



RESEARCH ARTICLE

MAGNETO-HYDRODYNAMIC COUPLE STRESS SQUEEZE FILM LUBRICATION OF
ROUGH ANNULAR PLATES

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ABSTRACT

In this paper, the combined effect of surface roughness and transverse magnetic field on the performance characteristic of the annular plates lubricated with conducting couple stress fluid (CCSF) has been studied. On the basis of the Christensen Stochastic model, the generalized stochastic Reynold's equation is derived. Modified equations for the non-dimensional pressure, load-carrying capacity and squeeze film time are derived. The results are presented both numerically and graphically and compared with conducting smooth surface case. It is observed the surface roughness effects are more pronounced for couple stresses as compared to non-conducting Newtonian fluid (NCNF) in the presence of magnetic field.

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INTRODUCTION

In recent years, the magneto hydrodynamic lubrication phenomenon has many industrial applications, because of increased use of liquid metal lubricants in high temperature. There is a possibility of increasing the load-carrying capacity by using lubricants in the presence of an applied magnetic field. Further, the above result has been used in modifying the squeeze-film action of bearing of an externally applied magnetic field. Many workers have made investigations on the hydrodynamic lubrication of rough surfaces using stochastic approaches. The effect of surface roughness on MHD lubrication flow between rectangular plates was analyzed by Bujurke and Kudenatti (2007). Effect of surface roughness on MHD squeeze film characteristics between finite rectangular plates is studied by Bujurke and Naduvinamani *et al.* (2011). Stochastic model for hydrodynamic lubrication of rough surfaces is studied by Christensen (1969). Naduvinamani *et al.* (2010) have undertaken a detailed study of hydromagnetic squeeze film between porous rectangular plates and reported that bearing characteristics such as pressure distribution, loadcapacity, and squeezing time seem to increase for increasing the Hartman number. Naduvinamani and Rajashekar (2012) studied the MHD couple stress squeeze-film

characteristics between a sphere and a plane surface. They found that the MHD squeeze-film pressure, load-carrying capacity, increases in the presence of externally applied magnetic field and for large values of Hartmann number, the response time also increases. The combined effect of MHD and couple stress has been studied by many authors (2011, 2014) and it is found that effect of couple stress in the presence of transverse magnetic field is significant on the squeeze film characteristics of the bearings. It is known that, the theory of couple stress fluid by Stokes (1966), is a generalization of viscous fluid theory with couple stresses and body couples. Couple stress fluids are consequence of the assumption that, the interaction of one part of the body on another across a surface is equivalent to a force and momentum distribution. It consists of rigid randomly oriented particles suspended in a viscous medium such as electro-rheological fluids and synthetic fluids. The MHD bearings with conducting fluids possess numerous advantages over the conventional bearings. Recently, Lin *et al.* (2012) studied hydro magnetic non-Newtonian cylindrical squeeze film and its application to circular plates by derivation of modified lubrication equation. They found that the improved characteristics are further emphasized for circular plates operating with a larger magnetic field parameter and non-Newtonian parameter. The effect of surface roughness on MHD conducting couple stress squeeze-film characteristics between the annular plates has not been studied so far. Hence, in this paper an attempt has been made

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to study the combined effect of surface roughness and conducting couple stresses on the MHD squeeze-film characteristics between annular plates. Expressions for the MHD squeeze-film pressure, load carrying capacity and the time height relation are obtained. Results are compared numerically with the corresponding conventional case for different values of Hartmann number M_0 studied by Lin (2012).

Mathematical Formulation of the Problem

Consider a squeezing flow between two rough annular plates approaching each other with squeezing velocity $w(= \frac{dH}{dt})$, where H is the film thickness between the two plates, p is the film pressure, u and w are the velocity components, μ is the lubricant viscosity, η is material constant responsible for couple stresses, σ being the conductivity of the lubricant. A uniform transverse magnetic field B_0 is applied to the bearing in the Z -direction as shown in the Figure 1. It is assumed that the fluid film is thin, the body forces and the body couples are negligible. The third term in Eq.[1] is a Lorentz body force coming from $J_y B_0$ under the assumption that the induced magnetic field is much less than the applied magnetic field as described by Kuzma (1964). Then, from Ohm's law the axial current density becomes $J_y = \sigma B_0 u$. Therefore the term $\sigma B_0^2 u$ appears in the Eq.[1].

