



RESEARCH ARTICLE

PLACEMENT STRATEGY OF PIEZOELECTRIC PATCHES FOR MULTIMODE ACTIVE VIBRATION CONTROL OF FIXED-FIXED BEAM USING STRAIN TOPOLOGICAL PATTERN

Khot, S. M. and \*Aqleem A. Siddiqui

Fr. C. Rodrigues Institute of Technology, Vashi, Navi Mumbai, India

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ABSTRACT

Placement of sensor actuator pair at proper location is very important for active vibration control of structures. A large number of research work by various researchers have been carried out for the optimization of actuators and sensors location, based on different approaches. There are two broad approaches. In one approach the optimization of sensor/actuator location is combined with controller parameters, whereas in the second approach, the optimal locations are obtained independently of the controller parameters. In order to get the best vibration control, the placement of the piezopatches should be in the region of maximum strain location. This paper presents, a controller independent, simple optimal piezoelectric patch placement strategy, on fixed-fixed beams, based on modal strain topological pattern. The modal strain patterns obtained from FE analysis of beams, are utilized to devise a strategy for identifying the locations of piezopatches. The effectiveness of strategy adopted for optimal location is compared with the results obtained from a controller dependent placement methodology employed by previous researchers. It is found that the proposed methodology is in agreement with the work carried out earlier.

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INTRODUCTION

Active control of vibrations of structures using piezoelectric sensors and actuators had been an important research area in last few decades. In active vibration control of structures, misplaced piezopatches leads to lack of controllability, which results in reduction of the performance of the control system. Identification of optimal location of piezoelectric patches for effectively controlling the vibration is very challenging. Earliest research work carried by Crawley and De Luis (1987) presented the analytical and experimental development of piezoelectric patches as elements of intelligent structures, i.e. structures with distributed actuators, sensors and processing. The optimal location of the actuator for a structure is the position at which the strain energy of the structure is highest. A comprehensive analysis for controlling structural vibrations of flexible beams with the use of piezoelectric actuators was presented by Baz and Poh (1988) using a Modified Independent Modal Space Control method as a viable algorithm for controlling multi-mode vibration. A methodology for actuator placement and sizing for suppression of vibration in uniform beams was proposed by Devasia et al. (1992). They derived objective functions for optimal placement and sizing of

piezoelectric actuators in uniform beams considering various closed loop performance. Kincaid and Padula (1999) presented a detailed survey of actuator and sensor placement problems from a wide range of engineering disciplines for variety of applications. In the survey, combination of optimization like tabu search and genetic algorithmic methods were recommended for finding sets of actuators and sensors that maximize the performance. Aldraihem et al. (2000) formulated an optimization problem for general beam for location of as many piezopatches as desired. Their proposed controller independent criteria is based on modal cost and controllability index as criterion for optimization. Narayanan and Balamurugan (2003) studied the active vibration control performance of these piezolaminated structures using classical control methods and modern control methods such as LQR. They detected that the LQR control scheme is very effective in controlling the vibration of beams. Their study was conducted using a piezolaminated steel beam. Yang and Kiong (2005) used maximization of dissipation of energy for optimization of control structure. Actuator placement and size of piezopatches were integrated in the design optimization of vibration control system. They used integer-real-encoded genetic algorithm for determination of optimal placement of piezopatches. Various researchers have used Genetic Algorithm for optimization of placement problems. Wherein, Rader et al. (2007) used the Frequency Response Function, obtained from FEA model, for formulating optimization function using Genetic Algorithm for

\*Corresponding author: Aqleem A. Siddiqui,  
Fr. C. Rodrigues Institute of Technology, Vashi, Navi Mumbai, India.  
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single as well as multi patch placement methodologies. Kumar and Narayanan (2008) used LQR performance as the objective function for optimization. Rao *et al.* (2008) formulated the problem of discrete optimal actuator locations in active vibration control of structures in the framework of a zero-one optimization problem. Dhuri and Seshu (2009) used a Multi Objective Genetic Algorithm for the identification and sizing of patches, on a stationary as well as a rotating beam. The fitness cost function for optimization using Genetic Algorithm was derived from the fundamental displacement equation by Khot and Khan (2015). Both the single patch as well as the multi-patch placement strategy was proposed by them. This paper deals with controller independent strategy for identification of optimal location for sensor/actuator patches on a fixed-fixed type of beam. Modal strain patterns are utilized for developing strategy for placement of multiple patches for multimode vibration control.

## MATERIALS AND METHODS

The methodology for identification of the optimal location for multiple sensor/actuator patches is presented in Fig. 1.

