



RESEARCH ARTICLE

FATIGUE LIFE PREDICTION OF INVOLUTE SPUR GEAR BY FINITE ELEMENT METHOD

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ABSTRACT

Gears are vital components of any automobiles, power generation systems and in heavy machinery industries, they need to have good fatigue properties such as fatigue life, endurance limit and fatigue strength for better life and performance of the equipment or machinery. The objective of this study is comparative simulation on zero-based cyclic loading conditions in the fatigue life analysis on general gear materials, and concludes which suits for the better purpose of usage. The Finite Element Method (FEM) has been performed, using ANSYS Workbench 16.2, on the gear models to observe the distribution of stress and life. Comparison was done between Structure steel, Chromium molybdenum Alloy Steel 4130, GG 20 and GG 40. The FEM results of stresses were compared with theoretical results from American Gear Manufacturing Association (AGMA) contact and bending stresses and Hertz contact stress equations. Fatigue life equation derived based on Goodman line theory was used to estimate the minimum gear life using S-N characteristic diagram and FEM for validation purpose. The comparison showed that there is an accepted agreement between the theoretical and FEM results especially in the trend of variation. There is some deviation between the results which may need more effort and modification to decrease this deviation.

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INTRODUCTION

Gears are the most common means of transmitting power in the modern mechanical engineering world. They vary from tiny size used in the watches to the large gears used in marine speed reducers; bridge lifting mechanism and others. They form vital elements of main and ancillary mechanism in many machines such as Automobiles, tractors, metal cutting machine tools, rolling mills, hoisting and transmitting machinery and marine engines etc. The four major failure modes in gear systems are tooth bending fatigue, contact fatigue, surface wear and scoring. If the gear is input or output unit, the applied load is considered as zero-base cycle (cycle ratio $R=0.0$) and if the gear is idler, the applied load is fully reversed ($R=-1.0$). In the present study, the zero-based cycle is considered. Tooth breakage is clearly the worst damage case, since the gear could have seriously hampered operating condition or even be destroyed. Because of this, the stress in the tooth should always be carefully studied in all practical gear application. The fatigue process leading to tooth breakage is divided into crack initiation and crack propagation period. However, the crack initiation period generally account for the most of service life, especially in high cycle fatigue.

The initial crack can be formed due to various reasons. The most common reasons are short-term overload, material defects, defects due to mechanical or thermal treatment and material fatigue. The initial crack then propagates under impulsive loading until some critical length is reached, when a complete tooth breakage occurs. The service life of a gear with a crack in the tooth can be determined experimentally or numerically (e.g. with finite element method). Stress analysis on the gear tooth may help to predict the critical points which lead to the crack initiation. The scoring type of failure is usually lubrication related and can be corrected by proper lubricant selection and/or changes in gear operating conditions (Borsof, 1959). Tooth breakage is caused by tooth loads that produce bending stresses above the endurance limit of the material (Seabrook, 1964). It is usually accepted that the endurance limit, if it does exist, can be predicted from available stress-life (S-N) curves for the material being used (Rating the Strength of Spur Gear Teeth, AGMA 1966). Predicting gear surface-pitting failures are similar to those used for predicting the bending fatigue limit (Surface Durability, 1965). According to the method of reference (Huffaker, AGMA 1950). The maximum surface contact stress (Hertz stress) should be limited to a value less than the surface endurance limit of the gear material. It is commonly believed that the gears would then have an infinite surface-pitting life. But, based on gearing studies (Huffaker, 1950 and Schilke,

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1967), and on rolling-element bearing-life studies (Bisson, 1964), there is no real evidence to support the concept of a surface-fatigue limit under normal operating conditions of bearings and gears. Rather, it appears that all gears, even if designed properly to avoid failure by scoring and tooth-bending fatigue, will eventually succumb to surface pitting in much the same way as rolling-element bearings. A method of predicting the surface-fatigue lives of low contact-ratio spur gears was presented (Coy, 1975; Coy, 1975 and Coy, 1976). The method was based on the commonly accepted Lundberg-Palmgren theory that has been used for many years to predict the lives of rolling-element bearings. An attempt has been made (Jyothirmai, 2014), to compare the performance of various helical gear systems for a given set of specification through an analytical approach based on AGMA standards as well as a Finite Element Analysis approach. The developed FEA model was validated against the analytical approach and was found to be very close. Stress analysis of involute tooth spur gear was conducted (Bekheet, 2017), to estimate the contact and bending stresses using AGMA and 2D modeling of FEM. The results showed that there is an accepted validation between the FEM and the analytical method. Shaik Kalam and Abdul Hasan predicted fatigue analysis of spur gear (Chaminda, 2013). They did static and dynamic analysis, and fatigue life estimation of test gear, which is contacting a master gear and assuming the loading on gear is random or constant amplitude by Finite Element package ANSYS. Karthik and Chaitanya determined fatigue analysis of truck wheel rim (Karthik, 2015), under fully reversed loading and predicted the life.

Most of the previous work did not pay attention to using the FEM to predict the fatigue life of gears. The estimation of allowable surface fatigue stress, contact stress, surface fatigue strength, tooth surface strength of gears and permissible bending stress have not received much attention. In addition to the stresses results, fatigue life can be estimated for every element in the finite element model and contour of life can be plotted.

compute the fatigue life of a component and to predict the damage areas in components. Necessary inputs for the fatigue analysis are the material properties, loading history and the mean stress criterion such as Goodman line, Gerber line, Soderberg line and ASME line. In this study, Goodman line criterion is used.

THEORETICAL BENDING AND CONTACT STRESSES

As described above, theoretical calculation of the tooth bending and contact stresses are used to validate the results of FEM. This theoretical analysis is based on the using of AGMA standard code and the Hertzian theory of contact stresses.