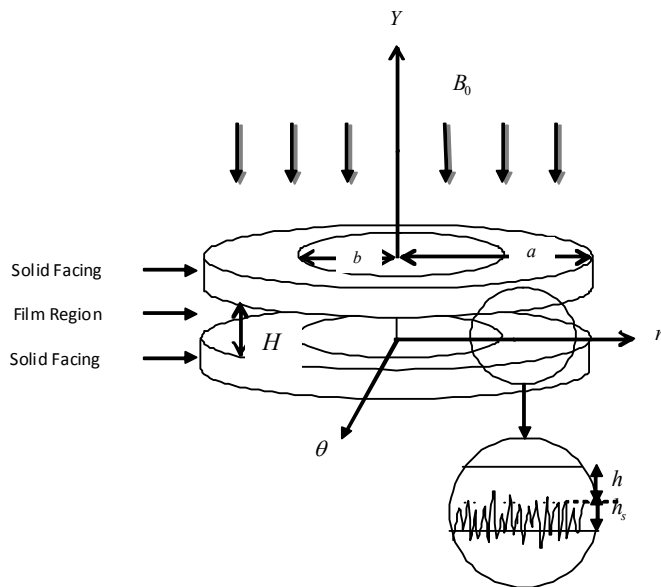


Figure 1. The physical configuration of rough annular plates in the presence of transverse magnetic field

Under the usual assumption of hydromagnetic lubrication theory applicable to thin films and Stokes theory for couple stresses, the continuity equation and the Magneto-hydrodynamic (MHD) momentum equations in polar form becomes

$$\frac{\partial^2 u}{\partial y^2} - \frac{\eta}{\mu} \frac{\partial^4 u}{\partial y^4} - \frac{\sigma B_0^2}{\mu} u = \frac{1}{\mu} \frac{\partial p}{\partial r} \dots\dots\dots (1)$$

$$\frac{\partial^2 v}{\partial y^2} - \frac{\eta}{\mu} \frac{\partial^4 v}{\partial y^4} - \frac{\sigma B_0^2}{\mu} v = \frac{1}{\mu} \frac{\partial p}{\partial \theta} \dots\dots\dots$$

$$(2) \quad \frac{\partial p}{\partial y} = 0 \dots\dots\dots (3)$$

$$\frac{1}{r} \frac{\partial}{\partial r} (ru) + \frac{1}{r^2} \frac{\partial v}{\partial \theta} + \frac{\partial w}{\partial y} = 0 \dots\dots\dots (4)$$

The relevant boundary conditions for the velocity components are

i) At the upper surface $y = H$

$$u = v = 0 \text{ (No Slip)}$$

$$\frac{\partial^2 u}{\partial y^2} = \frac{\partial^2 v}{\partial y^2} = 0 \text{ (Vanishing of couple stresses)} \dots\dots\dots (5a)$$

$$w = dH/dt \text{ (Squeezing Velocity)} \dots\dots\dots (5b)$$

ii) At the lower surface $y = 0$

$$u = v = 0 \text{ (No Slip)}$$

$$\frac{\partial^2 u}{\partial y^2} = \frac{\partial^2 v}{\partial y^2} = 0 \text{ (Vanishing of couple stresses)} \dots\dots\dots (6a)$$

$$w = 0 \dots\dots\dots (6b)$$

where $M_0 = B_0 h_0 (\sigma/\mu)^{1/2}$ is the Hartmann number.

The expressions for the radial component can be obtained by solving equation (1) subject to the boundary conditions eqns. (5) and (6) and is given by

$$u = -\frac{h_0^2}{\mu M_0^2} \frac{\partial p}{\partial r} \left\{ \frac{1}{(A^2 - B^2)} \left(\frac{B^2 \text{Cosh} \frac{A(2y-H)}{2l}}{\text{Cosh} \frac{AH}{2l}} - \frac{A^2 \text{Cosh} \frac{B(2y-H)}{2l}}{\text{Cosh} \frac{BH}{2l}} \right) + 1 \right\} \dots\dots\dots (7)$$

Similarly,

$$v = -\frac{h_0^2}{\mu M_0^2} \frac{\partial p}{\partial \theta} \left\{ \frac{1}{(A^2 - B^2)} \left(\frac{B^2 \text{Cosh} \frac{A(2y-H)}{2l}}{\text{Cosh} \frac{AH}{2l}} - \frac{A^2 \text{Cosh} \frac{B(2y-H)}{2l}}{\text{Cosh} \frac{BH}{2l}} \right) + 1 \right\} \dots\dots\dots (8)$$

Substituting (7) & (8) in (4) and integrating across the film thickness H gives

$$\frac{1}{r} \frac{\partial}{\partial r} \left\{ f(H, l, M_0) r \frac{\partial p}{\partial r} \right\} + \frac{1}{r^2} \frac{\partial}{\partial \theta} \left\{ f(H, l, M_0) \frac{\partial p}{\partial \theta} \right\} = \frac{dH}{dt} \dots\dots (9)$$

Where

$$f(H, l, M_0) = \frac{h_0^2}{\mu M_0^2} \left\{ \frac{2l}{A^2 - B^2} \left(\frac{B^2}{A} \tanh \frac{AH}{2l} - \frac{A^2}{B} \tanh \frac{BH}{2l} \right) + H \right\}$$

