



RESEARCH ARTICLE

BIPOLAR VAGUE A-IDEALS OF BCI-ALGEBRAS

*¹Cicily Flora, S., ¹Arockiarani, I. and ²Jafari, S.

¹Department of Mathematics, Nirmala College for Women, Coimbatore-18, India

²College of Vestsjaelland South, Herrestraede 11, 4200 slagelse, Denmark

ARTICLE INFO

Article History:

Received 06th July, 2017
Received in revised form
10th August, 2017
Accepted 28th September, 2017
Published online 17th October, 2017

ABSTRACT

The notion of bipolar vague A-ideals of BCI-algebra is introduced and their properties are investigated. Characterizations of bipolar vague A-ideals are given. Relationship between bipolar vague ideal and the newly defined ideals are analyzed. Also we studied some equivalent conditions for bipolar vague A-ideals. And we show that a bipolar vague subalgebra and a bipolar vague ideal. And Finally, we prove theorem in bipolar A-ideal using bipolar vague level cut.

Key words:

BCI-algebra, Bipolar vague ideal,
Subalgebra, Bipolar vague A-ideal,
Bipolar vague level-cut.

Copyright©2017, Cicily Flora et al. This is an open access article distributed under the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

Citation: Cicily Flora, S., Arockiarani, I. and Jafari, S. 2017. "Bipolar vague a-ideals of bci-algebras", International Journal of Current Research, 9, (10), 58514-58520.

INTRODUCTION

Several authors from time to time have made a number of generalizations of Zadeh's fuzzy set theory (Zadeh, 1965). The notions of BCK/BCI-algebra were introduced by Iseki (1980) and were extensively investigated by many researchers. Liu and Meng (2004) introduced the notion of q-ideals and a-ideals in BCI-algebras. The study of fuzzy algebraic structure was started with the introduction of the concept of fuzzy sub-group in 1971 by Rosenfeld (1971). In 1993, Jun (1993, 1994) applied in BCI algebra. Of these, the notion of vague set theory was introduced by Gau and Buehrer (1993). Using this vague set, Biswas (2006) studied vague groups. Further Ramakrishnan and Eswarlal (2008, 2009) continued the study of vague algebra by studying the characterizations of cyclic groups in terms of vague groups, vague normal groups, vague normalizer, vague centralizer, vague ideals, normal vague ideals etc. Ganeshree Selvachandran (2012, 2016) introduced vague soft rings and vague soft ideals and studied some of their properties. Also Alhazaymeh (2012) introduced the concept of possibility vague soft set. Jun and Park (2007) studied vague ideals and vague deductive systems in subtraction algebras. In (Arsham Bourmand Saied, 2009), the concept of vague BCK/BCI-algebras is discussed. Lee (2004, 2000) introduced an extension of fuzzy sets named bipolar -valued fuzzy sets. Based on notion of bipolar- valued fuzzy sets, Jun and Song (2008) and Lee (2009) discussed subalgebra and ideals of BCH-algebras. The concept of bipolar vague fuzzy translation and bipolar-valued fuzzy S-extensions of a bipolar valued fuzzy subalgebra in BCK/BCI-algebra was introduced by Jun et al. (2009) In this paper, we apply the concept of bipolar vague A-ideals to BCI-algebras and investigate its properties. Also we discuss the relations among bipolar vague subalgebras, bipolar vague ideals and bipolar vague A-ideals.

Preliminaries

Definition 2.1 (Young Bae Jun et al., 2007) An algebra $(X; *, 0)$ of type $(2,0)$ is called a BCI-algebra if it satisfies the following conditions:

- (i) $((x * y) * (x * z)) * (z * y) = 0,$
- (ii) $(x * (x * y)) * y = 0,$

*Corresponding author: Cicily Flora, S. Department of Mathematics, Nirmala College for Women, Coimbatore-18, India

- (iii) $x * x = 0$,
 (iv) $x * y = 0, y * x = 0 \Rightarrow x = y$ for all $x, y, z \in X$

We can define a partial order ' \leq ' on X by $x \leq y$ if and only if $x * y = 0$. Any BCI-algebra X has the following properties:

- (1) $x * 0 = x$.
 (2) $(x * y) * z = (x * z) * y$.
 (3) $x \leq y \Rightarrow x * z \leq y * z, z * y \leq z * x$ for all $x, y, z \in X$

Definition 2.2 (Zadeh, 1965): Let X be a nonempty set. A fuzzy set A drawn from X is defined as $A = \{ \langle x, \sim_A(x) \rangle : x \in X \}$, where $\sim_A : X \rightarrow [0,1]$ is the membership function of the fuzzy set A .

