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RESEARCH ARTICLE

Q-FUZZY BI-IDEALS AND Q-FUZZY STRONG BI-IDEALS OF NEAR-RINGS

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ABSTRACT

In this paper we introduce the notation of Q-fuzzy bi-ideal and Q-fuzzy strong bi-ideal of a near-ring. We have discussed some of their theoretical properties in detail and obtain some characterization.

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INTRODUCTION

Zadeh [10] introduced the concept of sets in 1995. The notions ofQ-fuzzyideal and anti-fuzzy N-subgroup of a near-ring were introduced by Kim, Jun and Yon [Kuyng, 2005; Liu, 19824]. In this paper, we introduce thenotion of aanti-fuzzy strong bi-ideal of a near-ring We establish that every anti-fuzzy left N-subgroup or anti-fuzzy left ideal of a near-ring is aanti-fuzzy strong bi-ideal of a near-ring and also we establish that every left permutable fuzzy right N-subgroup or left permutableanti-fuzzy right ideal of a near-ring is aanti-fuzzy strong bi-ideal of a near-ring. But the converse is not necessarily true as shown by an example. Further, we discuss the properties of anti-fuzzy strong bi-ideal of a near-ring and provide example. Throughout this paper N will denote a right near-ring unless otherwise specified.

2. Preliminaries

Definition: 2.1

A nonempty set N together with two binary operations "+" and "·" is called be a near-ring [Gunter, 1983] if it satisfies the following axioms:

- (N,+) is a group.
- (N,·) is a semi group.
- $(x + y)\cdot z=(x\cdot z)+y\cdot z$, for every $x, y, z\in N$.

Note: 2.2

- Let X beanear-ring. Given two subsets A and B of X, $AB = \{ab/a\in A,b\in B\}$. Also we define another operation "∗"$A*B = \{a(b+i)-ab/a,b\in A,i\in B\}$.
- **(ii)** $0x = 0$. In general $x0 \neq 0$, for some x in N.

Definition: 2.3

A near-ring N is called **zero-symmetric**, if $x0 = 0$, for all x in N.

Definition: 2.4

A subgroup A of (N,+) is called a **bi-ideal** of near-ring N if $ANA\cap(AN)*A\subseteq A$.

Definition: 2.5

An element $a\in N$ is said to be regular if for each $a\in N$, $a = aba$, for some$b\in N$

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Definition: 2.6

A near-ring N is said to be left permutable near-ring if abc = bac, for all a,b,c in N.

Definition: 2.7

A function A from a non-empty set X to the unit interval [0,1] is called a fuzzy subset of N [14].

Notation: 2.8

Let A and B be two Q-fuzzy subsets of a semi group N. We define the relation $\subseteq$between A and B, the intersection and product of A and B, respectively as follows:

- **(i)**$A\subseteq B$ if $A(x,q) \leq B(x,q)$, for all $x\in N$ and $q\in Q$,
- **(ii)** $(A\cap B)(x,q) = \min\{A(x,q), B(x,q)\}$, for all $x\in N$ and $q\in Q$,
- $(A\circ B)(x,q) = \begin{cases} \sup_{x=yz}\{\min\{A(y,q), B(z,q)\}\} & \text{if } x = yz, \text{ for all } y, z\in N \text{and } q\in Q, \\ 0 & \text{otherwise} \end{cases}$

It is easily verified that the "product" of fuzzy subsets is associative. Throughout this paper, N will denote a near-ring unless otherwise specified. We denote by k_I the characteristic function of a subset I of N. The characteristic function of N is denoted by **N**, that is, **N** : $N\times Q\rightarrow [0,1]$ mapping every element of N to1.

Definition: 2.9

A function A: $N\times Q\rightarrow [0,1]$ is called a Q-fuzzy set.

Definition: 2.9

A Q-fuzzy subset A of a group (N,+) is said to be a Q-fuzzy subgroup of N if for all $x,y\in N$ and $q\in Q$,

- $A(x+y, q) \geq \min\{ A(x, q) , A(y, q) \}$
- $A(-x, q) = A(x, q)$,

Or equivalently $A(x - y, q) \geq \min\{A(x, q) , A(y, q)\}$.