The AGMA equation for bending stress is given by, (Budynas-Nisbett, 2006).

$$\sigma_b = F_t K_o K_v K_s \frac{1}{B \times m} \frac{K_H K_B}{J} \quad (1)$$

The AGMA equation for contact stresses at the contact point on the tooth surface are given by, (Budynas-Nisbett, 2006).

$$\sigma_c = \sigma_x = -C_p \sqrt{F_t K_o K_v K_s \frac{K_m K_F}{B \times d_p I}} \quad (2)$$

$$\sigma_y = 2\nu\sigma_x$$

See Figure 1. K_o , K_v , K_s , K_m , K_B , K_F , are AGMA correction factors for loading condition, dynamic, size, load distribution, rim thickness, and contact surface conditions modification respectively. Since the study is static conditions, all of these factors are assumed 1 for simplicity. C_p is the material elastic constant factor calculated from the following equation:

$$C_p = \sqrt{\frac{1}{\pi \left(\frac{(1-\nu_1^2)}{E_1} + \frac{(1-\nu_2^2)}{E_2} \right)}} \quad (3)$$

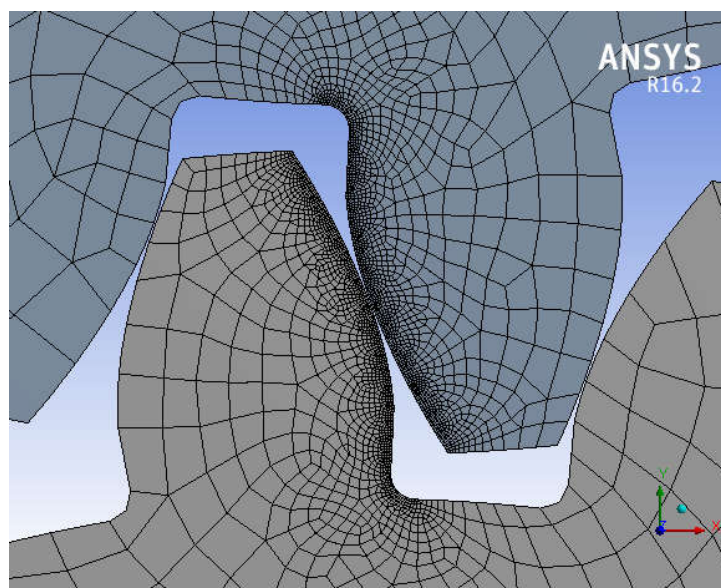


Figure 1. Teeth contact points

Geometry information provided by FE results define how an applied load is provided by FE results for each load case applied independently. Data have to be provided for the desired fatigue analysis method. The fatigue analysis is used to

ν_1 , E_1 , ν_2 and E_2 are poisson's ratio and elastic modulus for gear 1 and gear 2 respectively. $I = \frac{\sin \varphi \bullet \cos \varphi}{2m_N} \frac{m_G}{m_G + 1}$, m_N is

the contact ratio = 1 for spur gear, m_G is the reduction ratio which is 1 for equal diameter of driver and driven gears as assumed in this paper. J is the gear geometry factor (Budynas-Nisbett, 2006). Also, Hertz theory of contact stress is used considering the contact gear teeth as cylinders with a radii equal to the involute profile radii at the contact points. The Hertz equation is given by (Peter, 2004).

$$\sigma_c = \sigma_x = - \sqrt{\frac{F(1 + R_1/R_2)}{R_1 B \pi \left(\frac{(1 - \nu_1^2)}{E_1} + \frac{(1 - \nu_2^2)}{E_2} \right) \sin \phi}} \quad (4)$$

$$\sigma_y = \nu \sigma_x$$

See Figure 1. Where R_1 and R_2 are the respective radii of tooth profile curvature at the contact point. Thus

$$R_1 = R_2 = r_p \sin \phi$$

$F = T/r_p$ is the tangential force. Other parameters are defined in Tables 1 and 2. MatLAB R13a is used to write a code for calculating the bending and contact stresses.

Theoretical estimation of fatigue life

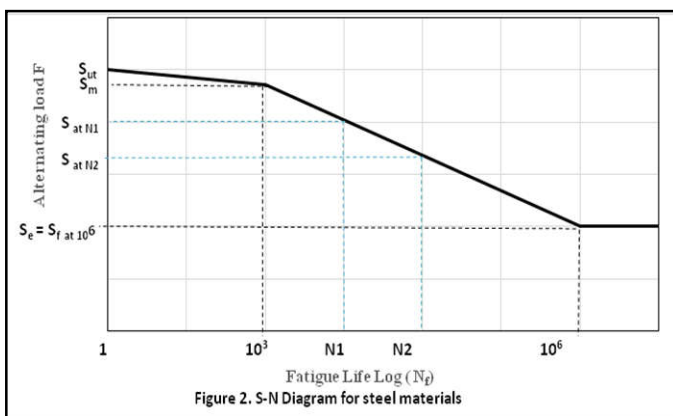
To estimate the fatigue life of the gear, fatigue data of materials must be provided. The S-N diagram shown in Figure 2 is the important data. S-N diagram, Figure 2, in the range of $10^3 < N < 10^6$ is mathematically modeled by the following equation: [15]

$$S_f = S_a = aN^b \quad (5)$$

Where;

$$a = \frac{(f \cdot S_{ut})^2}{S_e};$$

$$b = -\frac{1}{3} \log \left(\frac{f \cdot S_u}{S_e} \right) \quad (6)$$



$f = 0.78$ to 0.9 strength ratio at ($N = 10^3$) which depends on the ultimate strength of the material S_{ut} . For fully reversed cycle ($R = -1.0$, see Figure 3a) where, amplitude stress is σ_a , the fatigue life is estimate from the following equation:

$$N = \left(\frac{\sigma_a}{a} \right)^{1/b} \quad (7)$$

For low cycle fatigue $1 < N < 10^3$, $a = S_{ut}$, $b = \log(f)/3$, and the fatigue strength is given by:

$$S_f \geq S_{ut} N^{\log(f)/3} \quad (8)$$

For zero based cycle ($R = 0.0$, see Figure 3b), mean stress = amplitude stress ($\sigma_a = \sigma_m$), therefore a mean stress theory of fatigue such as Goodman line (Figure 4) can be used to drive the life equation. The equation of Goodman line is given by:

$$\frac{S_a}{S_e} + \frac{S_m}{S_{ut}} = 1 \quad (9)$$

From equations 5 and 9 the fatigue life for any cycle with a fluctuating ratio A is given by:

$$N = \left[\frac{1}{a} \left(\frac{A \cdot S_a \cdot S_{ut}}{A \cdot S_{ut} - S_a} \right) \right]^{1/b} \quad (10)$$

In this equation, the calculated σ_a from the applied loading cycle is substituted to S_a to calculate the fatigue life at the point where σ_a is applied. A is the fluctuating ratio defined as $S_a/S_m = \sigma_a/\sigma_m$ and σ_a , σ_m are the amplitude and mean stresses of the loading cycle. For the zero-based cycle $A = 1$. MatLAB code has been written to estimate the stresses values from equations 1 to 4 at the contact point and at the root of the tooth, see Figure 1. These component of stresses are used to estimate the equivalent stress at each point using the Von Mises and Maximum Shear Stress theories. The equivalent stresses is considered as the maximum stress σ_{max} of the zero-based cycle. The amplitude and mean stresses are determined as $\sigma_a = \sigma_m = \sigma_{max}/2$. Then the value of σ_a is substituted to S_a into equation 10 to calculate the life N at the point of interest.