Let $f(h_s)$ be the probability density function of the stochastic film thickness h_s . Taking the stochastic average of (9) with respect to $f(h_s)$ the averaged modified Reynolds type equation is obtained in the form

$$\frac{1}{r} \frac{\partial}{\partial r} \left\{ E(f(H, l, M_0)) r \frac{\partial E(p)}{\partial r} \right\} + \frac{1}{r^2} \frac{\partial}{\partial \theta} \left\{ E(f(H, l, M_0)) \frac{\partial E(p)}{\partial \theta} \right\} = \frac{dH}{dt}$$

$$E(*) = \int_{-\infty}^{\infty} (*) f(h_s) dh_s$$

(11)

Following Christensen (1969), we assume that

$$f(h_s) = \begin{cases} \frac{35}{32c^7} (c^2 - h_s^2)^3 & -c < h_s < c \\ 0 & \text{elsewhere} \end{cases}$$

Where $\bar{\sigma} = \frac{c}{3}$ is the standard deviation.

In the context of Christensen stochastic theory for the hydrodynamic lubrication of rough surfaces, two types of one-dimensional surface roughness patterns are considered viz., radial roughness pattern and azimuthal roughness pattern. For one-dimensional radial roughness pattern, the roughness structure has the form of long, narrow ridges and valleys running in the radial direction (i.e. they are straight ridges and valley passing through $z = 0, r = 0$ to form star pattern), in the case the film thickness takes the form

$$H = h + h_s(\theta, \xi)$$

And the stochastic modified Reynolds equation (10) takes the form

$$\frac{1}{r} \frac{\partial}{\partial r} \left\{ E(f(H, l, M_0)) r \frac{\partial E(p)}{\partial r} \right\} + \frac{1}{r^2} \frac{\partial}{\partial \theta} \left\{ \frac{1}{E(1/(f(H, l, M_0)))} \frac{\partial E(p)}{\partial \theta} \right\} = \frac{dH}{dt}$$

For the one dimensional azimuthal roughness, the bearing surfaces have the form of long narrow ridges and valleys running in the θ -direction (i.e. they are circular ridges and valleys on the flat plate that are concentric on $(z = 0, r = 0)$). In this case the film thickness assumes the form

$$H = h + h_s(r, \xi)$$

The modified Reynolds type equation (10) takes the form

$$\frac{1}{r} \frac{\partial}{\partial r} \left\{ \frac{1}{E(1/(f(H, l, M_0)))} r \frac{\partial E(p)}{\partial r} \right\} + \frac{1}{r^2} \frac{\partial}{\partial \theta} \left\{ E(f(H, l, M_0)) \frac{\partial E(p)}{\partial \theta} \right\} = \frac{dH}{dt}$$

For an axisymmetric case, these equations reduce to

$$\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial E(p)}{\partial r} \right) = \frac{\mu M_0^2 dh/dt}{h_0^2 G(H, l, M_0, c)}$$

$$G(H, l, M_0, c) = \begin{cases} E(f(H, l, M_0)) & \text{For radial roughness} \\ \{E(1/(f(H, l, M_0)))\}^{-1} & \text{For azimuthal roughness} \end{cases}$$

$$E(f(H, l, M_0)) = \frac{35}{32c^7} \int_{-c}^c f(H, l, M_0) (c^2 - h_s^2)^3 dh_s$$

$$E\left(\frac{1}{f(H, l, M_0)}\right) = \frac{35}{32c^7} \int_{-c}^c \frac{(c^2 - h_s^2)^3}{f(H, l, M_0)} dh_s$$

Solution of equation (17) using boundary conditions

$$E(p(a)) = E(p(b)) = 0$$

$$E(p) = -\frac{\mu M_0^2 dh/dt (a^2 - b^2)}{4h_0^2 G(H, l, M_0, c)} \left(\frac{\log r/b}{\log a/b} - \frac{(r/b)^2 - 1}{(a/b)^2 - 1} \right)$$

The expressions for the pressure distribution and the load-supporting capacity in dimensionless form are

$$l^* = \frac{2l}{h_0}, \psi = \frac{k\delta}{h_0^3}, h^* = \frac{h}{h_0}, H^* = \frac{H}{h_0}, r^* = \frac{r}{b}, \alpha = \frac{a}{b}$$

$$P^* = -\frac{h_0^3 E(p)}{\mu (dh/dt) (a^2 - b^2) \pi} = \frac{M_0^2}{4\pi G(H^*, l^*, M_0, C)} \left\{ \frac{\log r^*}{\log \alpha} - \frac{(r^*)^2 - 1}{(\alpha)^2 - 1} \right\}$$