Note: 2.10

If A is a Q-fuzzy subgroup of a group N, then $A(0, q) \geq A(x, q)$ for all $x\in N$ and $q\in Q$.

Definition: 2.11

A Q-fuzzy subset A of N is called a Q-fuzzy subnear-ring of N if for all $x,y\in N$ and $q\in Q$

- $A(x - y, q) \geq \min\{A(x, q) , A(y, q)\}$
- $A(xy, q) = \min\{A(x, q) , A(y, q)\}$

Definition: 2.12

A Q-fuzzy subset A of N is said to be a Q-fuzzy two-sided N-subgroup of N if

- A is a Q-fuzzy subgroup of (N,+),
- **(ii)**$A(xy, q)\geq A(x, q)$, for all $x,y\in N$ and $q\in Q$,
- $A(xy, q)\geq A(y, q)$, for all $x,y\in N$ and $q\in Q$.
- If A satisfies (i) and (ii), then A is called a Q-fuzzy right N-subgroup of N. If A satisfies (i) and (iii), then A is called a Q-fuzzy left N-subgroup of N.

Definition: 2.13

A Q-fuzzy subset A of N is said to be a Q-fuzzy ideal of N if

- A is a Q-fuzzy subnear-ring of N,
- $A(y+x-y, q) = A(x, q)$, for all $x, y\in N$ and $q\in Q$,
- $A(xy, q)\geq A(x, q)$, for all $x, y\in N$ and $q\in Q$,
- $A(a(b+i)-ab, q)\geq A(i, q)$, for all $a, b, i,\in N$ and $q\in Q$.

If A satisfies (i) and (ii) and (iii) then A is called a Q-fuzzy right ideal of N. If A satisfies (i), (ii) and (iv), then A is called a Q-fuzzy left ideal of N. In case of zero-symmetric, If A satisfies (i), (ii) and $A(xy, q)\geq A(y, q)$, for all $x, y\in N$, $q\in Q$ and A is called a Q-fuzzy left ideal of N.

Q-Fuzzy Bi-ideals of Near-Rings

Definition: 3.1.1

A Q-fuzzy subgroup A of N is called a Q-fuzzy bi-ideal of N if for all $x\in N$ and $q\in Q$, $((A\circ N\circ A)\cap(A\circ N)*A))(x, q) \leq A(x, q)$

Example: 3.1.1.1

Let N= {0,a,b,c} be the Klein's four group. Define multiplication in N as follows:

+	0	a	b	c
0	0	a	b	c
a	a	0	c	b
b	b	c	0	a
c	c	b	a	0

•	0	a	b	c
0	0	0	0	0
a	0	0	a	0
b	0	0	b	0
c	0	0	c	0

Then (N,+, •) is a near-ring (see([25], p.407, scheme 4). We define an Q-fuzzy set A in N as follows: $A(0, q) = 0.8$, $A(a, q) = 0.6$, $A(b, q) = 0.3 = A(c, q)$. Then $(A\circ\mathbf{N}\circ A)(0,q) = 0.3$, $(A\circ N\circ A)(a, q) = 0.3$, $(A\circ\mathbf{N}\circ A)(b, q) = 0.3$,$(A\circ\mathbf{N}\circ A)(c, q) = 0.3$. Therefore A is aQ-fuzzy bi-ideal of N.

+	0	a	b	c
0	0	a	b	c
a	a	0	c	b
b	b	c	0	a
c	c	b	a	0

•	0	a	b	c
0	0	0	0	0
a	0	0	a	0
b	0	0	b	0
c	0	0	c	0

Example: 3.1.1.2

Let N={0,a,b,c} be the Klein's four group. Define multiplication in N as follows:

Then (N,+, •) is a near-ring (see([25], p.407, scheme 4). We define an Q-fuzzy set A in N as follows: $(A\circ\mathbf{N}\circ A)(b, q) = 0.7$,$(A\circ\mathbf{N}\circ A)(c, q) = 0.2$. Here $(A\circ\mathbf{N}\circ A)(a, q) = 0.7 \nless A(0,q) = 0.3$. Hence A is not a Q-fuzzy bi-ideal of N.