Definition 2.3 (Lee, 2000): Let X be the universe. Then a bipolar valued fuzzy sets, A on X is defined by positive membership function \sim_A^+ , that is $\sim_A^+ : X \rightarrow [0,1]$, and a negative membership function \sim_A^- , that is $\sim_A^- : X \rightarrow [-1,0]$. For the sake of simplicity, we shall use the symbol $A = \{ \langle x, \sim_A^+(x), \sim_A^-(x) \rangle : x \in X \}$.

Definition 2.4 (Gau and Buehrer, 1993): A vague set A in the universe of discourse U is a pair $[t_A, f_A]$ where $t_A : U \rightarrow [0,1]$, $f_A : U \rightarrow [0,1]$ are the mappings (called truth membership function and false membership function respectively) where $t_A(x)$ is a lower bound of the grade of membership of x derived from the evidence for x and $f_A(x)$ is a lower bound on the negation of x derived from the evidence against x and $t_A(x) + f_A(x) \leq 1$ for all $x \in U$.

Definition 2.5 (Gau and Buehrer, 1993): The interval $[t_A(x), 1 - f_A(x)]$ is called the vague value of x in A , and it is denoted by $V_A(x)$. That is $V_A(x) = [t_A(x), 1 - f_A(x)]$.

Definition 2.6 (Gau and Buehrer, 1993): A vague set A of U with $t_A(x) = 1$ and $f_A(x) = 0 \quad \forall x \in U$, is called the unit vague set of U . A vague set A of U with $t_A(x) = 0$ and $f_A(x) = 1 \quad \forall x \in U$, is called the unit vague set of U .

Definition 2.7 (Gau and Buehrer, 1993): Let A be a non-empty set and the vague set A and B in the form $A = \{ \langle x, t_A(x), 1 - f_A(x) \rangle : x \in X \}$, $B = \{ \langle x, t_B(x), 1 - f_B(x) \rangle : x \in X \}$. Then

- (i) $A \subseteq B$ if and only if $t_A(x) \leq t_B(x)$ and $1 - f_A(x) \leq 1 - f_B(x)$
 (ii) $A \cup B = \max\{t_A(x), t_B(x)\}$ and $\max\{1 - f_A(x), 1 - f_B(x)\}$.
 (ii) $A \cap B = \min\{t_A(x), t_B(x)\}$ and $\min\{1 - f_A(x), 1 - f_B(x)\}$.
 (iv) $\bar{A} = \{ \langle x, f_A(x), 1 - t_A(x) \rangle : x \in X \}$.

Definition 2.8 (Cicily Flora and Arockiarani, 2016): Let X be the universe of discourse. A bipolar-valued vague set A in X is an object having the form $A = \{ \langle x, [t_A^+(x), 1 - f_A^+(x)], [-1 - f_A^-(x), t_A^-(x)] \rangle : x \in X \}$ where $[t_A^+, 1 - f_A^+] : X \rightarrow [0,1]$ and $[-1 - f_A^-, t_A^-] : X \rightarrow [-1,0]$ are the mapping such that $t_A^+ + f_A^+ \leq 1$ and $-1 \leq t_A^- + f_A^-$. The positive membership degree $[t_A^+(x), 1 - f_A^+(x)]$ denotes the satisfaction region of an element x to the property corresponding to a bipolar-valued set A and the negative membership degree $[-1 - f_A^-(x), t_A^-(x)]$ denotes the satisfaction region of x to some implicit counter property of A . For a sake of simplicity, we shall use the notion of bipolar vague set $v_A^+ = [t_A^+, 1 - f_A^+]$ and $v_A^- = [-1 - f_A^-, t_A^-]$.