FINITE element analysis

The objective of the current study is to calculate the fatigue life of a spur gear using total life method. To investigate the effect of mean stress on fatigue life with the help of the S-N curve, mean stress criterion such as Goodman line theory is used. The Von Mises stress and Maximum Shear stress theories are implemented to calculate the equivalent stress used to estimate the loading cycle as fully reversed or zero-based cycle. ANSYS then use the calculated cycle stress to estimate the fatigue life from the given S-N diagram of the material. This scenario is made at each node of the model for which the contour of fatigue life, damage, or safety factor are drawn. Fatigue damage is defined as the design life divided by the available life from the analysis. A damage of greater than 1 indicates the part will fail from fatigue before the design life is reached. The safety factor (SF) may be shown as a contour plot of the FS with respect to a fatigue failure at a given design life. The maximum FS reported is 15. By investigating the contour plots, the critical location on the body is defined and then a recommendation is given. In this study the fatigue life is considered and compared with that estimated values using the MatLAB for validation purpose. ANSYS is assuming that the life is equal to or greater than 10^6 if the applied equivalent alternating stress is less than or equal the endurance limit of the material.

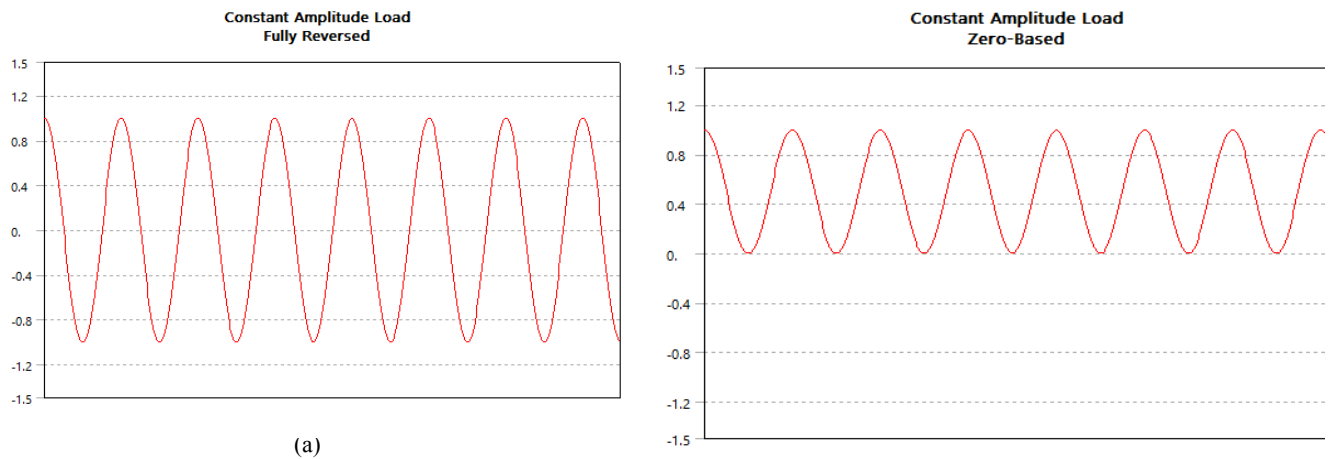


Figure 3. Fully reversed and zero-based cycles of load

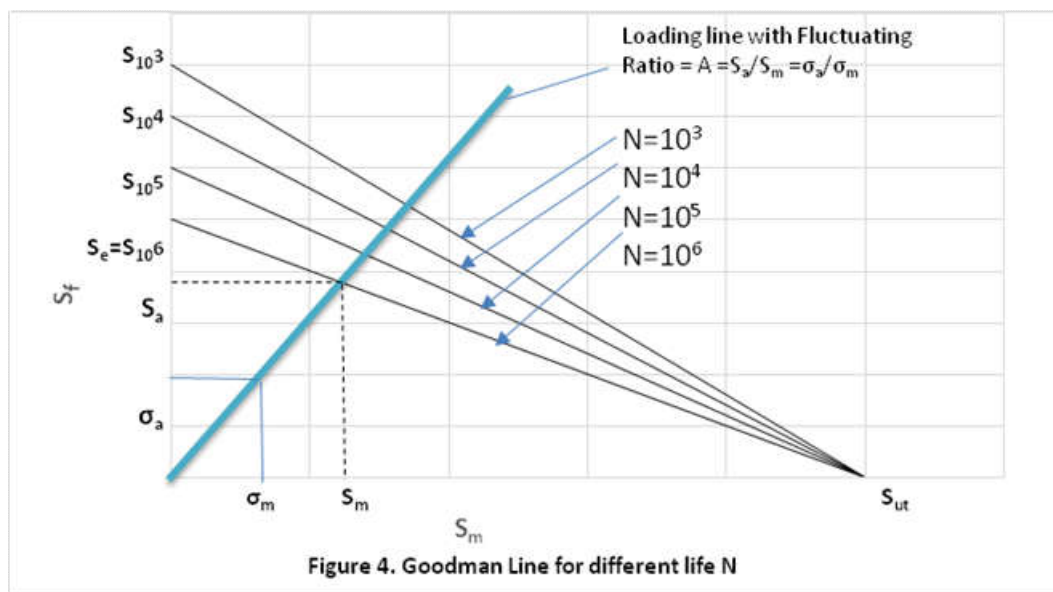


Figure 4. Goodman Line for different life N

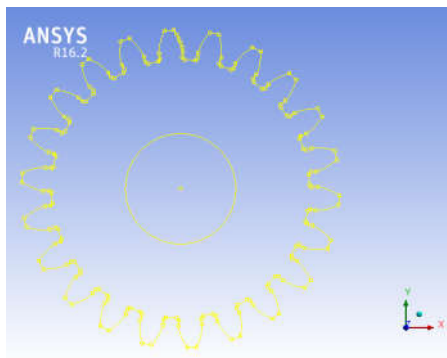


Figure 5 (a) Sketch of gear teeth

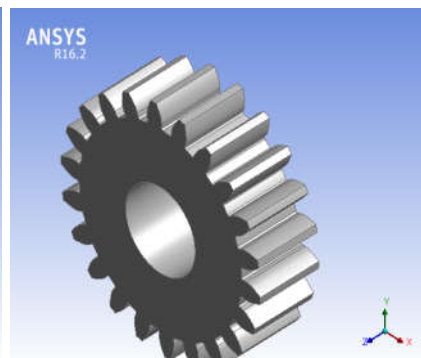


Figure 5(b) 3D gear

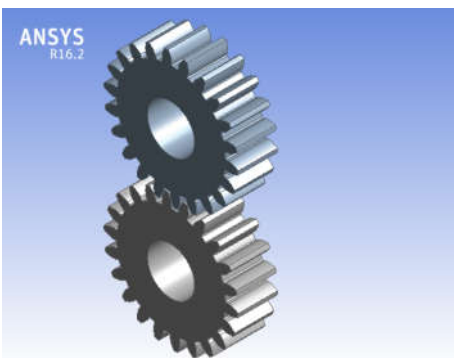


Figure 5(c) 3D gears set

In the following section, the geometry of the tested gears and the materials specification are detailed which are used in the MatLAB code and the FE model.

