

Proof

We know that $E^* = U^0\left(Px.\frac{a}{\left[a^{1+\rho}+Py^{\frac{\rho}{1-\rho}}+b^{1+\rho}+Px^{\frac{\rho}{1-\rho}}\right]} Py^{\frac{1}{1-\rho}} + Py.\frac{b}{\left[a^{1+\rho}+Py^{\frac{\rho}{1-\rho}}+b^{1+\rho}+Px^{\frac{\rho}{1-\rho}}\right]} Px^{\frac{1}{1-\rho}}\right)$ Thus

x= $\frac{\partial E}{\partial Px} = \frac{U^0.a.}{\left[a^{1+\rho}+Py^{\frac{\rho}{1-\rho}}+b^{1+\rho}+Px^{\frac{\rho}{1-\rho}}\right]} Py^{\frac{1}{1-\rho}}$

And y= $\frac{\partial E}{\partial Px} = \frac{U^0.b.}{\left[a^{1+\rho}.Py^{\frac{\rho}{1-\rho}}+b^{1+\rho}.Px^{\frac{\rho}{1-\rho}}\right]} Px^{\frac{1}{1-\rho}}$

Hence the Lemma

Lemma: Sum of elastic share of each commodity to total utility is unity (i.e. the degree of homogeneity).

Proof- Let we consider a utility function $U = A[ax^{\rho} + by^{\rho}]^{\frac{1}{\rho}}$

Then $kU = A[a(kx)^{\rho} + b(ky)^{\rho}]^{\frac{1}{\rho}}$ -which is linearly homogeneous?

Now $e_x = \frac{x}{U}\frac{\partial U}{\partial x} = \frac{x}{A[ax^{\rho}+by^{\rho}]^{\frac{1}{\rho}}}.A\ U^0 = \frac{1}{\rho}.[ax^{\rho} + by^{\rho}]^{\frac{1}{\rho}-1}$a. $\rho.x^{\rho-1}$

$= \frac{ax^{\rho}}{[ax^{\rho}+by^{\rho}]}$

And lly, lly $e_y = \frac{y}{U}\frac{\partial U}{\partial y} = \frac{y}{A[ax^{\rho}+by^{\rho}]^{\frac{1}{\rho}}}.A\ U^0 = \frac{1}{\rho}.[ax^{\rho} + by^{\rho}]^{\frac{1}{\rho}-1}$b. $\rho.y^{\rho-1}$

$= \frac{by^{\rho}}{[ax^{\rho}+by^{\rho}]}$

Thus $e_x + e_y = \frac{ax^{\rho}}{[ax^{\rho}+by^{\rho}]} + \frac{by^{\rho}}{[ax^{\rho}+by^{\rho}]} = \frac{(ax^{\rho}+by^{\rho})}{ax^{\rho}+by^{\rho}} = 1$

Hence the Lemma.

Statement: The Lagrangian multipliers of utility and expenditure functions are inversely related. From utility maximization we get

Ux - μ Px =0; or, UxM = μ Px

And, Uy - μ Py =0; or, UyM = μ Py

$\frac{\partial U}{\partial \mu}$ = M- Px. X - Py. Y =0

Combining two equations (i) & (ii) we get

$\frac{Ux}{Px} = \frac{Uy}{Py} = \mu$

Or, $Ux = \mu\ Px$ & $Uy = \mu\ Py$

From the budget equation we get,

M=Px. X - Py. Y Or, $Px.\frac{\partial x}{\partial M} + Py.\frac{\partial y}{\partial M} = 1$ or, $Px.x_M + Py.y_M = 1$

Therefore, $Ux.x_M + Uy.y_M = \mu(Px.x_M + Py.y_M)$

Thus $\mu = \frac{Ux.x_M+Uy.y_M}{Px.x_M+Py.y_M} = \frac{Ux}{Px} = \frac{Uy}{Py}$

Now $\frac{Ux}{Px}$ = the additional utility of Ux can be gained by an additional increment of x. Again the marginal cost of this extra x isPx. Hence the MU per rupee expenditure on x is $\frac{Ux}{Px}$. Similarly the MU per rupee expenditure on y is $\frac{Uy}{Py}$. From utility maximization we get the optimum values x*=$\phi(Px, Py, M)$& y*=θ(Px, Py, M)

Thus U^*= U ($\phi(Px, Py, M)$, y*=θ(Px, Py, M))Or, $\frac{\partial U^*}{\partial M} = \frac{\partial U}{\partial x}.\frac{\partial x}{\partial M} + \frac{\partial U}{\partial y}.\frac{\partial y}{\partial M} = Ux.\frac{\partial x}{\partial M} + Uy.\frac{\partial y}{\partial M} = \mu\ Px.\frac{\partial x}{\partial M} + \mu\ Py.\frac{\partial y}{\partial M} = \mu(Px.\frac{\partial x}{\partial M} + Py.\frac{\partial y}{\partial M})$ = μ.1= μ.

By non-satiation $\frac{\partial U^*}{\partial M} > 0$.

From the cost (expenditure) model we get,

$Px - \lambda\, Ux = 0\ or, \lambda = \frac{Px}{Ux}$

$Py - \lambda\, Uy = 0\ or, \lambda = \frac{Py}{Uy}$ Therefore, $\lambda = \frac{Px}{Ux} = \frac{Py}{Uy} = \frac{1}{\mu}$

Interpretation of Lagrange Multiplier

In the production process λ is the MC of output and in consumer behaviour μ is MU of money income. MU of money income is inversely related to the MC of production, i.e. $\frac{\partial U^*}{\partial M} = \frac{1}{\frac{\partial M}{\partial U}}$. It is a trivial one because these two are calculated from two different angles. For utility maximisation μ is used and for cost minimisation λ is used. The relation between μ and λ is such that $\mu = \frac{1}{\lambda}$. This leads to the conclusion that with the increase in the MU of the money income the MC of the production process decreases. When consumers find it very satisfactory in spending more money in consumption then producers are also encouraged in the production process.

Conclusion

The study concludes that there is an inter-linkage between the utility, cost and production. As we maximize utility and MU>0, the corresponding MC<0. This implies that this will be profitable for the producer and will motivate to produce more of that commodity. But in the long-run it will be the minimum point of the LAC curve which exhibits the CRS. Thus if we get any utility function of a commodity we can derive the corresponding demand cost and production functions. This is definitely an improvement of the consumer behaviour theory.

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