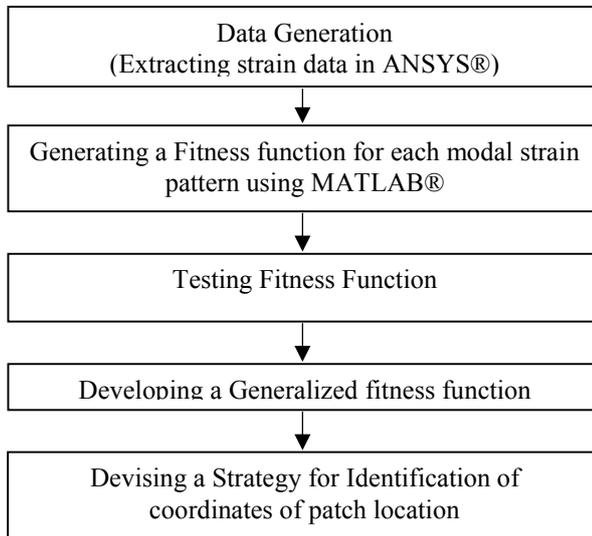


Fig. 1. Methodology for present study

Data Generation involves the extraction of modal strain data of the fixed beam under study. Initially a beam of dimension 500mm X 25mm X 5mm, as shown in Fig.2, was modeled in ANSYS®, using SOLID45 element.

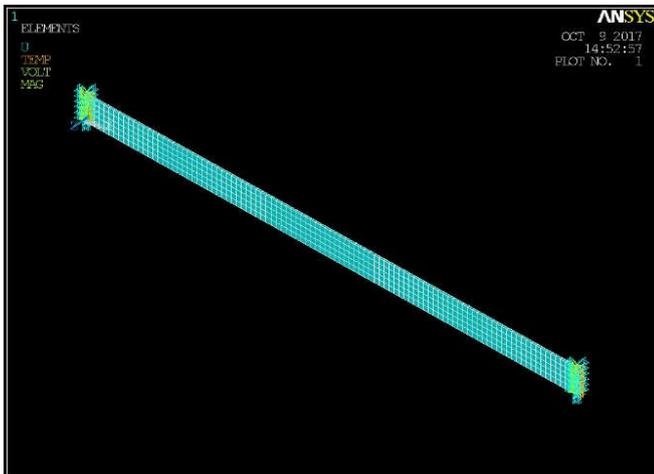


Fig. 2. Fixed Beam modeled in Ansys®

Modal analysis of the beam was performed and the modal displacement and modal strain data for six modes were extracted. The normalized displacement and strain data, extracted from ANSYS® was plotted in MATLAB® as shown in Figs. 3 and 4.

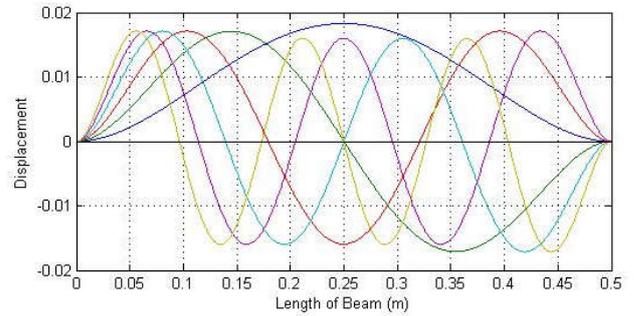


Fig. 3. Displacement Plot for 6 Modes

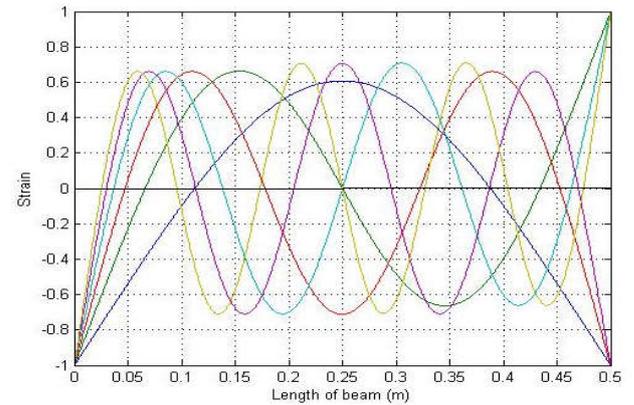


Fig. 4. Strain Plot for 6 Modes

Based on the modal strain data available, a fitness function was obtained using MATLAB®. These fitness function was generated using centering and scaling option and tested for its accuracy using POLYVAL function in MATLAB®. The fitness function obtained for each modal strain pattern is a polynomial equation which is presented in equation (1) to (6).

Polynomial equation for first mode:

$$e1 = -0.195 + 1.802x + 1.586x^2 - 20.767x^3 + 20.767x^4 \quad (1)$$

Polynomial equation for second mode:

$$e2 = 0.528 - 6.624x - 54.781x^2 + 618.72x^3 - 1637x^4 + 1309.600x^5 \quad (2)$$

Polynomial equation for third mode:

$$e3 = -0.992 + 10.086x + 462.99x^2 - 6280.15x^3 + 28017.61x^4 - 52169.61x^5 + 34779.94x^6 \quad (3)$$

Polynomial equation for fourth mode:

$$e4 = -1.534 - 2.854x + 2248.754x^2 - 37420.7x^3 + 241800.8x^4 - 747574.084x^5 + 1108267x^6 - 633295.35x^7 \quad (4)$$

