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RESEARCH ARTICLE

RATIO CUM MEDIAN BASED MODIFIED RATIO ESTIMATORS WITH KNOWN QUARTILES

[1,*]Srija and [2]Jambulingam Subramani

Department of Mathematics, MS University, Tirunelveli, Tamilnadu, India
Department of Statistics, Pondicherry University, R V Nagar, Kalapet, Puducherry – 605014, India

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ABSTRACT

In this paper, some ratio cum median based modified ratio estimators with known quartiles of the auxiliary variable have been proposed. The performance of the proposed class of estimators is assessed with that of simple random sampling without replacement (SRSWOR) sample mean, ratio estimator and modified ratio estimators in terms of variance/mean squared errors. The performance of proposed class of estimators is illustrated with the help of certain natural population available in the literature. The percentage relative efficiency of the proposed class of estimators with respect to SRSWOR sample mean, ratio estimator and some of the existing modified ratio estimators are also obtained.

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INTRODUCTION

The main objective of sampling is to estimate the population mean of the study variable on the basis of selecting a random sample of size n from the population of sizeN. In this connection, a finite population $U = \{U_1, U_2, \dots, U_N\}$ of N distinct and identifiable units has been considered for the estimation of the finite population mean. Let Y(X) denote the study (auxiliary) variable taking values $Y_i(X_i), i = 1,2,\dots,N$ and is measured onU_i. Ratio estimator is used to improve the precision of the estimator based on SRSWOR sample mean by making use of the information of auxiliary variable which is positively correlated with that of the study variable. For a detailed discussion on the ratio estimator and its related problems the readers are referred to the text books by Cochran (1977) and Murthy (1967).

The efficiency of the ratio estimator can be improved further with the help of known parameters of the auxiliary variable such as, correlation coefficient, coefficient of variation, Skewness, Kurtosis, Quartiles etc. The resulting estimators are called in literature as modified ratio estimators. See for example Adepoju and Shittu (2013), Das and Tripathi (1978), Diana, Giordan and Perri (2011), Gupta and Shabbir(2008), Kadilar and Cingi (2004), Kadilar and Cingi (2006a, 2006b), Koyuncu (2012), Koyuncu and Kadilar (2009), Shittu and Adepoju (2013), Singh and Agnihotri (2008), Singh and Tailor(2003), Sisodia and Dwivedi (1981), Subramani and Kumarapandiyan (2012a, 2012b) and the references cited there in. Recently a new median based ratio estimator that uses the population median of the study variableYhas been introduced by Subramani (2013). From the median based ratio estimator, themedian based modified ratio estimators are developed by Subramani and Prabavathy (2014a, 2014b, 2015). Recently Jayalakshmi et.al (2016), Srijaet.al.andSubramani et.al (2016) have introduced some ratio cum median based modified ratio estimators for estimation of finite population mean with known parameters of the auxiliary variable such as kurtosis, skewness, coefficient of variation and correlation coefficient and their linear combinations. In this paper, some more ratio cum median based modified ratio estimators with known quartiles of the auxiliary variable and their linear combinations are introduced. Before discussing about the proposed estimators, we present the notations to be used and are as follows:

***Corresponding author:* Srija,**
Department of Mathematics, MS University, Tirunelveli, Tamilnadu, India.

- $f = n/N$, Sampling fraction
- $\delta = \frac{1-f}{n}$, finite population correction
- $\bar{X}, \bar{Y}$ – Population means
- $\bar{x}, \bar{y}$ – Sample means
- S_X, S_y – Population standard deviations
- S_{xy} – Population covariance between X and Y
- $C_X(C_y)$ – Co-efficient of variation of X (Y)
- $\rho = \frac{S_{xy}}{S_x S_y}$ – Co-efficient of correlation between X and Y
- $\beta_1 = \frac{\mu_3^2}{\mu_2^3}$, Skewness of the auxiliary variable
- $\beta_2 = \frac{\mu_4}{\mu_2^2}$, Kurtosis of the auxiliary variable where $\mu_r = \frac{1}{N}\sum_{i=1}^{N}(X_i - \bar{X})^r$
- Q_1 – First(lower) quartile of the auxiliary variable
- Q_3 - Third(upper) quartile of the auxiliary variable
- $M\ (m)$ – Population (sample) Median of the study variable
- $B(.)$ – Bias of the estimator
- $MSE(.)$ – Mean squared error of the estimator
- $V(.)$ – Variance of the estimator
- $\bar{y}$ – Simple random sampling without replacement (SRSWOR) sample mean
- $\hat{\bar{Y}}_R$ – Ratio estimator
- $\hat{\bar{Y}}_M$ – Median Based Ratio Estimator
- $\hat{\bar{Y}}_{p_j}$ – j^{th} Proposed median based modified ratio estimator of $\bar{Y}$
- Q_1 – First(lower) quartile of the auxiliary variable