Where

$$G(H^*, l^*, M_0, C) = \begin{cases} E(F(H^*, l^*, M_0)) & \text{For radial roughness} \\ \{E(1/(F(H^*, l^*, M_0)))\}^{-1} & \text{For azimuthal roughness} \end{cases}$$

$$E(F(H^*, l^*, M_0)) = \frac{35}{32C^7} \int_{-C}^C F(H^*, l^*, M_0) (C^2 - h_s^{*2})^3 dh_s^*$$

$$E\left(\frac{1}{F(H^*, l^*, M_0)}\right) = \frac{35}{32C^7} \int_{-C}^C \frac{(C^2 - h_s^{*2})^3}{F(H^*, l^*, M_0)} dh_s^*$$

$$F(H, l^*, M_0) = \frac{l^*}{A^{*2} - B^{*2}} \left(\frac{B^{*2}}{A^*} \tanh \frac{A^* H^*}{l^*} - \frac{A^{*2}}{B^*} \tanh \frac{B^* H^*}{l^*} \right) + H^*$$

$$E(W) = 2\pi \int_b^a E(p) r dr$$

$$E(W) = -\frac{M_0^2 \mu \left(\frac{dh}{dt}\right) (a^2 - b^2)^2 \pi \left\{ \left(\frac{a}{b}\right)^2 + 1 \right.}{8h_0^2 G(H, l, M_0, c) \left. \left\{ \left(\frac{a}{b}\right)^2 - 1 \right. - \frac{1}{\log \frac{a}{b}} \right\}} \right\} \dots\dots\dots (22)$$

$$W^* = -\frac{E(W)h_0^3}{\mu \left(\frac{dh}{dt}\right) (a^2 - b^2)^2 \pi^2} = \frac{M_0^2}{8\pi G(H^*, l^*, M_0, C) \left\{ \left(\alpha\right)^2 + 1 \right. \left. \left\{ \left(\alpha\right)^2 - 1 \right. - \frac{1}{\log \alpha} \right\}} \right\} \dots\dots\dots (23)$$

$$t = -\frac{M_0^2 \mu \pi (a^2 - b^2)^2}{8E(W)h_0^2} \left\{ \left(\frac{a}{b}\right)^2 + 1 \right. \left. \left\{ \left(\frac{a}{b}\right)^2 - 1 \right. - \frac{1}{\log \frac{a}{b}} \right\}} \right\} \int_{h_0}^{h_1} \frac{1}{G(h, l, M_0, c)} dh$$

$$T^* = \frac{E(W)h_0^2 t}{\mu \pi^2 (a^2 - b^2)^2} = -\frac{M_0^2}{8\pi} \left\{ \left(\alpha\right)^2 + 1 \right. \left. \left\{ \left(\alpha\right)^2 - 1 \right. - \frac{1}{\log \alpha} \right\}} \right\} \int_1^{h_1^*} \frac{1}{G(h^*, l^*, M_0, C)} dh^* \dots\dots\dots (24)$$

Where $h_1^* = \frac{h_1}{h_0}$

RESULTS AND DISCUSSION

The combined effect of surface roughness and MHD on the conducting couplestress fluid lubrication between annular plates is analyzed on basis of Christensen’s stochastic theory (1969) for two types of roughness patterns. The squeeze film characteristics are analyzed with respect to three non-dimensional parameters namely the Hartmann number M_0^* , conducting couplestress parameter l^* and the roughness parameter C .

Non-dimensional Squeeze Film Pressure

Figure 2, 3, 4 depicts the variation of non-dimensional mean pressure P^* with the axial co-ordinate r^* as a function of the roughness, magnetic, and couplestress parameter C, M_0, l^* for both roughness patterns. It is observed that at $C=0$, the radial roughness pattern coincides with the azimuthal roughness pattern. Further, the increase in P^* is more pronounced for azimuthal roughness pattern as compared to radial roughness pattern with increasing values of C, M_0, l^* .

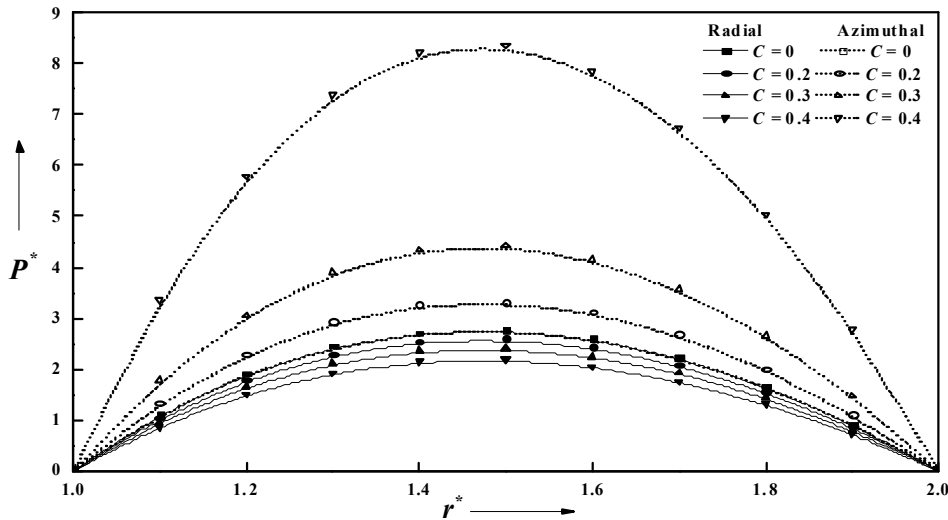


Fig. 2 Variation of non-dimensional P^* with r^* for different values of C with $M_0 = 3, h^* = 0.5, l^* = 0.3, \alpha = 2$.