Spur gear geometry and modeling

To satisfy the fundamental law of gearing, tooth profile is usually cut to an involute curve (Peter, 2004). The construction of the tooth profile gear teeth are fully described in reference (Bekheet, 2017).

MatLAB code has been written to calculate the involute curve coordinates x and y . The curve is then constructed using ANSYS design modeler or any CAD software. Full number of gear teeth sketch are then constructed using array command as shown in Figure 5 (a). This sketch is then extruded to form the 3D spur gear body as shown in Figure 5(b). Using the translation command, the set of gear is constructed as shown in Figure 5 (c). If the required model is 2D, a surface is constructed from the gear teeth sketch using the concept of the generating surface from a sketch. The thickness of the surface

is then defined in mechanical modeler of ANSYS before conducting the analysis. The modeled spur gear dimensions are shown in Table 1 and material characteristics are given in Table 2. Different materials properties are considered to study the effect of elastic modulus on the gear tooth contact stresses, deformation and life.

Table 1. Spur gear specification

Descriptions	Data	Descriptions	Data
Number of teeth	22	Base circle diameter	$d_b = d_p \cos \phi$
Module	3 mm	Addendum circle diameter d_a	$d_a = (N+2)*m$
Pitch circle diameter $d_p = d_g = N*m$	66 mm	Addendum a	$a=m$
Face width B	21 mm	Deddendum b	$b=1.25 * m$
Deddendum circle diameter d_d	$d_d = d_p - 2*b$		

Table 2. Materials properties

Material	Elastic Modulus E [GPa]	Poisson's Ratio ν	Ultimate Strength S_{ut} [MPa]	Strength factor f	Endurance Limit S_e [MPa]
Structure steel	200	0.3	490	0.85	$0.5 S_{ut}$
Chromium –Molybdenum Alloy Steel 4130	206	0.29	670	0.85	$0.6 S_{ut}$
Gray Cast Iron GG-20	91	0.25	140	0.85	$0.4 S_{ut}$
Gray Cast Iron GG-30	130	0.25	275	0.85	$0.4 S_{ut}$

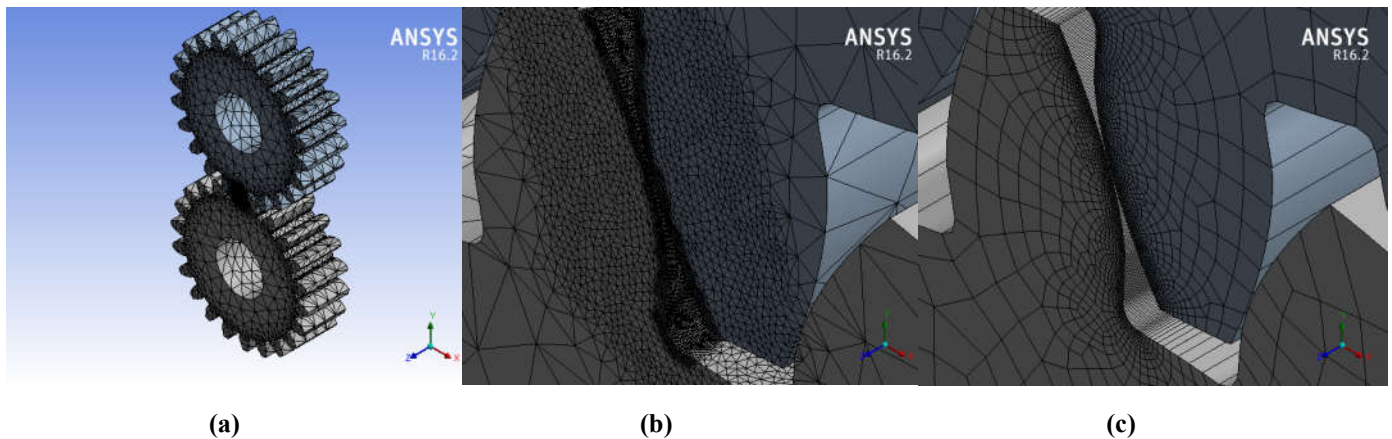


Figure 6. Gear set mesh

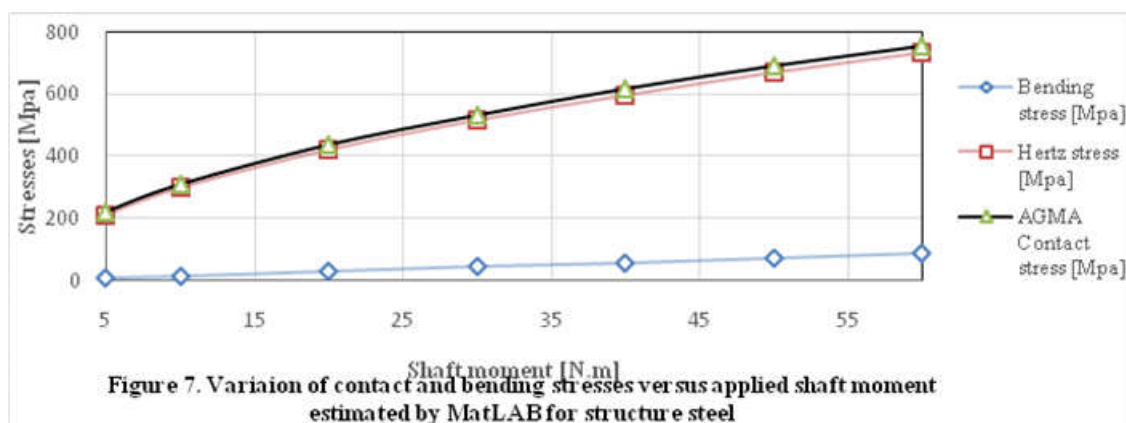


Figure 7. Variation of contact and bending stresses versus applied shaft moment estimated by MatLAB for structure steel

The constructed gears are meshed using Tetrahedron solid element and care must be taken for the refinement process of the mesh which is affecting the results. Figure 6 (a and b) shows the meshing of the gear and mesh refinement for 3D model. Refinement must be done on 3 steps; face meshing, refinement and contact zone refinement. Refinement of level 3 and refinement of contact zone with element size of 0.2 mm are implemented. In this scenario for 3D model a number of nodes of 4945966 and number of element of 3408682 are

achieved which are huge numbers and the CPU time is 143 minutes. On the other hand, the 2D model is meshed and refined as shown in Figure 6(c) with edge refinement of element size of 0.04mm but the number of elements in z direction is only one. The total number of nodes is 15133 and number of elements is 4733.