1.1.Existing Estimators

In case of SRSWOR, the sample mean is used to estimate population mean which is an unbiased estimator. The SRSWOR sample mean together with its variance is given below:

$$\bar{y} = \frac{1}{n}\sum_{i=1}^{n} \tag{1}$$

$$V(\bar{y}) = \frac{(1-f)}{} \tag{2}$$

where $f = \frac{n}{N}, S_y^2 = \frac{1}{N-1}\sum_{i=1}^{N}(Y_i - \bar{Y}$

The ratio estimator for estimating the population mean of the study variable is defined as

$$\hat{\bar{Y}}_R = \frac{\bar{y}}{\bar{x}}\bar{X} = \hat{R}\bar{X} \tag{3}$$

The mean squared error of $\hat{\bar{Y}}_R$ is given below:

$$MSE\left(\hat{\bar{Y}}_R\right) = \bar{Y}^2\{C'_{yy} + C'_{xx} - 2C'_{yx}\} \tag{4}$$

where $C'_{yy} = \frac{V(\bar{y})}{\bar{Y}^2}, C'_{xx} = \frac{V(\bar{x})}{\bar{X}^2}, \ C'_{yx} = \frac{Cov(\bar{y},\bar{x})}{\bar{X}\bar{Y}}$

The modified ratio estimator $\hat{\bar{Y}}_i$ with known parameter λ_i of the auxiliary variable for estimating the finite population mean $\bar{Y}$ is defined as

$$\hat{\bar{Y}}_i = \bar{y}\left[\frac{\bar{X}+\lambda_i}{\bar{x}+\lambda_i}\right] \tag{5}$$

The mean squared error of $\hat{\bar{Y}}_i$ is as follows:

$$MSE\left(\hat{\bar{Y}}_R\right) = \delta\bar{Y}^2\left(C_y^2 + \theta_i^2 C_x^2 - 2\rho\theta_i C_x C_y\right) \tag{6}$$

PROPOSED ESTIMATORS

In this section, some more ratio cum median based modified ratio estimators with known linear combinations of the known parameters of the auxiliary variable like First Quartile Q_1 and Third Quartile Q_3 in line with the ratio cum median based modified ratio estimators by Jayalakshmi et.al (2016), Subramani et.al (2016) and Srija et.al (2016). The proposed estimators together with their mean squared errors are given below:

Case i : The proposed estimator with known First Quartile Q_1 is

$$\hat{\bar{Y}}_{p_1} = \bar{y}\left\{\alpha_1\left(\frac{M+Q_1}{m+Q_1}\right) + \alpha_2\left(\frac{\bar{X}+Q_1}{\bar{x}+Q_1}\right)\right\} \tag{7}$$

Case ii: The proposed estimator with known Third Quartile Q_3 is

$$\hat{\bar{Y}}_{p_2} = \bar{y}\left\{\alpha_1\left(\frac{M+Q_3}{m+Q_3}\right) + \alpha_2\left(\frac{\bar{X}+Q_3}{\bar{x}+Q_3}\right)\right\} \tag{8}$$

DERIVATION OF BIAS AND MEAN SQUARED ERRORS OF PROPOSED ESTIMATORS

The following theorem 2.0, will be useful to derive the bias and the mean squared errors of the proposed ratio cum median based modified ratio estimators with known quartiles and their functions given in (7) to (8).