Theorem: 3.1.3

Let $\{A_i: i\in I\}$ be any family of Q-fuzzy bi-ideals in a near-ring N. Then
$A = \bigcap_{i\in I} A_i$is a Q-fuzzy bi-ideal of N, where I be an index set.

Proof:

Let $\{A_i : i \in I\}$ be any family of Q-fuzzy bi-ideals of N. Now for all x, y∈N, and q∈Q ,

$A(x - y, q) = \inf A_i(x - y, q)$
$\geq \inf\{\min\{A_i(x, q), A_i(y, q) / i \in I\}\}$
(since A_i is a Q-fuzzy subgroup of N)
$= \min\{\inf A_i(x, q), \inf A_i(y, q) / i \in I\}$
$= \min\{\bigcap_{i\in I} A_i (x, q), \bigcap_{i\in I} A_i(y,q)\}$
$= \min\{A(x, q), A(y, q)\}$
This implies $A(x - y, q) \geq \min\{A(x, q), A(y, q)\}$

Therefore A is a Q-fuzzy subgroup of N.
Now for all x∈N, and q∈Q, Since $A = \bigcap_{i\in I} A_i$, for every i∈I, we have

$((A^\circ N^\circ A)\cap(A^\circ N)*A))\ (x, q) \leq ((A_i{}^\circ N^\circ A_i)\cap(A_i{}^\circ N)*A_i))\ (x, q) \leq A_i(x, q)$ for every i∈I.

(SinceA_i is a Q-fuzzy bi-ideal of N)

It follows that

$((A^\circ N^\circ A)\cap(A^\circ N)*A))\ (x, q) \leq \inf\{A_i(x, q) : i\in I\} = (\bigcap_{i\in I} A_i)(x,q) = A(x,q)$
Thus $((A^\circ N^\circ A)\cap(A^\circ N)*A))\ (x, q) \leq A(x, q)$.

Hence A is a Q-fuzzy bi-ideal of N.

Theorem: 3.1.4

Let A be any Q-fuzzy bi-ideal of a near-ring N. Then $A(xay, q) \geq \min\{A(x, q), A(y, q)\}$ for all x, y ∈N and q∈Q.

Proof

Assume that A is a Q- fuzzy bi-ideal of a near-ring N.
Then we know that $(A\circ \mathbf{N}\circ A)(a, q) \leq A(a, q)$.
Let x, a, y be any elements of N and q∈Q. Then
$A(xay, q) \geq (A\circ\mathbf{N}\circ A)(xay, q)$
$= \sup_{xay = x_1x_2} \min\{(A\circ\mathbf{N})(x_1, q), A(x_2, q)\}$
$\geq \min\{(A\circ\mathbf{N})(xa, q), A(y, q)\}$
$= \min\{\sup_{xa = z_1z_2} \min\{A(z_1, q), \mathbf{N}(z_2, q)\}, A(y, q)\}$
$\geq \min\{\min\{A(x, q), N(a, q)\}, A(y, q)\}$
$= \min\{\min\{A(x, q), 1\}, A(y, q)\}$
$= \min\{A(x, q), A(y, q)\}$
This shows that $A(xay, q) \geq \min\{A(x, q), A(y, q)\}$.

Theorem: 3.1.5

Let N be a regular near-ring. If A be any Q-fuzzy b-ideal of N, then

$A(a, q) = (A\circ\mathbf{N}\circ A)(a, q)$.