Definition 2.9 (Arockiarani and Cicily Flora, 2016): A bipolar vague set $A = (X; v_A^+, v_A^-)$ in X is called a bipolar vague subalgebra of X if it satisfies:

$$v_A^+(x * y) \geq \min\{v_A^+(x), v_A^+(y)\}$$

$$v_A^-(x * y) \leq \max\{v_A^-(x), v_A^-(y)\}$$

for all $x, y \in X$, that is

$$t_A^+(x * y) \geq \min\{t_A^+(x), t_A^+(y)\}$$

$$1 - f_A^+(x * y) \geq \min\{1 - f_A^+(x), 1 - f_A^+(y)\}$$

$$t_A^-(x * y) \leq \max\{t_A^-(x), t_A^-(y)\}$$

$$1 - f_A^-(x * y) \leq \max\{1 - f_A^-(x), 1 - f_A^-(y)\}$$

Definition 2.10 (Arockiarani and Cicily Flora, 2016): A bipolar vague set $A = (X; v_A^+, v_A^-)$ of a BCK algebra X is called a bipolar vague ideal of X if the following conditions are true:

- (I) $v_A^+(0) \geq v_A^+(x)$ and $v_A^-(0) \leq v_A^-(x)$
- (II) $v_A^+(x) \geq \min\{v_A^+(x * y), v_A^+(y)\}$ and $v_A^-(x) \leq \max\{v_A^-(x * y), v_A^-(y)\}$

3. Bipolar vague A-ideal

Definition 3.1: A bipolar vague set $A = (X; V_A^+, V_A^-)$ in X is called a bipolar vague A-ideal of X if it satisfies :

- (i) $V_A^+(0) \geq V_A^+(x)$
- (ii) $V_A^-(0) \leq V_A^-(x)$
- (iii) $V_A^+(y * x) \geq \min\{V_A^+((x * z) * (0 * y)), V_A^+(z)\}$
- (iv) $V_A^-(y * x) \leq \max\{V_A^-((x * z) * (0 * y)), V_A^-(z)\}$ for all $x, y, z \in X$.

Example 3.2: Consider a BCI-algebra $X = \{0, a, b, c\}$ with the following Cayley table:

*	0	a	b	c
0	0	a	b	c
a	a	0	c	b
b	b	c	0	a
c	c	b	a	0

Define a bipolar vague set $A = (X; V_A^+, V_A^-)$ in X by

X	0	a	b	c
V_A^+	(0.6, 0.8)	(0.6, 0.8)	(0.3, 0.4)	(0.3, 0.4)
V_A^-	(-0.6, -0.4)	(-0.6, -0.4)	(-0.4, -0.3)	(-0.4, -0.3)

Then $A = (X; V_A^+, V_A^-)$ is a bipolar vague A-ideal of X .

Theorem 3.3: If $A = (X; V_A^+, V_A^-)$ is a bipolar vague A-ideal of X , then $V_A^-(x) = V_A^-(0 * x)$ and $V_A^+(x) = V_A^+(0 * x)$ for all $x \in X$.

Proof: Let $A = (X; V_A^+, V_A^-)$ be a bipolar vague a-ideal of X . Taking $y = z = 0$ in Definition 3.1 and using Definition 2.1(iii) and (1), we get $V_A^-(0 * x) \leq V_A^-(x)$ and $V_A^+(0 * x) \geq V_A^+(x) \dots *$

Setting $x = z = 0$ in Definition 3.1 and using Definition 2.1(iii), (*) we have $V_A^-(y) = V_A^-(y * 0) \leq V_A^-(0 * (0 * y)) \leq V_A^-(0 * y)$ and $V_A^+(y) = V_A^+(y * 0) \geq V_A^+(0 * (0 * y)) \geq V_A^+(0 * y)$ for all $y \in X$.

Hence, $V_A^-(x) = V_A^-(0 * x)$ and $V_A^+(x) = V_A^+(0 * x)$ for all $x \in X$.

Theorem 3.4: Every bipolar vague A-ideal of X is both a bipolar vague subalgebra of X and a bipolar vague ideal of X .