Theorem 2.0: In SRSWOR, ratio cum median based modified ratio estimator

$\hat{\bar{Y}}_{p_i} = \bar{y}\left\{\alpha_1\left(\frac{M+T_i}{m+T_i}\right) + \alpha_2\left(\frac{\bar{X}+T_i}{\bar{x}+T_i}\right)\right\}$ where $\alpha_1 + \alpha_2 = 1$, for the known parameter T_i is not an unbiased estimator for its population mean $\bar{Y}$ and its bias and MSE are respectively given as:

$$B\left(\hat{\bar{Y}}_{p_i}\right) = \bar{Y}\left\{\alpha_1\left(\theta_i^2 C'_{mm} - \theta_i C'_{ym} - \theta_i \frac{B(m)}{M}\right) + \alpha_2\left(\varphi_i^2 C'_{xx} - \varphi_i C'_{yx}\right)\right\}$$

$$MSE\left(\hat{\bar{Y}}_{p_i}\right) = \bar{Y}^2\left\{C'_{yy} + \alpha_1^2\theta_i^2 C'_{mm} + \alpha_2^2\varphi_i^2 C'_{xx} - 2\alpha_1\theta_i C'_{ym} - 2\alpha_2\varphi_i C'_{yx} + 2\alpha_1\alpha_2\theta_i\varphi_i C'_{xm}\right\}$$

$$\text{where } \theta_i = \frac{M}{M+T_i}, \varphi_i = \frac{\bar{X}}{\bar{X}+T_i}, \; C'_{xm} = \frac{Cov(\bar{x}, m)}{M\bar{X}}$$

Proof:

Consider $\hat{\bar{Y}}_{p_i} = \bar{y}\left[\alpha_1\left(\frac{M+T_i}{m+T_i}\right) + \alpha_2\left(\frac{\bar{X}+T_i}{\bar{x}+T_i}\right)\right]$

where $\alpha_1 + \alpha_2 = 1$

$\bar{y}$: The SRSWOR sample mean of the study variable Y
m: The sample median of the study variable Y
M: The population median of the study variable Y
T_i : The known parameter or ratio of parameters of the auxiliary variable X

Let $e_0 = \frac{\bar{y}-\bar{Y}}{\bar{Y}}$, $e_1 = \frac{m-M}{M}$ and $e_2 = \frac{\bar{x}-\bar{X}}{\bar{X}}$

$\Rightarrow E(e_0) = 0;\ E(e_1) = \frac{\bar{M}-M}{M} = \frac{B(m)}{M}$ and $E(e_2) = 0$
$\Rightarrow E(e_0^2) = \frac{V(\bar{y})}{\bar{Y}^2};\ E(e_1^2) = \frac{V(m)}{M^2};\ E(e_2^2) = \frac{V(\bar{x})}{\bar{X}^2};$
$E(e_0 e_1) = \frac{Cov(\bar{y},m)}{\bar{Y}M};\ E(e_0 e_2) = \frac{Cov(\bar{y},\bar{x})}{\bar{Y}\bar{X}};\ E(e_1 e_2) = \frac{Cov(\bar{x},m)}{\bar{X}M}$

The estimator $\hat{\bar{Y}}_{p_i}$ can be written in terms of e_0, e_1 and e_2 as

$$\hat{\bar{Y}}_{p_i} = \bar{Y}(1+e_0)\left\{\alpha_1\left(\frac{M+T_i}{M(1+e_1)+T_i}\right) + \alpha_2\left(\frac{\bar{X}+T_i}{\bar{X}(1+e_2)+T_i}\right)\right\}$$

$$\Rightarrow \hat{\bar{Y}}_{p_i} = \bar{Y}(1+e_0)\left\{\alpha_1\left(\frac{M+T_i}{(M+T_i)+Me_1}\right) + \alpha_2\left(\frac{\bar{X}+T_i}{(\bar{X}+T_i)+\bar{X}e_2}\right)\right\}$$

$$\Rightarrow \hat{\bar{Y}}_{p_i} = \bar{Y}(1+e_0)\left\{\alpha_1\left(\frac{1}{1+\left(\frac{M}{M+T_i}\right)e_1}\right) + \alpha_2\left(\frac{1}{1+\left(\frac{\bar{X}}{\bar{X}+T_i}\right)e_2}\right)\right\}$$

$$\Rightarrow \hat{\bar{Y}}_{p_i} = \bar{Y}(1+e_0)\left\{\alpha_1\left(\frac{1}{1+\theta_i e_1}\right) + \alpha_2\left(\frac{1}{1+\varphi_i e_2}\right)\right\};\text{ where } \theta_i = \frac{M}{M+T_i}, \varphi_i = \frac{\bar{X}}{\bar{X}+T_i}$$

$$\Rightarrow \hat{\bar{Y}}_{p_i} = \bar{Y}(1+e_0)\{\alpha_1(1+\theta_i e_1)^{-1} + \alpha_2(1+\varphi_i e_2)^{-1}\}$$