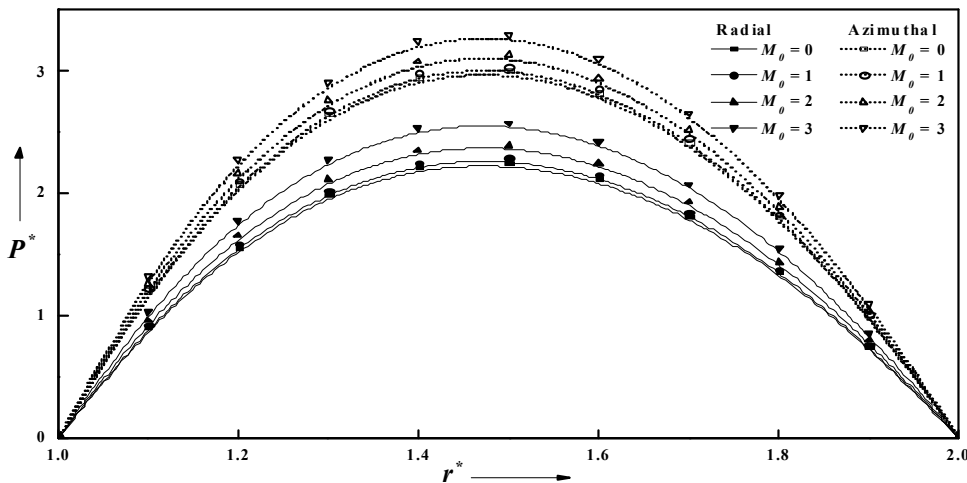


Fig. 3 Variation of non-dimensional P^* with r^* for different values of M_0 with $C = 0.2, h^* = 0.5, l^* = 0.3, \alpha = 2$.

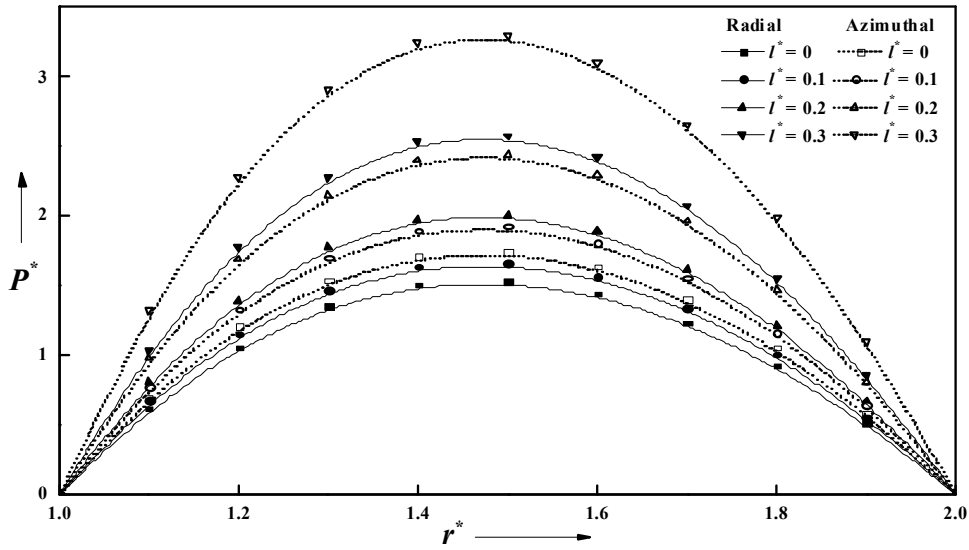


Fig. 4 Variation of non-dimensional P^* with r^* for different values of l^* with $M_0 = 3, C = 0.2, h^* = 0.5, \alpha = 2$.