The CPU time is 10 s which is about 0.1% of the CPU time of 3D model. This lead to a more accurate results in 3D than that in 2D as will be seen in later sections.

RESULTS AND DISCUSSIONS

In this section the results of MatLAB algorithm and the FE modeling are analyzed and discussed. Bending and contact stresses calculated by AGMA and Hertz equations and fatigue

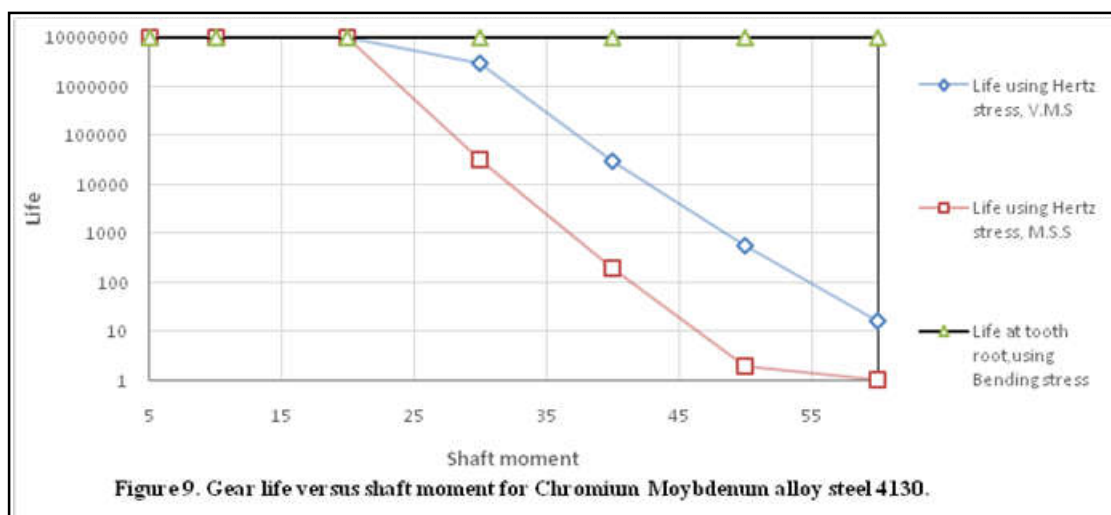
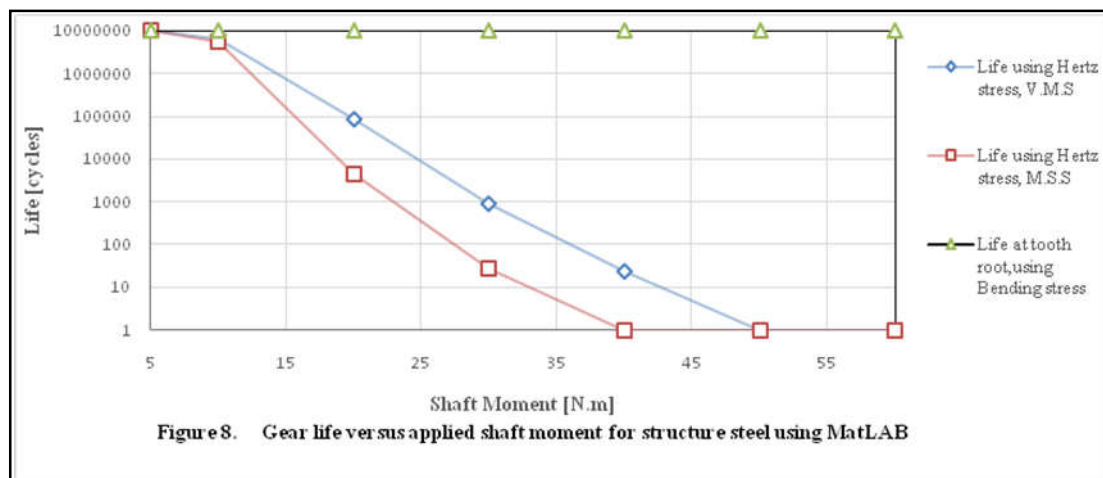
life calculated by the derived life equation 10 are compared with the results of FEM.

Theoretical results using MatLAB

Figure 7 shows the bending and contact stresses calculated by MatLAB. The used geometry is the same geometry used by (Bekheet, 2017) except the gear face width B . A 21 mm width is used in the present model while that was used previously is 7 mm. As expected, the stresses of the current model is less than those obtained by (Bekheet, 2017). The fatigue life estimated based on the equivalent Von Mises Stress (V.M.S) and Maximum Shear Stress (M.S.S) theories at the contact point and the normal bending stresses at the tooth root. Since the bending stress is low compared with the contact stress the life based on bending is infinity, see Figure 8. It can be seen that the life at the contact point is decreasing with the increasing of the applied shaft moment. For moment greater than 40 N.m the life is almost zero which indicate that crack may be initiated. The maximum shear theory is more conservative compared with the V.M.S theory since it shows less life, therefore, it will be considered through the rest of this study. Figure 9 shows the life for Chromium Molybdenum alloy steel 4130. As the ultimate strength of this material is 670 MPa it can resist more moment which may reach 50 N.m compared with 40 N.m for structure steel. A comparison of the gear life for the 4 materials examined is shown in Figure 10. The GG 20 is the weakest material.

Finite element results

The 3D modeling results of equivalent V.M.S, normal stress in y direction (bending), contact pressure and minimum life are shown in Figures 11 for structure steel and applied moment of 20 N.m. The variation of the V.M.S, bending and contact stresses with the applied shaft moment are shown in Figure 12. Similar results have been obtained for 2D model. The results of 2D shows less accuracy than 3D compared with the analytical results as shown in Figures 13 and 14. The bending stress at tooth root show very good agreement for 2D-FEM and 3D-FEM (38 MPa) and some deviation than the analytical result 30 MPa, see Figure 14. For contact stress, 422 MPa is obtained from MatLAB, 326 MPa from 3D-FEM and 255 MPa from 2D-FEM. It is clear that 3D is more accurate which shows a deviation of about 22.7% while the deviation is 40% for 2D-FEM, see Figure 13. This may be due to, in the 2D model, the total width of the gear (21mm) is considered as one element but in 3D model it is 105 elements. This affects the life of the gear. In 2D model the life is 1.9×10^4 , for 3D model is 6.5×10^4 , and analytically it is about 0.43×10^4 which are closed since the life is estimated by millions cycles. Because of 3D-FEM is more accurate, it was decided to use it for the rest of the study. The minimum fatigue life determined for each model are found at the location of maximum contact stresses as shown in Fig. 11 (d). The minimum life estimated based on contact stress using equivalent V.M.S and M.S.S together with the life estimated based on maximum normal bending stress



are shown in Figure 15. Life based on bending stress is above 10^6 cycles for all tested shaft moment. On the other hand, the life estimated based on contact stress are very close for both V.M.S and M.S.S and decreased from 10^6 at 10 N.m to few thousands at 60 N.m. The results are close to those obtained theoretically shown in Figures 8 and 9.