Neglecting the terms of higher order, we have

$$\hat{\bar{Y}}_{p_i} = \bar{Y}(1+e_0)\{\alpha_1(1-\theta_i e_1 + \theta_i^2 e_1^2) + \alpha_2(1-\varphi_i e_2 + \varphi_i^2 e_2^2)\}$$

$$\hat{\bar{Y}}_{p_i} = \bar{Y}\{\alpha_1(1+e_0-\theta_i e_1 - \theta_i e_0 e_1 + \theta_i^2 e_1^2) + \alpha_2(1+e_0-\varphi_i e_2 - \varphi_i e_0 e_2 + \varphi_i^2 e_2^2)\}$$

$$\Rightarrow \hat{\bar{Y}}_{p_i} = \bar{Y}\{1+e_0 + \alpha_1(-\theta_i e_1 - \theta_i e_0 e_1 + \theta_i^2 e_1^2) + \alpha_2(-\varphi_i e_2 - \varphi_i e_0 e_2 + \varphi_i^2 e_2^2)\}$$

$$\hat{\bar{Y}}_{p_i} - \bar{Y} = \bar{Y}\{e_0 - \alpha_1(\theta_i e_1 + \theta_i e_0 e_1 - \theta_i^2 e_1^2) - \alpha_2(\varphi_i e_2 + \varphi_i e_0 e_2 - \varphi_i^2 e_2^2)\} \quad (12)$$

Taking expectations on both sides of (10) one can have,

$$E(\hat{\bar{Y}}_{p_i} - \bar{Y}) = \bar{Y}\{E(e_0) - \alpha_1 E(\theta_i e_1 + \theta_i e_0 e_1 - \theta_i^2 e_1^2) - \alpha_2 E(\varphi_i e_2 + \varphi_i e_0 e_2 - \varphi_i^2 e_2^2)\}$$

$$\Rightarrow E(\hat{\bar{Y}}_{p_i} - \bar{Y}) = \bar{Y}\left\{\alpha_1\left(\theta_i^2\frac{V(m)}{M^2} - \theta_i\frac{B(m)}{M} - \theta_i\frac{Cov(\bar{y}, m)}{\bar{Y}M}\right) + \alpha_2\left(\varphi_i^2\frac{V(\bar{x})}{\bar{X}^2} - \varphi_i\frac{Cov(\bar{y},\bar{x})}{\bar{Y}\bar{X}}\right)\right\}$$

$$\Rightarrow B(\hat{\bar{Y}}_{p_i}) = \bar{Y}\left\{\alpha_1\left(\theta_i^2 C'_{mm} - \theta_i C'_{ym} - \theta_i\frac{B(m)}{M}\right) + \alpha_2(\varphi_i^2 C'_{xx} - \varphi_i C'_{yx})\right\}$$

Squaring on both sides of (10), neglecting the terms of higher order and taking expectation on both sides one can get,

$$MSE(\hat{\bar{Y}}_{p_i}) = E(\hat{\bar{Y}}_{p_i} - \bar{Y})^2 = \bar{Y}^2 E\{e_0 - \alpha_1\theta_i e_1 - \alpha_2\varphi_i e_2\}^2$$

$$E(\hat{\bar{Y}}_{p_i} - \bar{Y})^2 = \bar{Y}^2\{E(e_0^2) + \alpha_1^2\theta_i^2 E(e_1^2) + \alpha_2^2\varphi_i^2 E(e_2^2) - 2\alpha_1\theta_i E(e_0 e_1) - 2\alpha_2\varphi_i E(e_0 e_2) + 2\alpha_1\alpha_2\theta_i\varphi_i E(e_1 e_2)\}$$