Proof: $A = (X; V_A^+, V_A^-)$ be a bipolar vague a-ideal of X . Using Definition 3.1, Theorem 3.3 we have

$$V_A^-(x) = V_A^-(0 * x) \leq \max\{V_A^-((x * z) * (0 * 0)), V_A^-(z)\}$$

$$= \max\{V_A^-(x * z), V_A^-(z)\} \text{ and}$$

$$V_A^+(x) = V_A^+(0 * x) \geq \min\{V_A^+((x * z) * (0 * 0)), V_A^+(z)\}$$

$$= \min\{V_A^+(x * z), V_A^+(z)\} \text{ for all } x, z \in X.$$

Hence $A = (X; V_A^+, V_A^-)$ is a bipolar vague ideal of X . Now for any $x, y \in X$. we obtain

$$V_A^-(x * y) \leq \max\{V_A^-((x * y) * x), V_A^-(x)\}$$

$$= \max\{V_A^-(0 * y), V_A^-(x)\} = \max\{V_A^-(x), V_A^-(y)\} \text{ and}$$

$$V_A^+(x * y) \geq \min\{V_A^+((x * y) * x), V_A^+(x)\}$$

$$= \min\{V_A^+(0 * y), V_A^+(x)\} = \min\{V_A^+(x), V_A^+(y)\}.$$

Therefore $A = (X; V_A^+, V_A^-)$ is a bipolar vague subalgebra of X .

The following example shows that the converse of the above need not be true.

Example 3.5: Let $X = \{0, a, b\}$ be a BCI-algebra with the following Cayley table:

*	0	a	b
0	0	b	a
a	a	0	b
b	b	a	0

Define a bipolar vague set $A = (X; V_A^+, V_A^-)$ in X by

X	0	a	b
V_A^+	(0.6,0.8)	(0.4,0.4)	(0.4,0.4)
V_A^-	(-0.8,-0.6)	(-0.7,-0.5)	(-0.7,-0.5)

Then $A = (X; V_A^+, V_A^-)$ is both a bipolar vague ideal and a bipolar vague subalgebra of X , but not a bipolar vague A-ideal of X .

Theorem 3.6: Let $A = (X; V_A^+, V_A^-)$ be a bipolar vague ideal of X . If the inequality $x * y \leq z$ holds in X . Then

$$V_A^-(x) \leq \max\{V_A^-(y), V_A^-(z)\}$$

$$V_A^+(x) \geq \min\{V_A^+(y), V_A^+(z)\}.$$

Proof: Let $x, y, z \in X$ be such that $x * y \leq z$. Then $(x * y) * z = 0$, and so

$$V_A^-(x) \leq \max\{V_A^-(x * y), V_A^-(y)\} \leq \max\{\max\{V_A^-((x * y) * z), V_A^-(z)\}, V_A^-(y)\}$$

$$= \max\{\max\{V_A^-(0), V_A^-(z)\}, V_A^-(y)\} = \max\{V_A^-(y), V_A^-(z)\} \text{ and}$$

$$V_A^+(x) \geq \min\{V_A^+(x * y), V_A^+(y)\} \geq \min\{\min\{V_A^+((x * y) * z), V_A^+(z)\}, V_A^+(y)\}$$

$$= \min\{\min\{V_A^+(0), V_A^+(z)\}, V_A^+(y)\} = \min\{V_A^+(y), V_A^+(z)\}.$$

Hence the proof.

Theorem 3.7: Let $A = (X; V_A^+, V_A^-)$ be a bipolar vague ideal of X . Then the following are equivalent.

(i) $A = (X; V_A^+, V_A^-)$ is a bipolar vague A-ideal of X .

(ii) $A = (X; V_A^+, V_A^-)$ satisfies the following conditions:

$$V_A^-(y * (x * z)) \leq V_A^-((x * z) * (0 * y))$$

$$V_A^+(y * (x * z)) \geq V_A^+((x * z) * (0 * y)) \text{ for all } x, y, z \in X.$$

(iii) $A = (X; V_A^+, V_A^-)$ satisfies the following conditions:

$$V_A^-(y * x) \leq V_A^-(x * (0 * y))$$

$$V_A^+(y * x) \geq V_A^+(x * (0 * y)) \text{ for all } x, y \in X.$$

Proof:

(i) \Rightarrow (ii) Assume that $A = (X; V_A^+, V_A^-)$ is a bipolar vague A-ideal of X and let $x, y, z \in X$ using Definition 3.1, we get $V_A^-(y * (x * z)) \leq \max\{V_A^-(((x * z) * 0) * (0 * y)), V_A^-(0)\} = V_A^-((x * z) * (0 * y))$ and $V_A^+(y * (x * z)) \geq \min\{V_A^+(((x * z) * 0) * (0 * y)), V_A^+(0)\} = V_A^+((x * z) * (0 * y))$.
 (ii) \Rightarrow (iii) taking $z=0$ in (ii) and using (1) induce (iii)
 (iii) \Rightarrow (i) Note that $(x * (0 * y)) * ((x * z) * (0 * y)) \leq x * (x * z) \leq z$ for all $x, y, z \in X$. It follows from (iii) and Theorem 3.6 that $V_A^-(y * x) \leq V_A^-(x * (0 * y)) \leq \max\{V_A^-((x * z) * (0 * y)), V_A^-(z)\}$ and $V_A^+(y * x) \geq V_A^+(x * (0 * y)) \geq \min\{V_A^+((x * z) * (0 * y)), V_A^+(z)\}$. Hence $A = (X; V_A^+, V_A^-)$ is a bipolar vague A-ideal of X .

Theorem 3.8: Assume that X is associative, i.e., X satisfies the following identity:

$$((x * y) * z = x * (y * z)). \text{ For all } x, y, z \in X, \text{ then every bipolar vague ideal is a bipolar vague A-ideal of } X.$$

Proof: Let $A = (X; V_A^+, V_A^-)$ be a bipolar vague ideal of a associative BCI-algebra X . Since $0 * x = x$ for all $x \in X$, it follows that $y * x = (0 * y) * x = (0 * x) * y = x * y = x * (0 * y)$ for all $x, y, z \in X$. Therefore $V_A^-(y * x) = V_A^-(x * (0 * y))$ and $V_A^+(y * x) = V_A^+(x * (0 * y))$ using Theorem 3.7, we conclude that $A = (X; V_A^+, V_A^-)$ is a bipolar vague A-ideal of X .

Theorem 3.9: Let $A = (X; V_A^+, V_A^-)$ be a bipolar vague A-ideal of X . Then the set $\Omega = \{x \in X \mid V_A^-(x) = V_A^-(0), V_A^+(x) = V_A^+(0)\}$ is an A-ideal of X .

Proof: Obviously, $0 \in \Omega$. Let $x, y, z \in X$ be such that $(x * z) * (0 * y) \in \Omega$ and $z \in \Omega$. Then $V_A^-(0) \leq V_A^-(y * x) \leq \max\{V_A^-((x * z) * (0 * y)), V_A^-(z)\} = V_A^-(0)$ and $V_A^+(0) \geq V_A^+(y * x) \geq \min\{V_A^+((x * z) * (0 * y)), V_A^+(z)\} = V_A^+(0)$ by using Definition 3.1. It follows that $V_A^-(y * x) = V_A^-(0)$ and $V_A^+(y * x) = V_A^+(0)$. That is, $y * x \in \Omega$. Therefore Ω is an A-ideal of X .

Theorem 3.10: If $A = (X; V_A^+, V_A^-)$ and $B = (X; V_B^+, V_B^-)$ be two bipolar vague A-ideal of BCI-algebra X , then $A \cap B$ is a bipolar vague A-ideal of X .

