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RESEARCH ARTICLE

STUDY OF EVEN-EVEN Sr ISOTOPES USING ENERGY DENSITY FUNCTIONAL APPROACH

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ABSTRACT

The ground state properties of the atomic nuclei provide valuable information on their structure. The investigation of atomic nuclei is one of the fundamental and mostly pursued topics of the natural sciences, owing to its direct as well as indirect impact on human life. Atomic nuclei being quantum many-body systems, render the investigation of their properties among the challenging disciplines of the natural sciences. It is because of the fact that the single particle and the collective behavior of the constituents is at play almost on the same scales, making them interesting laboratories for the investigation of the different phenomena. These phenomena are also responsible for the varied shapes of atomic nuclei i.e., spherical, prolate, oblate etc. These shape changes take place from spherical doubly-magic (closed shell) nuclei, as one moves away from the valley of stability towards the drip lines, one can find the diversity in shapes of atomic nuclei and hence variation in their size, resulting in the change in the rms radii. In the present work the Sr isotopes have been investigated using the microscopic Energy Density Functional approach. The HFBTHO solver has been employed for calculation of binding energies, charge radii and the two-neutron separation energies which are in agreement with the experimentally measured values.

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INTRODUCTION

Atomic nucleus is a mysterious quantum many-body system, whose properties are difficult to characterize as compared to the properties of macroscopic objects. Nuclear physicists choose relatively small number of measurable properties of quantum systems to specify the overall characteristic of the entire nucleus. Investigations on these properties and moreover using them as bridges to reveal the maximum possible information on atomic nucleus, form the basis of ongoing research in nuclear physics. The ground state properties like the binding energies, charge radii, separation energies and nuclear deformations provide vital information on the characteristic behavior of nuclei. The rms radii is one of the static property of nucleus which characterized the nuclear extension in space. The nuclear charge radius is one of the most obvious and important nuclear parameters that give information about the nuclear shell model and the influence of effective interactions on nuclear structure. Experimental information on root-mean- square (rms) nuclear charge radii can be derived from different sources which include electron (e^-) scattering and muonic X-ray energies. The results from electron scattering (e^-) experiments are expressed in terms of the rms radius, and, for some nuclei, in parameters of the Fermi- distribution (Hofstadter *et al.*, 1953 and de Vries *et al.*, 1987). Muonic X-ray energies probe somewhat different moments of nuclear distribution, the so called Barrett moments $r= k\ e^{-\alpha r}$, nevertheless the results are expressed in terms of r^2 also (Engfer *et al.*, 1974; Friecke, 1995). Optical and K_α X-ray isotope shifts are sensitive to the same nuclear parameters and provide valuable complementary information on the changes in mean square (ms) radii. The K_α X-ray results are easier to interpret; however, these measurements can be performed only on stable isotopes since the experimental method requires several tens of milligrams of target. The same refers to experiments with μ^- atoms and electron scattering e^-. It is possible to measure optical isotope shifts (OIS) with negligible quantities of radioactive atoms, inclusive single ones, with lifetimes down to 1ms, and thus give access to long chains of radioactive isotopes extending far off stability (Otten *et al.*, 1989; Kluge *et al.*, 2010). These methods are sensitive to different properties of the nuclear ground-state charge distributions. Therefore a combination of data from different experimental methods generally yields more detailed and accurate knowledge of the nuclear radii than is available from any single method alone. The data on the size of nuclei (Hofstadter *et al.*, 1953) constitute one of the most precise and

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extensive arrays of experimental information available for the interpretation of nuclear phenomena. The pioneering works by using various methods such as electron scattering and muonic atomic spectroscopy (Vries *et al*., 1987), indicated that a nuclides near the β-stability line behaviors like a solid sphere with constant density. There has been an enormous progress in determination of sizes for exotic nuclides, entwined by the realization of radioactive ion beams facilities in last few decades (Engfer *et al*., 1974) and fast developments in ultra-high-sensitive laser spectroscopy techniques (Friecke *et al* 1995; Otten *et al*., 1989). These techniques have contributed to reveal, for example, the neutron skin effect (Kluge *et al*., 2010) and the nuclear shape variances (Moller *et al*., 1995, Smolanczuk and Dobaczewski, 1994) when moving far away from the stability line. This information is among the strongest motivations for the next generations of nuclear physics facilities, in which electron-scattering experiments (Dobaczewski *et al*., 1996) on unstable nuclei are under way. These state-of-the -art techniques are believed to open a new era of the nuclear physics and the knowledge of nuclear ground state properties play a very important role in understanding complex atomic nuclei.