After a little algebra, the mean squared error of $\hat{\bar{Y}}_{p_i}$ is obtained as

$$MSE(\hat{\bar{Y}}_{p_i}) = \bar{Y}^2\left\{\frac{V(\bar{y})}{\bar{Y}^2} + \alpha_1^2\theta_i^2\frac{V(m)}{M^2} - 2\alpha_1\theta_i\frac{Cov(\bar{y},m)}{\bar{Y}M} + \alpha_2^2\varphi_i^2\frac{V(\bar{x})}{\bar{X}^2} - 2\alpha_2\varphi_i\frac{Cov(\bar{y},\bar{x})}{\bar{Y}\bar{X}} + 2\alpha_1\alpha_2\theta_i\varphi_i\frac{Cov(m,\bar{x})}{M\bar{X}}\right\}$$

hat is, $MSE(\hat{\bar{Y}}_{p_i}) = \bar{Y}^2\{C'_{yy} + \alpha_1^2\theta_i^2 C'_{mm} + \alpha_2^2\varphi_i^2 C'_{xx} - 2\alpha_1\theta_i C'_{ym} - 2\alpha_2\varphi_i C'_{yx} + 2\alpha_1\alpha_2\theta_i\varphi_i C'_{xm}\}$

Hence the proof.

The derivation of bias and the mean squared errors of the proposed ratio cum median based modified ratio estimators with known quartiles and their functions given in (7) to (8) are presented in the following theorems.

Theorem 2.1: In SRSWOR, ratio cum median based modified ratio estimator

$\hat{\bar{Y}}_{P_1} = \bar{y}\left\{\alpha_1\left(\frac{M+Q_1}{m+Q_1}\right) + \alpha_2\left(\frac{\bar{X}+Q_1}{\bar{x}+Q_1}\right)\right\}$ where $\alpha_1 + \alpha_2 = 1$, for the known parameter Q_1 is not an unbiased estimator for its population mean $\bar{Y}$ and its bias and MSE are respectively given as:

$$B(\hat{\bar{Y}}_{p_1}) = \bar{Y}\left\{\alpha_1\left(\theta_i^2 C'_{mm} - \theta_i C'_{ym} - \theta_i\frac{B(m)}{M}\right) + \alpha_2(\varphi_i^2 C'_{xx} - \varphi_i C'_{yx})\right\}$$

$$MSE(\hat{\bar{Y}}_{p_1}) = \bar{Y}^2\{C'_{yy} + \alpha_1^2\theta_i^2 C'_{mm} + \alpha_2^2\varphi_i^2 C'_{xx} - 2\alpha_1\theta_i C'_{ym} - 2\alpha_2\varphi_i C'_{yx} + 2\alpha_1\alpha_2\theta_i\varphi_i C'_{xm}\},$$

$$\text{where } \theta_i = \frac{M}{M+Q_1}, \varphi_i = \frac{\bar{X}}{\bar{X}+Q_1}$$

Proof: By replacing $T_i = Q_1$ in Theorem 2.0 the proof follows.

Theorem 2.2: In SRSWOR, ratio cum median based modified ratio estimator

$\hat{\bar{Y}}_{P_2} = \bar{y}\left\{\alpha_1\left(\frac{M+Q_3}{m+Q_3}\right) + \alpha_2\left(\frac{\bar{X}+Q_3}{\bar{x}+Q_3}\right)\right\}$ **where** $\alpha_1 + \alpha_2 = 1$**, for the known parameter** Q_3 **is not an unbiased estimator for its population mean** $\bar{Y}$ **and its bias and MSE are respectively given as:**

$$B\left(\hat{\bar{Y}}_{P_2}\right) = \bar{Y}\left\{\alpha_1\left(\theta_i^2 C'_{mm} - \theta_i C'_{ym} - \theta_i \frac{B(m)}{M}\right) + \alpha_2\left(\varphi_i^2 C'_{xx} - \varphi_i C'_{yx}\right)\right\}$$

$$MSE\left(\hat{\bar{Y}}_{P_2}\right) = \bar{Y}^2\{C'_{yy} + \alpha_1^2\theta_i^2 C'_{mm} + \alpha_2^2\varphi_i^2 C'_{xx} - 2\alpha_1\theta_i C'_{ym} - 2\alpha_2\varphi_i C'_{yx} + 2\alpha_1\alpha_2\theta_i\varphi_i C'_{xm}\},$$

where $\theta_i = \frac{M}{M+Q_3}, \varphi_i = \frac{\bar{X}}{\bar{X}+Q_3}$

Proof: By replacing $T_i = Q_3$ in Theorem 2.0 the proof follows.