Proof: $V_A^-(0) \leq V_A^-(x)$ and $V_B^-(0) \leq V_B^-(x)$ for all $x \in X$. $\max\{V_A^-(0), V_B^-(0)\} \leq \max\{V_A^-(x), V_B^-(x)\} = V_{A \cap B}^-(0) \leq V_{A \cap B}^-(x)$ for all $x \in X$. To verify second condition, $V_A^-(y * x) \leq \max\{V_A^-((x * z) * (0 * y)), V_A^-(z)\}$ and $V_B^-(y * x) \leq \max\{V_B^-((x * z) * (0 * y)), V_B^-(z)\}$. $\max\{V_A^-(y * x), V_B^-(y * x)\} \leq \max\{\max\{V_A^-((x * z) * (0 * y)), V_A^-(z)\}, \max\{V_B^-((x * z) * (0 * y)), V_B^-(z)\}\}$
 $V_{A \cap B}^-(y * x) \leq \max\{\max\{V_A^-((x * z) * (0 * y)), V_B^-((x * z) * (0 * y))\}, \max\{V_A^-(z), V_B^-(z)\}\}$
 $V_{A \cap B}^-(y * x) \leq \max\{V_{A \cap B}^-((x * z) * (0 * y)), V_{A \cap B}^-(z)\}$. And $V_A^+(0) \geq V_A^-(x)$ and $V_B^+(0) \geq V_B^-(x)$ for all $x \in X$. $\min\{V_A^+(0), V_B^+(0)\} \geq \min\{V_A^+(x), V_B^+(x)\} = V_{A \cap B}^+(0) \geq V_{A \cap B}^+(x)$ for all $x \in X$. To verify second condition, $V_A^+(y * x) \geq \min\{V_A^+((x * z) * (0 * y)), V_A^+(z)\}$ and $V_B^+(y * x) \geq \min\{V_B^+((x * z) * (0 * y)), V_B^+(z)\}$. $\min\{V_A^+(y * x), V_B^+(y * x)\} \geq \min\{\min\{V_A^+((x * z) * (0 * y)), V_A^+(z)\}, \min\{V_B^+((x * z) * (0 * y)), V_B^+(z)\}\}$
 $V_{A \cap B}^+(y * x) \geq \min\{\min\{V_A^+((x * z) * (0 * y)), V_B^+((x * z) * (0 * y))\}, \min\{V_A^+(z), V_B^+(z)\}\}$
 $V_{A \cap B}^+(y * x) \geq \min\{V_{A \cap B}^+((x * z) * (0 * y)), V_{A \cap B}^+(z)\}$ for all $x, y, z \in X$. Therefore $A \cap B$ is a bipolar vague A-ideal of X .

Definition 3.11: For a bipolar vague set $A = (X; V_A^+, V_A^-)$ in X and $(\Gamma, S) \in [0,1]$ and $(\sim, \chi) \in [-1,0]$, the positive (Γ, S) -cut and negative (\sim, χ) -cut are denoted by $A_{(\Gamma, S)}^+$ and $A_{(\sim, \chi)}^-$, and are defined as follows:
 $A_{(\Gamma, S)}^+ = \{x \in X \mid t_A^+(x) \geq \Gamma \text{ and } 1 - f_A^+(x) \geq S\}$ and $A_{(\sim, \chi)}^- = \{x \in X \mid t_A^-(x) \leq \sim \text{ and } -1 - f_A^-(x) \leq \chi\}$, respectively

with $\Gamma + S \leq 1$ and $\sim + X \geq -1$. The bipolar level-cut of $A = (X; V_A^+, V_A^-)$ denoted by A_{cut} is denoted to be the set $A_{cut} = \langle A_{(\Gamma, S)}^+, A_{(\sim, X)}^- \rangle$

Theorem 3.12: A bipolar vague set $A = (X; V_A^+, V_A^-)$ in X is a bipolar vague a ideal of X if and only if for all $(\Gamma, S) \in [0, 1]$ and $(\sim, X) \in [-1, 0]$, the non-empty positive (Γ, S) -cut and the non-empty negative (\sim, X) -cut are bipolar vague A-ideals of X .