The properties of the nuclei close to the drip lines play a very important role in understanding the astrophysical r-process, s-process and other processes of relevance to the physics community. Therefore to extract the information on these processes near the drip lines, where the very short life-time of theses nuclear species renders their experimental analysis intractable by the present experimental techniques, it is necessary to rely upon the state-of-the-art theoretical techniques which are able to reproduce the experimental data available near the valley of stability and at the same time make it possible to underpin the properties of nuclei near drip lines, by extrapolating the investigations close to the particle drip line regions. The one- and two-particle separation energies provide the important information on the stability of particular system. The calculation on particle separation energies helps to locate the particle drip lines and gives information on the shell structure of the system under investigation. One can further investigate the existence of number of isotopes of particular nuclear specie once the particle drip lines are located, hence defining the limits of existence on the nuclear landscape. The absence of electric charge in case of neutrons prevents the coulomb interaction between them making the neutron potential well shallower compared to that of protons. Due to which particular nuclear specie is capable to exist in large number of isotopes. In particular this results in the location of the neutron drip line farther away from the valley of stability than the proton drip line. The exciting new phenomena are observed on the neutron-rich side including neutron halos, low-density neutron matter and neutron skins. Consequently, neutron drip-line systems are characterized by unusually large N/Z ratios. The outer zone of these nuclei is expected to constitute essentially a new form of a many-body system, rich in neutron matter. The bounds of neutron stability are not known experimentally, except for the lightest nuclei. Theoretically, because of their sensitivity to approximations used, parameter values and the interactions, predicted drip lines are strongly model dependent. The placement of the one-neutron drip line, defined by the condition $S_n(Z,N) = B_n(Z,N)-B_n(Z,N-1)= 0$, is determined by the binding energy difference between two neighboring isotopes. Similarly, the the position of the two-neutron drip line is obtained by the condition of vanishing of the two-neutron separation energy expressed as $S_{2n}(Z,N)=B_n(Z,N)-B_n(Z,N-2) = 0$. The masses, and binding energies are not known experimentally near the neutron drip lines, in order to extrapolate far from stability, the large-scale mass calculations are usually used. These calculations are based upon the techniques where parameters are optimized to reproduce known atomic masses. However it is by no means obvious whether the particle number dependence obtained from global calculations at extreme values of N/Z is correct. Calculations for nuclei far from stability have strong astrophysical implications, especially in the context of the r-process mechanism (Kratz *et al*., 1993 and Chen *et al*., 1995), in addition to strong theoretical and experimental interest in nuclear physics aspects of exotic nuclei.

Different theoretical models have been proposed to explain observed data are available which have a capability to reproduce the experimental data very well in different regions of the nuclear landscape. However, there is no unique theory available at present which will be able to reproduce the whole experimental data available at present in different regions in the nuclear chart. Density Functional Theory is the promising theoretical frame work which can be applied across the whole nuclear chart (Bender *et al* 2003, Ring and Schuck, 1980, Stoitsov *et al*., 2005). Density Functional Theory have its roots in the Kohn-Sham density functional theory, which shows the existence of the Universal Energy Density Functional for the system of interacting fermions. The basic formulation of the Density Functional Theory goes back to 1964 seminal papers by Kohn and Sham (Hohenberg *et al*., 1964 and Kohn *et al*., 1965, Kohn, 1999).

MATERIALS AND METHODS

On the theoretical front current ab-initio no-core shell model calculations are limited to mass region $A \leq 16$. Configuration interaction techniques (i.e. shell model) are viable up to medium mass regions only due to the explosion of dimensionality of the configuration space with increase in the number of valance particle on top of closed shell. Coupled cluster methods have limited applicability in the vicinity of light- and medium mass double magic nuclei. Nuclear density functional theory (DFT) can be applied trough the whole nuclear landscape. The main idea of DFT is to describe an interacting system of fermions via its densities rather than the many-body wave function. The energy of the many body system can be written as a density functional, and the ground state energy is obtained through the variational procedure. The energy functional is a three-dimensional spatial integral of local energy density H that is a real, scalar, time-even, and isoscalar function of local densities and their derivatives. The energy density consists of a kinetic term, the Skyrme energy functional representing the effective nuclear interaction between nucleons, and the Coulomb term. A Skyrme functional depends on a number of local densities: nucleonic densities, kinetic densities, spin densities, spin-kinetic densities, current densities, tensor-kinetic densities, and spin-current densities (Perlinska *et al*., 2004).