Proof

Let A be a Q-fuzzy bi-ideal of N and 'a' be any element of N and q∈Q. Since N is regular, there exists an element x in N such that a = axa, we have

$(A\circ\mathbf{N}\circ A)(a, q) = (A\circ\mathbf{N}\circ A)(axa, q)$
$= \sup_{axa = x_1x_2} \min\{(A\circ\mathbf{N})(x_1, q), A(x_2, q)\}$
$\geq \min\{(A\circ\mathbf{N})(ax, q), A(a, q)\}$
$= \min\{\sup_{ax = yz} \min\{A(y, q), \mathbf{N}(z, q)\}, A(a, q))\}$
$\geq \min\{\min\{A(a, q), \mathbf{N}(x, q)\}, A(a, q)\}$
$= \min\{\min\{A(a, q), 1\}, A(a, q)\}$
$= A(a, q)$

This shows that $(A\circ\mathbf{N}\circ A)(a, q) \geq A(a, q)$.
Since A is a Q-fuzzy bi-ideal of N, we have $(A\circ\mathbf{N}\circ A)(a, q) \leq A(a, q)$.

Therefore $A(a, q) = (A\circ\mathbf{N}\circ A)(a, q)$.

3.2 Q-fuzzy strong bi-ideals of Near-Rings

§3.2We shall now give the precise definition of a Q-fuzzy strong bi-ideal and illustrate this concept with suitable examples.

+	0	1	2	3
0	0	1	2	3
1	1	2	3	0
2	2	3	0	1
3	3	0	1	2

•	0	1	2	3
0	0	0	0	0
1	0	1	2	3
2	0	2	0	2
3	0	3	2	1

Definition: 3.2.1

A Q-fuzzy bi-ideal A of N is called a Q-fuzzy strong bi-ideal of N if

$(\mathbf{N}\circ A\circ A)(a, q) \leq A(a, q)$.

Example: 3.2.1.1

Let N={0,a,b,c} be a near-ring with two binary operations "+" and "•" is defined as follows.

+	0	a	b	c
0	0	a	b	c
a	a	0	c	b
b	b	c	0	a
c	c	b	a	0

•	0	a	b	c
0	0	0	0	0
a	0	0	a	0
b	0	0	b	0
c	0	0	c	0

Then (N,+, •) is a near-ring (see([25], p.407, scheme 4). We define a Q-fuzzy set A in N as follows: A(0, q) = 0.8, A(a, q) = 0.6, A(b, q) = 0.3 = A(c, q). Then $(A^\circ\mathbf{N}^\circ A)(0,q) = 0.3$, $(A^\circ\mathbf{N}^\circ A)(a, q) = 0.3$, $(A^\circ\mathbf{N}^\circ A)(b, q) = 0.3$,$(A^\circ\mathbf{N}^\circ A)(c, q) = 0.3$, $(\mathbf{N}\circ A\circ A)(0, q) = 0.3$, $(\mathbf{N}\circ A\circ A)(a, q) = 0.3$,$(\mathbf{N}\circ A\circ A)(b, q) = 0.3$, $(\mathbf{N}\circ A\circ A)(c) = 0.3$. Hence A is a Q-fuzzy strong bi-ideal of N.

Remark: 3.2.2

Every Q-fuzzy strong bi-ideal is a fuzzy bi-ideal but the converse is not true.

Example: 3.2.2.1

Let N={0, 1, 2, 3} be the Klein's four group. Define multiplication in N as follows:

Then (N,+, •) is a near-ring (see([25], p.407, scheme 11). We define an Q-fuzzy set A in N as follows: A(0, q) = 0.9, A(1, q)

= 0.7 = A(2, q) , A(3, q) = 0.4. Then (A∘**N**∘A)(0, q) = 0.9, (A∘**N**∘A)(1, q) = 0.7, (A∘**N**∘A)(2, q) = 0.7,(A∘**N**∘A)(3, q) = 0.4 Clearly A is a Q-fuzzy bi-ideal ofN. Also (**N**∘A∘A)(0, q) = 0.9, (**N**∘A∘A)(1, q) = 0.7,(**N**∘A∘A)(2, q) = 0.4, (**N**∘A∘A)(3, q) = 0.7$\nleq$ A(3, q) = 0.4. Therefore A is not aQ-fuzzy strong bi-ideal ofN.