Proof: Let $A = (X; V_A^+, V_A^-)$ is a bipolar vague A-ideal of X and assume that $A_{(\Gamma, S)}^+$ and $A_{(\sim, X)}^-$ are non-empty for $(\Gamma, S) \in [0, 1]$ and $(\sim, X) \in [-1, 0]$. Obviously, $0 \in A_{(\Gamma, S)}^+ \cap A_{(\sim, X)}^-$. Let $x, y, z \in X$ be such that $t_A^-((x * z) * (0 * y)) \in A_{(\sim, X)}^-$ and $t_A^-(z) \in A_{(\sim, X)}^-$ also $-1 - f_A^-((x * z) * (0 * y)) \in A_{(\sim, X)}^-$ and $-1 - f_A^-(z) \in A_{(\sim, X)}^-$. Then $t_A^-((x * z) * (0 * y)) \leq \sim$ and $t_A^-(z) \leq \sim$ also $-1 - f_A^-((x * z) * (0 * y)) \leq X$ and $-1 - f_A^-(z) \leq X$. It follows from Definition 3.1 That $t_A^-(y * x) \leq \max\{t_A^-((x * z) * (0 * y)), t_A^-(z)\} \leq \sim$ and $-1 - f_A^-(y * x) \leq \max\{-1 - f_A^-((x * z) * (0 * y)), -1 - f_A^-(z)\} \leq X$ so that $y * x \in A_{(\sim, X)}^-$. Now assume that $t_A^+((x * z) * (0 * y)) \in A_{(\Gamma, S)}^+$ and $t_A^+(z) \in A_{(\Gamma, S)}^+$ also $1 - f_A^+((x * z) * (0 * y)) \in A_{(\Gamma, S)}^+$ and $1 - f_A^+(z) \in A_{(\Gamma, S)}^+$. Then $t_A^+((x * z) * (0 * y)) \geq \Gamma$ and $t_A^+(z) \geq \Gamma$ also $1 - f_A^+((x * z) * (0 * y)) \geq S$ and $1 - f_A^+(z) \geq S$ it follows from Definition 3.1 that $t_A^+(y * x) \geq \min\{t_A^+((x * z) * (0 * y)), t_A^+(z)\} \geq \Gamma$ and $1 - f_A^+(y * x) \geq \min\{1 - f_A^+((x * z) * (0 * y)), 1 - f_A^+(z)\} \geq S$ so that $y * x \in A_{(\Gamma, S)}^+$. Therefore $A_{(\Gamma, S)}^+$ and $A_{(\sim, X)}^-$ are an A-ideal of X .

Conversely, suppose that the non-empty negative (\sim, X) -cut and the non-empty positive (Γ, S) -cut are A-ideals of X for every $(\Gamma, S) \in [0, 1]$ and $(\sim, X) \in [-1, 0]$. If $t_A^-(0) > t_A^-(a)$, $-1 - f_A^-(0) > -1 - f_A^-(a)$ or $t_A^+(0) < t_A^+(b)$, $1 - f_A^+(0) < 1 - f_A^+(b)$ for some $a, b \in X$, then $0 \notin A_{(t_A^-(a), -1 - f_A^-(a))}^-$ or $0 \notin A_{(t_A^+(a), 1 - f_A^+(a))}^+$. This is a contradiction. Thus $t_A^-(0) \leq t_A^-(x)$, $-1 - f_A^-(0) \leq -1 - f_A^-(x)$ and $t_A^+(0) \geq t_A^+(x)$, $1 - f_A^+(0) \geq 1 - f_A^+(x)$ for all $x \in X$. Assume that $t_A^-(b * a) > \max\{t_A^-((a * c) * (0 * b)), t_A^-(c)\} = \sim$ and $-1 - f_A^-(b * a) > \max\{-1 - f_A^-((a * c) * (0 * b)), -1 - f_A^-(c)\} = X$ for some $a, b, c \in X$. Then $(a * c) * (0 * b) \in A_{(\sim, X)}^-$ and $c \in A_{(\sim, X)}^-$ but $b * a \notin A_{(\sim, X)}^-$. This is impossible, and thus $t_A^-(y * x) \leq \max\{t_A^-((x * z) * (0 * y)), t_A^-(z)\}$ for all $x, y, z \in X$. If $t_A^+(b * a) < \min\{t_A^+((a * c) * (0 * b)), t_A^+(c)\} = \Gamma$ and $1 - f_A^+(b * a) < \min\{1 - f_A^+((a * c) * (0 * b)), 1 - f_A^+(c)\} = S$ for some $a, b, c \in X$. Then $(a * c) * (0 * b) \in A_{(\Gamma, S)}^+$ and $c \in A_{(\Gamma, S)}^+$ but $b * a \notin A_{(\Gamma, S)}^+$. This is impossible, and thus $t_A^+(y * x) \geq \min\{t_A^+((x * z) * (0 * y)), t_A^+(z)\}$ for all $x, y, z \in X$. Consequently $A = (X; V_A^+, V_A^-)$ is a bipolar vague A-ideal of X .