NOTE 2.1: The proposed estimators are written into a class of estimators with the population parameter T_i is

$$\hat{\bar{Y}}_{P_i} = \bar{y}\left\{\alpha_1\left(\frac{M+T_i}{m+T_i}\right) + \alpha_2\left(\frac{\bar{X}+T_i}{\bar{x}+T_i}\right)\right\} \quad (9)$$

where $\alpha_1 + \alpha_2 = 1, \quad i = 1,2$

The mean squared error of proposed estimator is given as

$$MSE\left(\hat{\bar{Y}}_{P_i}\right) = \bar{Y}^2\{C'_{yy} + \alpha_1^2\theta_i^2 C'_{mm} + \alpha_2^2\varphi_i^2 C'_{xx} - 2\alpha_1\theta_i C'_{ym} - 2\alpha_2\varphi_i C'_{yx} + 2\alpha_1\alpha_2\theta_i\varphi_i C'_{xm}\} \quad (10)$$

where $\theta_i = \frac{M}{M+T_i}, \varphi_i = \frac{\bar{X}}{\bar{X}+T_i}, \; C'_{xm} = \frac{Cov(\bar{x}, m)}{M\bar{X}},$

$T_1 = Q_1, T_2 = Q_3$

3. EFFICIENCY COMPARISON

In this section, the efficiencies of proposed estimators given in (15)are assessed with that of SRSWOR sample mean ratio estimator and modified ratio estimators in terms of variance/mean squared error. The results are as follows:

3.1 Comparison with that of SRSWOR sample mean

Comparing (10) and (2), it is noticed that the proposed estimators perform better than the SRSWOR sample mean if

$MSE\left(\hat{\bar{Y}}_{P_i}\right) \le V(\bar{y})$ i.e.,

$$\alpha_1^2\theta_i^2 C'_{mm} + \alpha_2^2\varphi_i^2 C'_{xx} + 2\alpha_1\alpha_2\theta_i\varphi_i C'_{xm} \le 2(\alpha_1\theta_i C'_{ym} + \alpha_2\varphi_i C'_{yx}) \quad (11)$$

3.2 Comparison with that of Ratio Estimator

Comparing (10) and (4), it is noticed that the proposed estimators perform better than the ratio estimator if $MSE\left(\hat{\bar{Y}}_{P_i}\right) \le MSE\left(\hat{\bar{Y}}_R\right)$ i.e.,

$$\alpha_1^2\theta_i^2 C'_{mm} + (\alpha_2^2\varphi_i^2 - 1)C'_{xx} + 2\alpha_1\alpha_2\theta_i\varphi_i C'_{xm} \le 2\left[\alpha_1\theta_i C'_{ym} + (\alpha_2\varphi_i - 1)C'_{yx}\right] \quad (12)$$

3.3 Comparison with that of Modified Ratio Estimators

Comparing (10) and (6), it is noticed that the proposed estimators perform better than the modified ratio estimators.

That is, $MSE\left(\hat{\bar{Y}}_{P_i}\right) \le MSE\left(\hat{\bar{Y}}_i\right)$ if

$$\alpha_1^2\theta_i^2 C'_{mm} + (\alpha_2^2 - 1)\varphi_i^2 C'_{xx} + 2\alpha_1\alpha_2\theta_i\varphi_i C'_{xm} \leq 2[\alpha_1\theta_i C'_{ym} + (\alpha_2 - 1)\varphi_i C'_{yx}] \quad (13)$$

4. NUMERICAL COMPARISON

In the section3, the conditions for the efficiency of proposed estimators given in (9) with that of existing estimators have been derived algebraically. To support it by means of numerical comparison, data of a natural population from Singh and Chaudhary (1986, page.177) has been considered.

Population Description

X= Area under Wheat in 1971 and Y= Area under Wheat in 1974
The population parameters computed for the above population is given below:

N= 34	n= 3	$\bar{Y}$= 856.4118
ρ = 0.4491	M= 767.5	$\bar{X}$= 208.8824
Q_1= 94.25	Q_3= 254.75	

The variance/mean squared error of the existing and proposed estimators at different values of $\alpha_1$and α_2 are given in the following table