Fatigue strength of materials is affected by many factors such as surface roughness, component size, notch stress concentration and surface treatment and many other conditions. In the current analysis, the fatigue strength factor K_f is assumed to account for all of these factors.

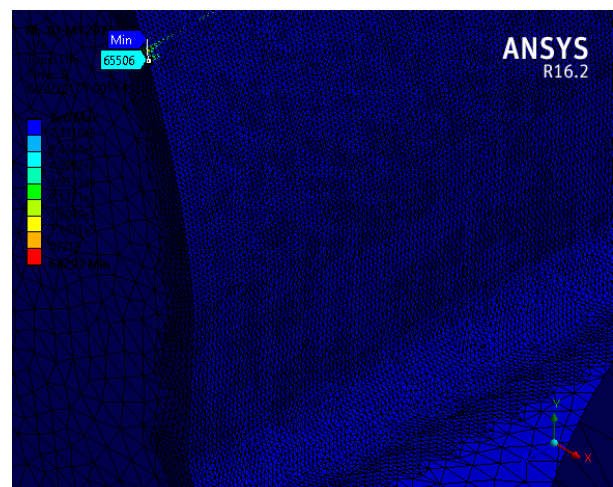
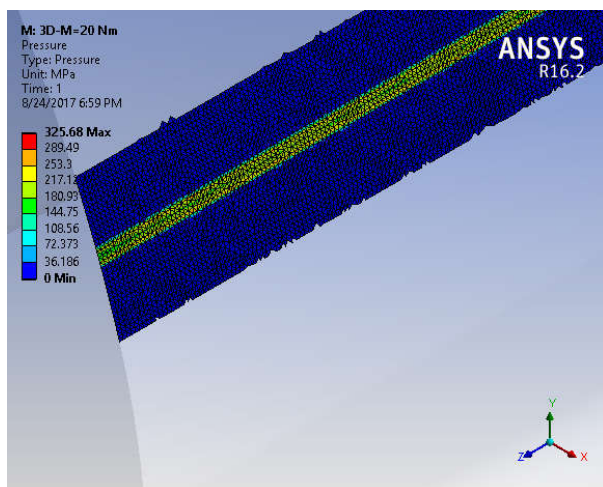
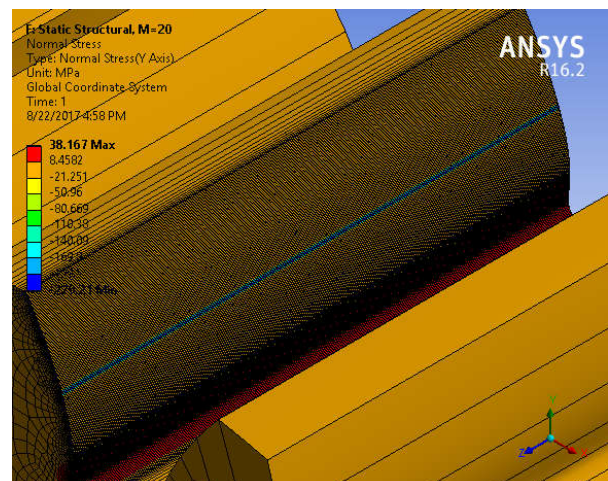