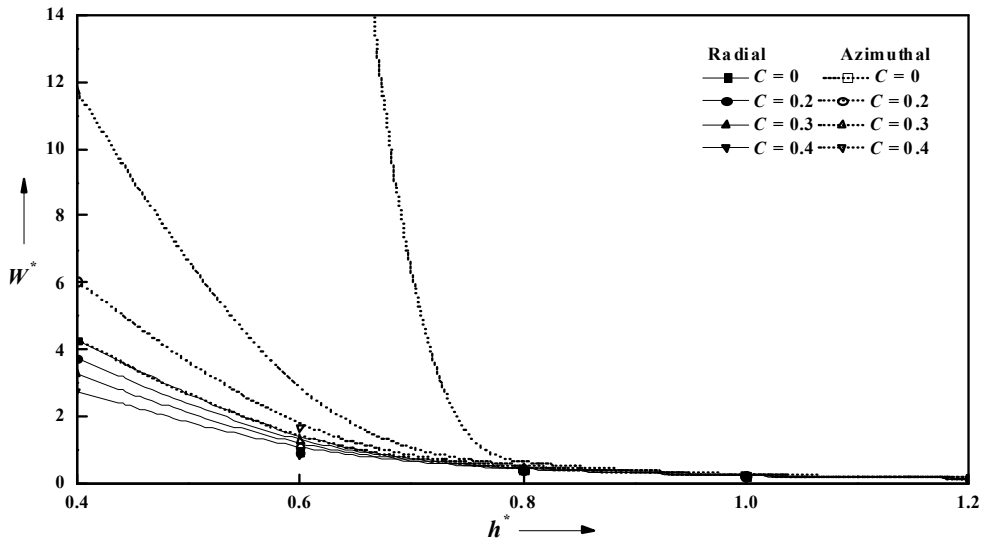


Fig. 5 Variation of non-dimensional W^* with h^* for different values of C with $M_0 = 3, l^* = 0.3, \alpha = 2$,

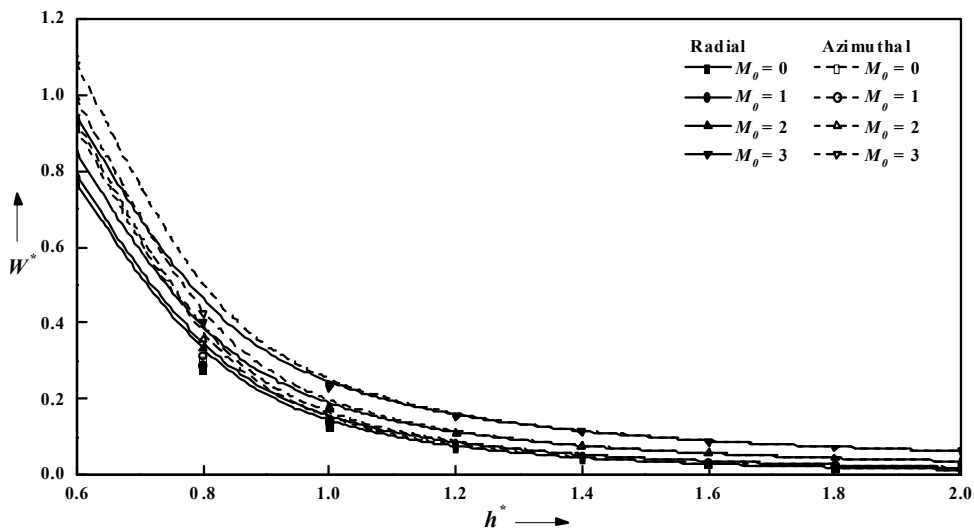


Fig. 6 Variation of non-dimensional W with h^* for different values of M_0 with $C = 0.2, l^* = 0.3, \alpha = 2$,

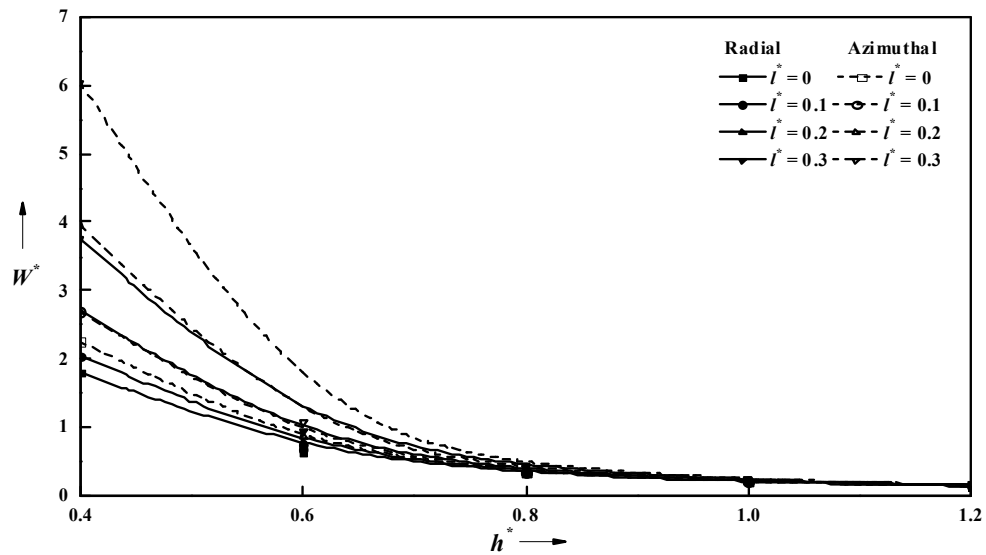


Fig. 7 Variation of non-dimensional W^* with h^* for different values of l^* with $C = 0.2, M_0 = 0.3, \alpha = 2,$