REFERENCES

- Alhazaymeh, S, Nasruddin Hassan, 2012. Possibility Vague soft set and its application in Decision making, *International Journal of Pure and Applied Mathematics*, 77(4), 549-563.
- Arockiarani. I and Cicily Flora. S. 2016. Positive Implicative Bipolar Vague Ideals In BCK- algebras, *International Research Journal of Pure Algebra*, 6(8), 1-7.
- Arsham Bourmand Saied, 2009. Vague BCK/BCI Algebras, *Opuscula Mathematica*, 2(29), 177-186.
- Cicily Flora. S. and Arockiarani. I. 2016. A new class of generalized bipolar vague sets, *International Journal of Information Research and Review*, 3(11), 3058-3065.
- Eswaralal. T, Vague ideals and Normal Vague ideals in semirings, *International Journal of Computational Cognition*, 6(3), 2008, 60-65.
- Gau W. L, Buehrer. D.J. 1993. "Vague sets", *IEEE Transactions on Systems, Man and Cybernetics*, 23(2), 610-614.
- Ganeshsree Selvachandran and Abdul Razak Salleh, 2012. Vague soft Rings and Vague Soft ideals, *International Journal of Algebra*, 6(12), 557-572.
- Ganeshsree Selvachandran and Abdul Razak Salleh, 2016. Rings and Ideals in a Vague soft settings, *Far East Journal of Mathematical Sciences*, 99(2), 279-300.
- Hakimuddin Khan, Musheer Ahmad and Ranjit Biswas, 2007. "On Vague Groups", *International Journal of Computational Cognition*, 5(1), 27-30.

- Iseki. K. 1980. On BCI-algebras Math. Sem. Notes, 8, 125-130.
- Jun Y.B. 1993. Closed fuzzy ideals in BCI-algebra, Math. Japonica, 38(1), 199-202.
- Jun. Y.B and Meng. J. 1994. Fuzzy P-ideals in BCI-algebra, Math. Japonica, 40(2), 271-282.
- Jun. Y.B and Park. C.H. 2007. Vague ideals of subtraction algebra, Int. Math. Forum, 2(59), 2919-2926.
- Jun. Y.B, Kim. H.S and Lee. K.J. 2009. Bipolar fuzzy translations in BCK/BCI-algebra, *J. of the Chungcheong Math. Soc.*, 22(3), 399-408.
- Jun. Y.B and Song. S.Z, M. Otelbaev and B. Akpayev, 2008. Subalgebras and closed ideals of BCH-algebras based on bipolar valued fuzzy sets, *Scientiae Mathematicae Japonicae online*, 2, 427-437.
- Lee. K.M. 2004. Comparison of interval-valued fuzzy sets, intuitionistic fuzzy sets and bipolar-valued fuzzy sets, *J. Fuzzy Logic Intelligent*, 14(2), 125-129.
- Lee. K.M. 2000. Bipolar-valued fuzzy sets and their operations, Proc. Int. Conf. on Intelligent Technologies, Bangkok, Thailand, 307-312.
- Lee. K.J. 2009. Bipolar fuzzy subalgebras and bipolar fuzzy ideals of BCK-algebras, *Bull. Malays. Math. Sci. Soc.*, 32(3), 361-373.
- Liu. Y.Y, Meng. J, Zhang. H and Yue. Z.C. 2004. Q-ideals and a-ideals in BCI-algebras, *Southeast Asian Bull. Math.*, 24, 243-253.
- Park. C.H. 2008. Vague deductive systems of subtraction algebras, *J. Appl. Math. Comput.*, 26, 427-436.
- Ranjit Biswas, 2006. "Vague groups", *International Journal of Computational Cognition*, 4(2), 20-23.
- Ramakrishana. N.A. 2009. Characterizations of cyclic in terms of Vague groups, *Int. Journal of Computational Cognition*, 66(1), 913-916.
- Rosenfeld, 1971. Fuzzy groups, *J. Math. Anal. Appl.*, 35, 512-517.
- Young Bae Jun, Chul Hwan Park, SeokZunSong, 2007. Intuitionistic fuzzifications of a-ideals in BCI-algebras, *International Mathematical Forum*, 2(59), 2937-2950.
- Zadeh L.A. 1965. "Fuzzy sets", *Information and Control*, 8, 338-353.