We investigate the Ground state properties of the Sr isotopes using the microscopic Energy Density Functional approach (Stoitsov *et al*., 2005 and Stoitsov, Dobaczewski *et al* 2005). One of the DFT solvers is the HFBTHO comes in its different versions with the current version HFBTHO-v200d available publicly on CPC library. It solves the Hartree-Fock-Bogoliubov equations in the axially symmetric deformed harmonic oscillator basis, taking into account the time-reversal symmetry, with Skyrme interaction (Skyrme, 1956). The Skyrme Interaction comes in different parameterizations fitted to various nuclear matter properties. Many differing parameterizations of the Skyrme force have been developed in the study of various nuclear properties. A unified framework to govern the whole nuclear chart is an undergoing enterprise in nuclear physics investigations. The type of Skyrme interaction SLY4 has been employed to take account of the particle-hole channel of the interaction taking into account the center of mass correction to the kinetic energy part of the functional. The particle-particle interaction has been taken care of by zero range pairing interaction of mixed surface volume type. The variation of the Energy Density Functional with respect to the density and the pairing tensor with the constraint on the number of particles in a system, leads to the self consistent system of HFB equations. In standard HFB calculations (Stoitsov *et al.*, 2013), the strength of the pairing force (assumed identical for protons and neutrons) is usually adjusted at a given cut-off energy of 60 MeV to the experimental value of the average neutron gap of 1.245 MeV in ^{120}Sn. The detailed account of the mathematical formalism can be found in the references (Ring and Schuck, 1980; Stoitsov *et al.*, 2005). HFB equations are solved iteratively with iterations started from some initial guess and iterated until solution converges.

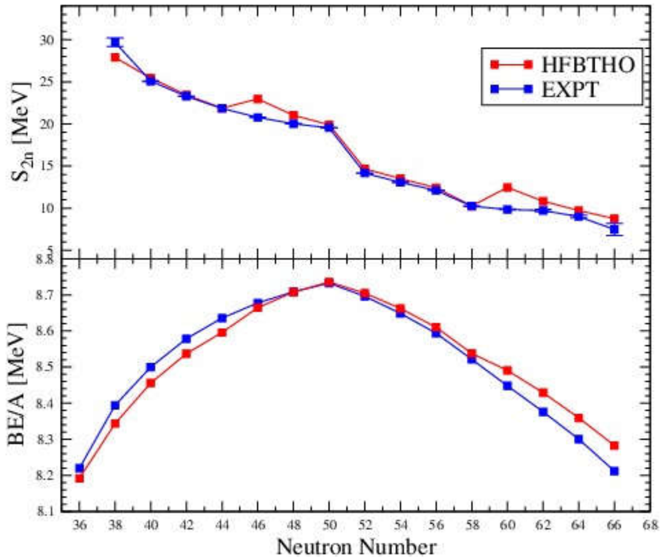

Fig. 1. Behavior of the binding energy per nucleon and two-neutron separation energies as mass number for even-even Sr isotopes

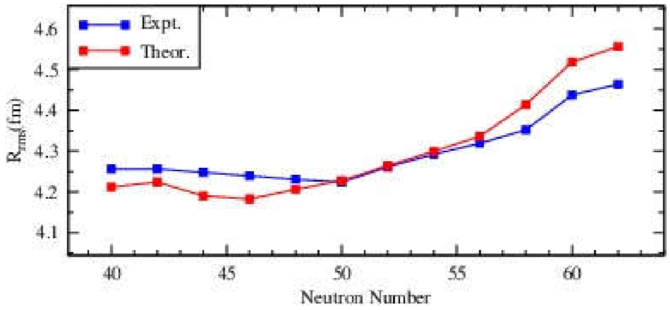

Fig. 2. Behavior of charge radii with increase in mass number for Sr isotopes

RESULTS AND DISCUSSION

The HFBTHO solver has been employed to calculate the ground state properties which include binding energies, charge radii and neutron separation energies of even-even Sr isotopes. The calculations have been performed with 14 oscillator shells and the basis deformation of 0.25. It can be observed from the plots that the calculated binding energies are in agreement with the experimental data around the valley of stability and deviated slightly as one goes away from the valley of stability. Indicating the sensitivity to approximations used, parameter values and the interactions. Further the calculated two-neutron separation energies are in fair agreement with the experimentally measured values and show a pronounced shell effect at the closed neutron shell at N = 50, observed as the sharp drop in the two-neutron separation energy plot. The calculated mean square charge radii, which are compared to the experimental values show a sharp increase near N = 60. The theoretical analysis suggests that the change in mean square charge radii for N = 60, 62 is due to the onset of a large static deformation as indicated in fig. 2, resulting in the shape variation and the phenomenon of shape coexistence. Here the nucleus can exist either in prolate configuration or in the oblate configuration, resulting in the abrupt changes in rms radii. The constrained calculations in this direction are under process and will be addressed separately in future.

Conclusion

The properties of the even even Sr isotopes have been investigated using the microscopic Energy Density Functional approach. The calculations have been performed using the HFBTHO-v200d, which solves the Skyrme Hartree-Fock-Bogoliubov equations in the axially deformed harmonic oscillator basis with time-reversal symmetry. The calculated results are in good agreement with the experimentally measured values. The trends in measured values of binding energies, two-neutron separation energies and the charge radii are followed well by the calculated values. The zero range pairing interaction has been employed with the pairing cut off energy upto 60 MeV of the quasi-particle excitation. The work is in progress to take into account the finite range pairing interaction in the Skyrme energy density functional approach.

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