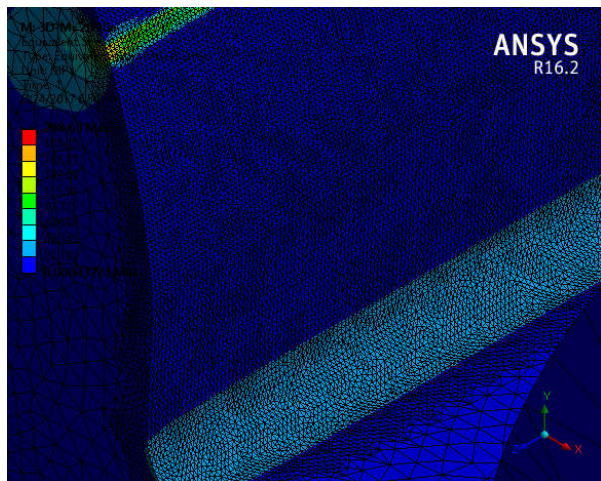
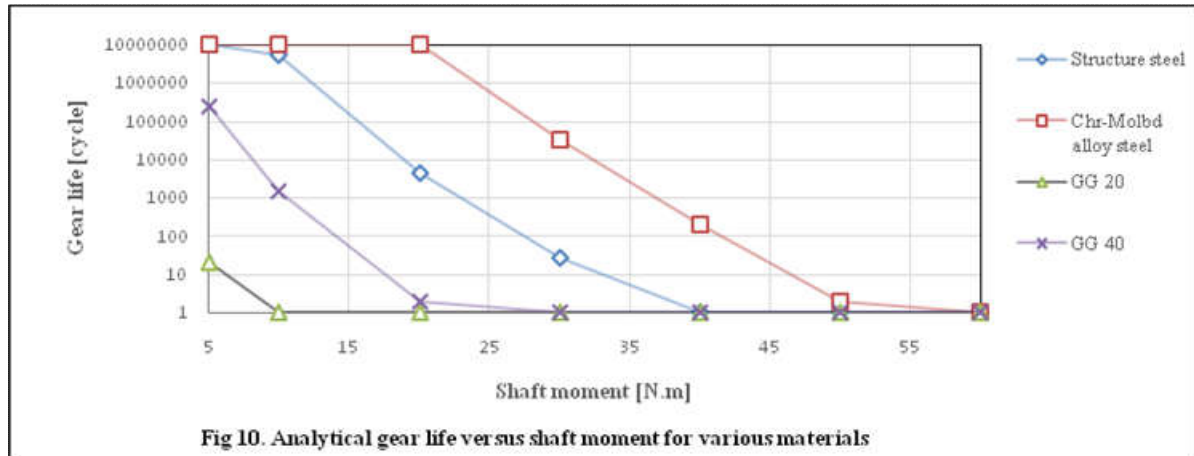
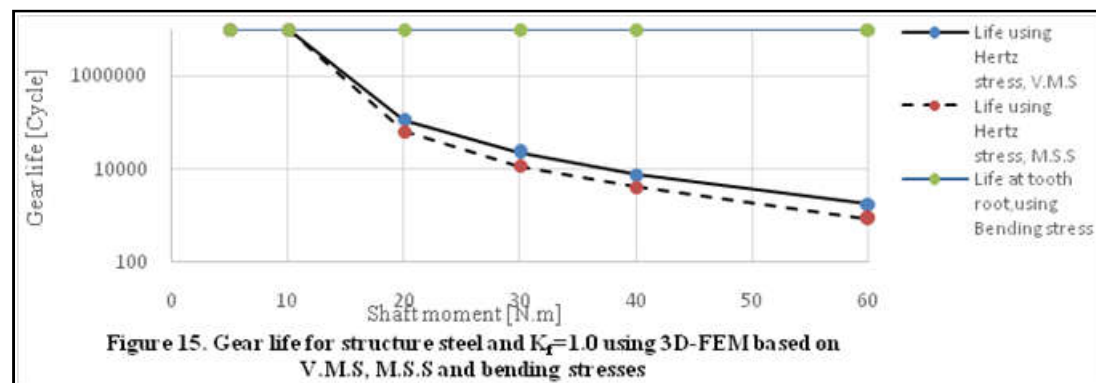
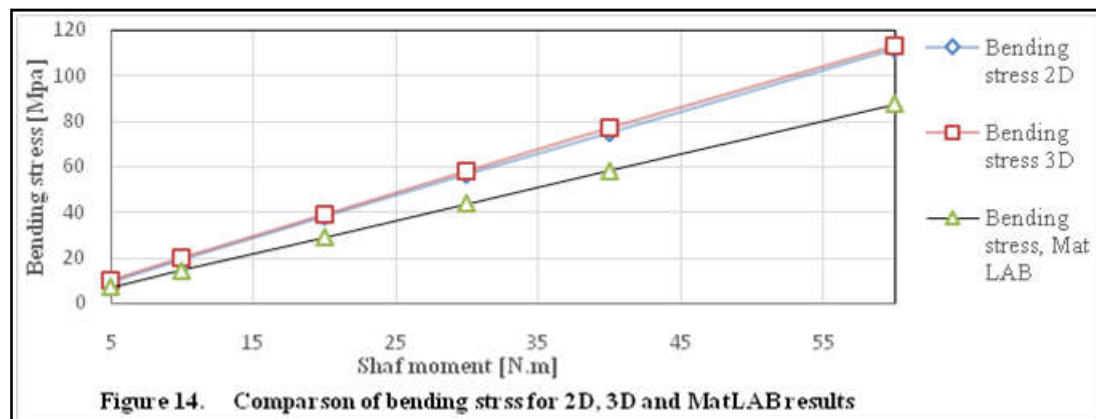
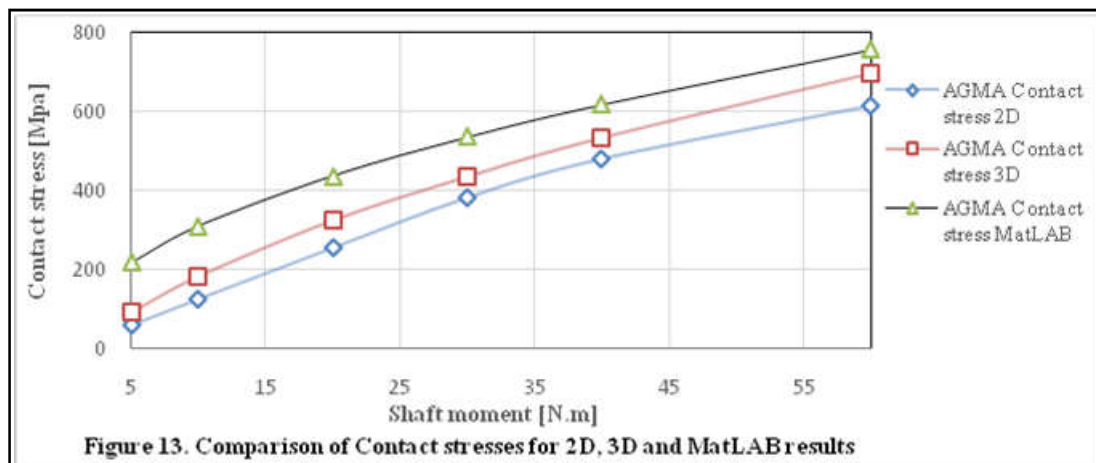
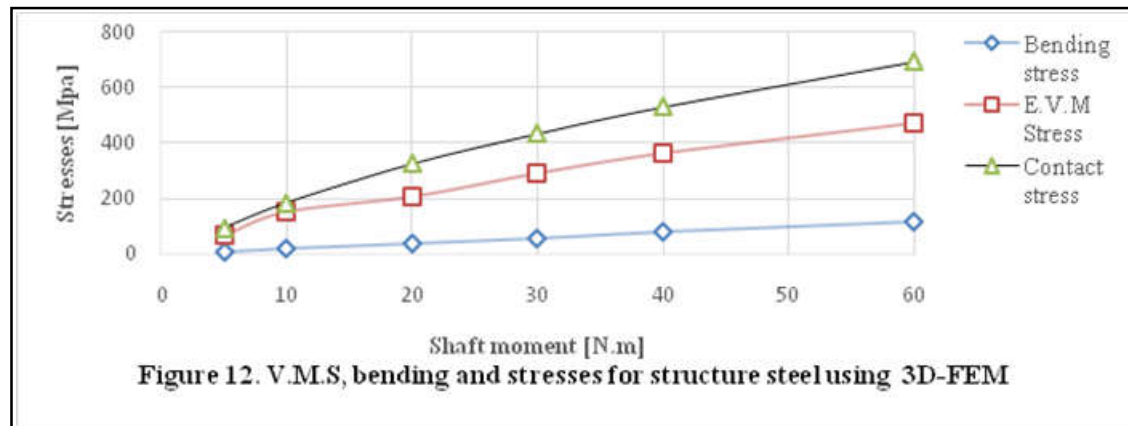
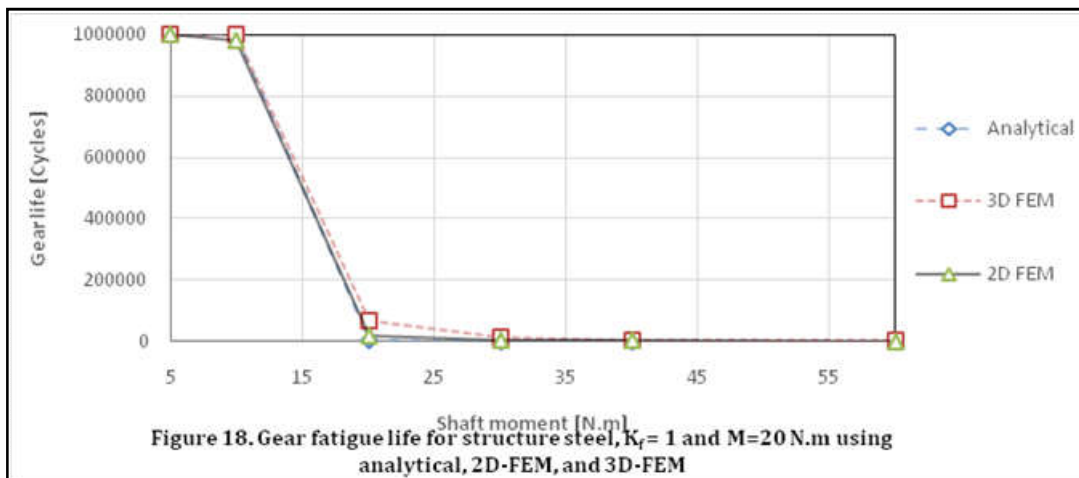
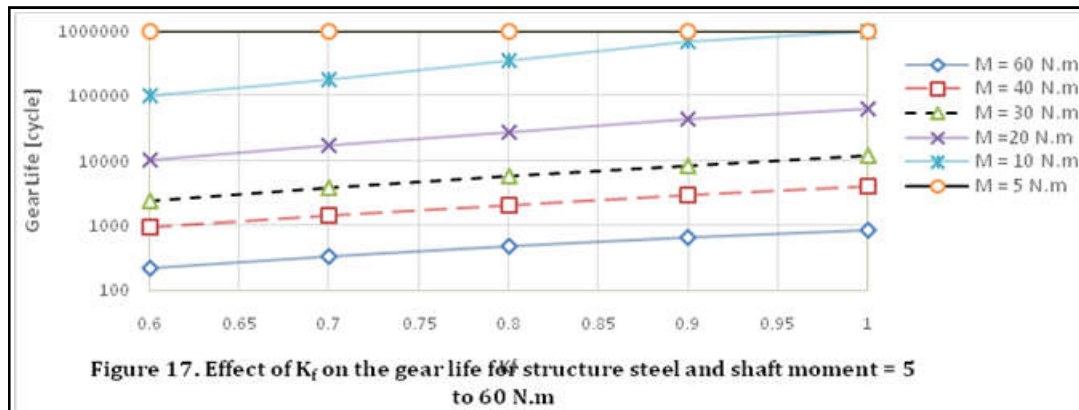
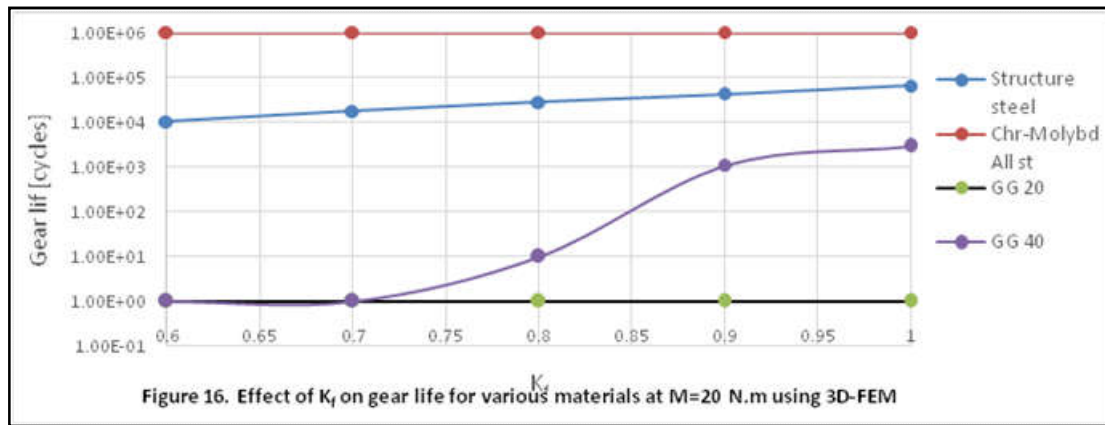