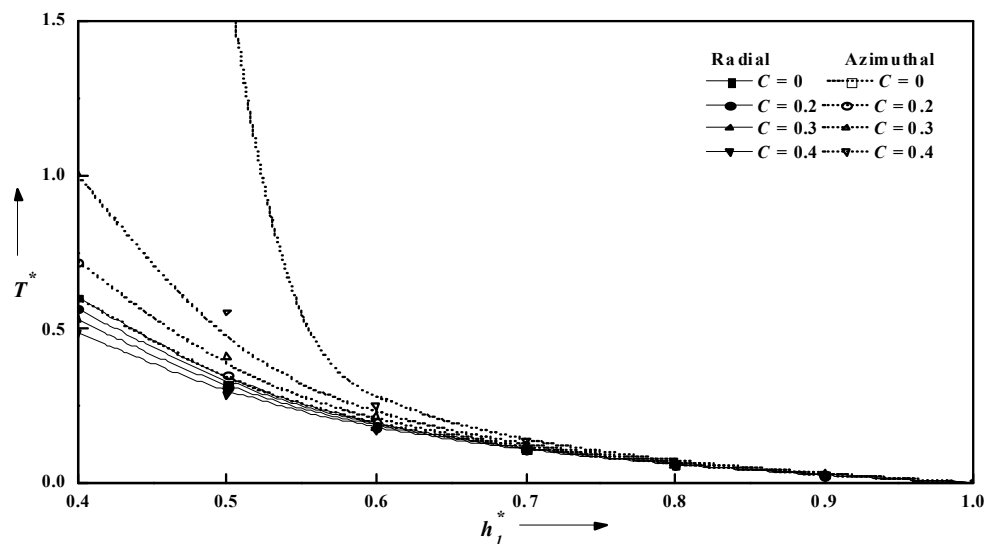


Fig. 8 Variation of non-dimensional T^* with h_1^* for different values of C with $M_0 = 3, l^* = 0.3, \alpha = 2,$

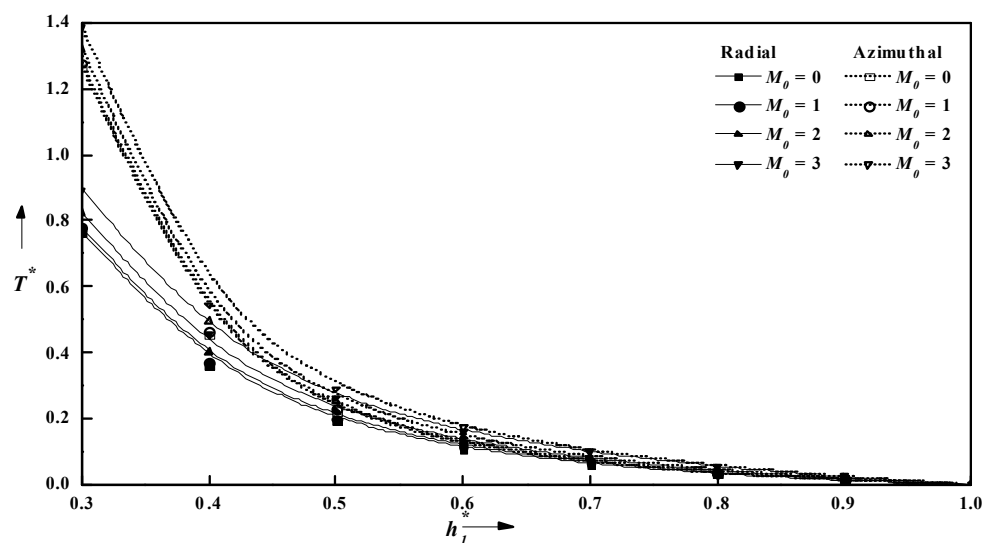


Fig. 9 Variation of non-dimensional T^* with h_1^* for different values of M_0 with $C = 0.2, l^* = 0.2, \alpha = 2,$