Table 4.1. Mean Squared Errors for different values of $\alpha_1 and\ \alpha_2$

Existing Estimators			
SRSWOR Sample mean		$\bar{y}$	163356.41
Ratio Estimator		$\hat{\bar{Y}}_R$	155579.71
Modified Ratio Estimators		$\hat{\bar{Y}}_1$	133203.49
		$\hat{\bar{Y}}_2$	131204.42
		$\hat{\bar{Y}}_3$	130523.62
		$\hat{\bar{Y}}_4$	134533.03
		$\hat{\bar{Y}}_5$	130420.55
Proposed Estimators			
α_1	α_2	$\hat{\bar{Y}}_{P_1}$	$\hat{\bar{Y}}_2$
0.1	0.9	123612.14	123008.79
0.2	0.8	115064.72	115572.39
0.3	0.7	107561.22	108895.23
0.4	0.6	101101.65	102977.31
0.5	0.5	95686.01	97818.62
0.6	0.4	91314.30	93419.16
0.7	0.3	87986.51	89778.94
0.8	0.2	85702.65	86897.96
0.9	0.1	84462.72	84776.21

From Table 4.1, it is observed that the proposed estimators discussed in (9) have less mean squared errors than the SRSWOR sample mean, ratio estimator and the modified ratio estimators. The percentage relative efficiencies (PRE) of the proposed estimators with respect to the existing estimators are obtained by using the formula $PRE(e,p) = \frac{MSE(e)}{MSE(p)} * 100$ and are given in the following table:

Table 4.2. PRE of proposed estimators with respect to SRSWOR sample mean

α_1	α_2	$\hat{\bar{Y}}_{P_1}$	$\hat{\bar{Y}}_2$
0.1	0.9	132.15	132.80
0.2	0.8	141.97	141.35
0.3	0.7	151.87	150.01
0.4	0.6	161.58	158.63
0.5	0.5	170.72	167.00
0.6	0.4	178.89	174.86
0.7	0.3	185.66	181.95
0.8	0.2	190.61	187.99
0.9	0.1	193.41	192.69

Table 4.3. PRE of proposed estimators with respect to Ratio Estimator

α_1	α_2	$\hat{\bar{Y}}_{P_1}$	$\hat{\bar{Y}}_2$
0.1	0.9	125.86	126.48
0.2	0.8	135.21	134.62
0.3	0.7	144.64	142.87
0.4	0.6	153.88	151.08
0.5	0.5	162.59	159.05
0.6	0.4	170.38	166.54
0.7	0.3	176.82	173.29
0.8	0.2	181.53	179.04
0.9	0.1	184.20	183.52

Table 4.4. PRE of proposed estimators with respect to Modified Ratio Estimators

α_1	α_2	$\hat{\bar{Y}}_{P_1}$	$\hat{\bar{Y}}_2$
0.1	0.9	107.76	106.66
0.2	0.8	115.76	113.53
0.3	0.7	123.84	120.49
0.4	0.6	131.75	127.41
0.5	0.5	139.21	134.13
0.6	0.4	145.87	140.45
0.7	0.3	151.39	146.14
0.8	0.2	155.43	150.99
0.9	0.1	157.71	154.77

From Tables 4.2, 4.3 and 4.4, it is observed that the PRE values of the proposed estimators with respect to SRSWOR sample mean, ratio estimator and modified ratio estimators are greater than 100 and hence we conclude that the proposed estimators are efficient estimators.

In fact the PREs are ranging from

- 132.15 to 193.41 for the case of SRSWOR sample mean
- 125.86 to 184.20 for the case of ratio estimator
- 106.66 to 157.71 for the case of modified ratio estimators

SUMMARY

In this paper we have proposed some more ratio cum median based modified ratio estimators with the known parameters such as quartiles Q_1 and Q_3 of the auxiliary variable and their linear combinations. The efficiencies of the proposed ratio cum median based modified ratio estimators are assessed algebraically as well as numerically with that of SRSWOR sample mean, ratio estimator and some of the modified ratio estimators. Further it is shown from the numerical comparison that the PREs of proposed ratio cum median based modified ratio estimators with respect to the existing estimators are more than 100. Hence the proposed ratio cum median based modified ratio estimators with known quartiles may be recommended for the use of practical applications.

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IJCRS has over 6 years of experience in Publishing, which was initiated upon the publish upon the publication of International Journal of Current Research, The IJCRLS has evolved into a prominent and trusted Publication in India, reporting on clinically, relevant advance in the understanding of normal function and disease conditions of the digestive system Meanwhile,, IJCRLS has emerged as an internationally renowned publication, with a broad scope of reporting basic and clinical research in esophageal, gastrointestinal, liver, pancreas and biliary tract conditions.

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