Figure 11. 3D FEM results for structure steel, applied moment = 20 N.m





The effect of K_f on the fatigue life is studied for all applied moments and tested materials. Values of K_f in the range 0.6 to 1.0 are investigated. The results are shown in Figure 16 for different materials and in Figure 17 for different applied shaft moments. It is clear that the life is very sensitive to K_f for GG 40 and less sensitive for structure steel. The GG 20 material is failed all values of k_f . The alloy steel 4130 live more than 10^6 cycles for all K_f values. The life of the gear is increased with the increase of K_f for all applied moment except very low moment, $M=5$ N.m, the gear experienced an infinity life. Very similar results have been achieved for 2D-FEM. The fatigue life estimated for structure steel, $M=20$ N.m and $K_f = 1.0$ using MatLAB, 2D-FM and 3D-FEM are shown in Figure 18. It can be seen from the figure that estimated life is very close for all methods.

Conclusion

The fatigue life estimation is important for design process of gears.

The total life method used in this study is able to estimate the fatigue life at the location of maximum contact stress and normal bending stress. The life estimated by FEM is in good agreement with that obtained analytically by AGMA and Hertz contact stress equations. The results also show that the fatigue life for all materials investigated is sensitive to the fatigue strength factor K_f and the GG 40 material is more sensitive. Although it is very expensive, the 3D-FEM is more accurate when it is fine meshed to the size of 0.2 mm compared with 0.04 mm used for 2D-FEM, therefore it is recommended to be used in design purpose. An important aspect of the fatigue process is plastic deformation. Fatigue cracks initiate from the plastic straining in localized regions. Significant localized plastic deformation is often present, therefore, cyclic strain-controlled fatigue method could better characterize the fatigue behavior of materials. The cyclic-controlled fatigue particularly in notched members where the significant localized plastic deformation is often present. In the crack initiation approach the plastic strain is directly measured and

quantified. The total-life approach does not account for plastic strain. In strain-life when the load history contains large over loads, significant plastic deformation can exist, particularly at stress concentrations and the load sequence effects can be significant. Therefore, it would be more accurate if similar study is done based on the strain-controlled fatigue.

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