Theorem: 3.2.3

Let N be a strongly regular near-ring. If A be any Q-fuzzy strong bi-ideal of N, then A = **N**∘A∘A**.**

Proof

Let A be a fuzzy strong bi-ideal of N and 'a' be any element of N**.** Then since N is strongly regular, there exists an element x in N such that a = xa^2, we have

$(\mathbf{N}\circ A\circ A)\,(a, q) = (\mathbf{N}\circ A\circ A)\,(xa^2,q)$

$= \sup_{xaa = x_1x_2} \min\{(N \circ A)(x_1, q), A(x_2, q)\}$

$\geq \min\{(N \circ A)(xa, q), A(a, q)\}$

$= \min\{\sup_{xa = yz} \min\{N(y, q), A(z, q)\}, A(a, q)\}$

$\geq \min\{\min\{N(x, q), A(a, q)\}, A(a, q)\}\}$

$= \min\{\min\{1,A(a, q), A(a, q)\}\}$

$= A(a, q).$

This means that **N**∘A∘A⊇A. Since A is a Q-fuzzy strong bi-ideal of N, we have **N**∘A∘A⊆A. Thus we have A = **N**∘A∘A**.**

Theorem: 3.2.4

Let R and L be a Q-fuzzy right N-subgroup and a Q-fuzzy left N-subgroup of N respectively. If A is any Q-fuzzy subgroup of N such that L∘ R ⊆A⊆ L∩R, then A is a Q-fuzzy strong bi-ideal of N.

Proof:

Assume that A is a Q-fuzzy subgroup of N, such that L∘ R ⊆A⊆ L∩R. Then **N**∘A∘A⊆**N**∘ (L∩R) ∘ (L∩R) ⊆ **N**∘L∘R (Since L∩R ⊆R and L∩R ⊆ L) ⊆ L∘R (Since **N**∘L ⊆ L) ⊆ A. This implies that **N**∘A∘A⊆A. And hence A is a Q-fuzzy strong bi-ideal o N.

Theorem: 3.2.5

Let N be a strongly regular near-ring. Then A∩B = A∘ B ∘B holds for every Q-fuzzy two-sided N-subgroup A of N and every Q-fuzzy strong bi-ideal B of N.

Proof

Let A be a Q-fuzzy two-sided N-subgroup and B be a Q-fuzzy strong bi-ideal ofN respectively, we have A∘ B ∘ B ⊆**N**∘ B ∘B ⊆B. Then A∘ B ∘ B ⊆A∘ **N** ∘**N**⊆A∘**N**⊆A.

Thus A∘ B ∘B ⊆A∩B. To prove the reverse inclusion, assume that 'a' be any element of N. Since N is strongly regular, there exists an element x in N such that a = xa^2 = ($xaxa^2$) = xxa^2xa^2. Since A is a fuzzy two-sided N-subgroup of N, we have A(xxa, q) ≥ A(xa, q)≥A(a, q). Since B is a fuzzy strong bi-ideal of N, we have B(a, q) = B(xa^2, q) ≥ min{B(a, q), B(a, q)} = B(a, q).

Then

$(A\circ B \circ B)(a,q) = \sup_{a = bc} \min\{\mathbf{A} \circ B)(b, q), B(c, q)\}$

$= \sup_{a=bc} \min\{ \sup_{b= b_1b_2} \min\{A(b_1, q), B(b_2, q)\}, B(c, q)\}$

$\geq \min\{(\mathbf{A} \circ B)(x^2a^2, q), B(xa^2, q)\}$

$= \min\{ \sup_{x^2a^2 = b_1b_2} \min\{A(b_1,q), B(b_2,q)\}, B(xa^2,q)\}$

$\geq \min\{\min\{A(xxa,q), B(a,q)\}, B(xa^2,q)\}$

$\geq \min\{\min\{A(a,q), B(a,,q)\}, B(a,q)\}$

$= \min\{A(a,q), B(a,q)\}$

$= (A\cap B)(a,q)$

And so A∘ B ∘ B ⊇A∩B

Thus A∩B = A∘ B ∘ B.

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