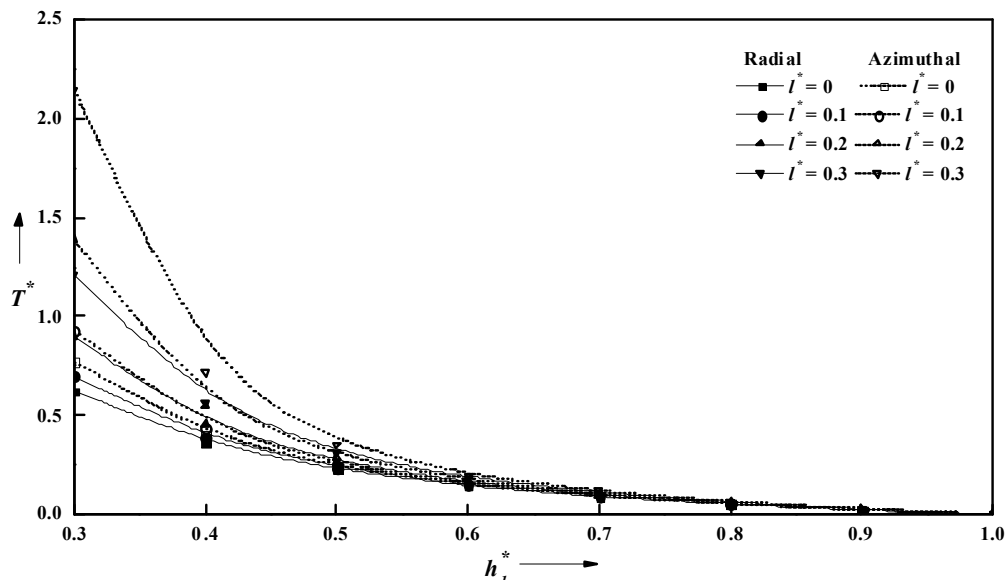


Fig. 10 Variation of non-dimensional T^* with h_1^* for different values of l^* with $C = 0.2, M_0 = 3, \alpha = 2$

Non-dimensional Load Carrying capacity

Figure 5,6,7 represents the variation of non-dimensional load carrying capacity w^* with h^* for different values of C, M_0, l^* . It is observed that with the increasing values of C, M_0, l^* there is significant increase in the load carrying capacity w^* . The effects are more prominent in rough annular plates for CCSF in the presence of transverse magnetic field. As seen from the figures that the load carrying capacity is significant with increasing values of C, M_0, l^* and the effects are more prominent for azimuthal roughness pattern than that of radial roughness pattern.

Non-dimensional Squeeze film time

Figure 8,9,10 represents Variation on non-dimensional squeeze film time T^* with h_1^* for different values of C, M_0, l^* . It is observed that with the increasing values of C, M_0, l^* there is significant increase in squeeze film time T^* .

It is observed that the effect of magnetic parameter is to enhance the squeezing effect for rough annular plates in the presence of CCSF as compared to smooth plates. It is observed that the effect of azimuthal/radial roughness patterns is to increase/decrease T^* as compared to conventional case (*i.e.* $C = 0$). The time of approach is more pronounced for azimuthal roughness patterns as compared to the radial roughness pattern.

Conclusion

The combined effect of surface roughness and MHD on the conducting couplestress squeeze-film characteristics between annular plates is presented on the basis of Christensen stochastic theory for rough surfaces(1969). From the above theoretical and numerical results the following conclusions can be drawn.

- In the presence of conducting couplestress fluid and externally applied transverse magnetic field, one dimensional radial (azimuthal) roughness patterns on the annular plates decrease (increase) the load-carrying capacity and response time as compared to the non-conducting case.
- As the surface asperity increases, the large amount of load is delivered in the bearing and enhances the response time of squeeze-film motion as compared to the smooth case.
- In the limiting case, as $C \rightarrow 0$ the result for both roughness patterns for one-dimensional annular plates can be reduced to smooth surface case.

Nomenclature

a radius of the annular plates

B_0 applied magnetic field in the z – direction

c maximum asperity deviation from the nominal film height

C dimensionless roughness parameter (c/h_0)

H film thickness

h_0 initial film thickness

h_1 film thickness at time t_1

h^* dimensionless film thickness

h_1^* dimensionless film thickness after time Δt

h_s stochastic film thickness

M_0 Magnetic Parameter $\left(= B_0 h_0 \left(\frac{\sigma}{\mu} \right)^{1/2} \right)$

l Couplestress fluids $\left(\frac{\eta}{\mu} \right)^{1/2}$

l^* Dimensionless parameter $\frac{2l}{h_0}$

r, θ, z Radial, angular and axial coordinates

u, v, w Velocity components in film region

E(P) Pressure in the film region

$$P^* \text{ dimensionless pressure} \left(= -\frac{E(p)h_0^3}{\mu(dH/dt)\pi a^2} \right)$$

t mean time of approach

$$T^* \text{ Dimensionless time of approach} \left(= -\frac{tE(W)h_0^2}{\mu\pi^2 a^4} \right)$$

$$V \text{ Squeezing Velocity} \left(-\frac{dH}{dt} \right)$$

$$W^* \text{ Dimensionless load carrying capacity} \left(= -\frac{E(W)h_0^3}{\mu(dH/dt)a^4\pi^2} \right)$$

σ Conductivity of fluid

$\bar{\sigma}$ standard deviation

η material constant responsible for couplestress

μ lubricant viscosity

ξ random variable

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