Polynomial equation for fifth mode:

$$e5 = -2.113 - 59.640x7850.053x^2 - 159431x^3 + 1380558x^4 - 6170804.38x^5 + 14914201x^6 - 18514854x^7 + 925E04x^8 \quad (5)$$

$$e6 = -2.698 - 200.545x + 21923.31x^2 - 535101x^3 + 5905188x^4 - 35309354x^5 + 1.22E08x^6 - 243E6x^7 + 260E06x^8 - 1.2E08x^9 \quad (6)$$

It is observed that the order of the polynomials for each mode is  $m+3$ , where  $m$  = mode number. The R-square and R-squared adjusted values of each of the fitness functions were observed to be between 0.9095 to 0.9096. The process of modal analysis of beams of length 750mm and 1000mm is repeated and their strain fitting equation is generated. It is observed that the pattern of strain topology and the order of polynomial for respective modes, remains the same for different lengths of the beam. Hence the generalized strain topological pattern in the polynomial form can be expressed as a function of  $x/l$ . The generalized polynomial function for strain in a beam of length 'l' can be expressed as shown in equation (7).

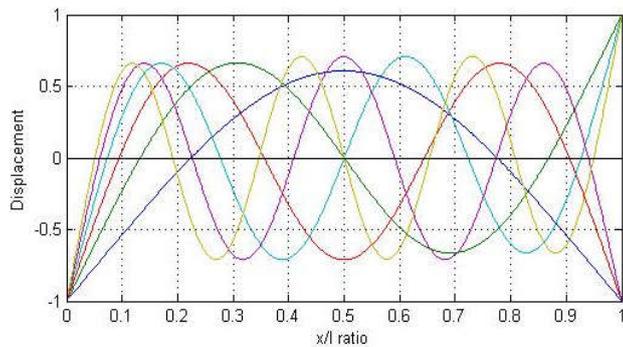
$$strain = \sum_{i=0}^{m+1} C_i \left(\frac{x}{l}\right)^i \tag{7}$$

Where  
 $m$  = mode number,  
 $C_i$  = coefficient of generalized polynomial equation

The values of  $C_i$  obtained for the first six modes is shown in Table 1.

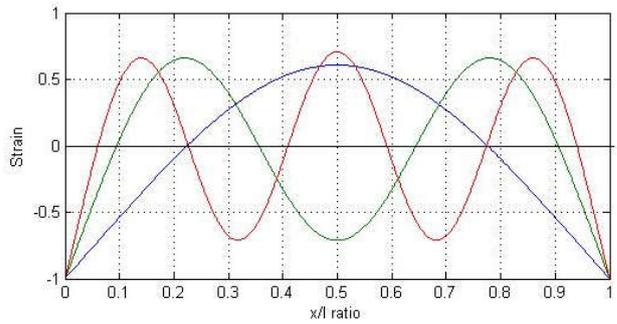
**Table 1. Coefficients for the generalized polynomial**

Mode	C0	C1	C2	C3	C4
1	-0.9524	4.2896	2.7273	-14.034	7.0168
2	-0.9336	5.7002	26.273	-143.42	188.86
3	-0.8886	4.2024	109.89	-734.77	1633.9
4	-0.819458	-1.639613	321.588136	-2647.310698	8527.47993
5	-0.733532	-13.005977	756.49731	-7585.625788	32694.96417
6	-0.643639	-30.193087	1520.633292	-18328.48547	100679.1706
Mode	C5	C6	C7	C8	C9
1					
2	-75.544				
3	-1519.8	506.59			
4	-13167.19601	9756.204039	-2787.486868		
5	-72918.10326	88031.90396	-54622.17389	13655.54347	
6	-300365.7673	518347.3103	-517032.3904	276699.8692	-61488.85982



**Fig. 5. Strain topological pattern from generalized equation for  $x/l$  ratio**

The generalized strain plot based on the normalized  $x/l$  ratio is plotted. Fig. 5 shows the strain topological pattern plot of the generalized polynomial function obtained from equation (7). The pattern thus obtained indicates that there are different regions where the maximum strain due to different modes are clustered. These clustered zones of maximum strain are in the vicinity of the antinode points of odd modes. Hence, if the path of strain topology is traced for the odd modes, then the peaks of this can be the location for placement of piezopatches. The modal strains of first three odd modes of the beam are shown in Fig. 6. From the pattern of the three odd modes, it is seen that if the outer profile of the upper topology is traced, then the different peak points obtained will be the optimal location of patches.



**Fig. 6. Strain pattern for first three odd modes**

**Strategy to identify patch location**

As discussed earlier if a strain topological plot of the odd modes is created, the trace of the intersecting profile will help in identifying the coordinates for patch placements. A methodology is proposed in the form of an algorithm, which will trace the outline of the intersecting profile of the odd mode strain patterns and will also identify the placement locations. The algorithm proposed for first six modes of vibrations is illustrated with the help of a flowchart shown in Fig. 7 and the steps for the same are enumerated below:

1. Assign a value 'n' for the number of data points desired.
2. Store the values of 'strain' obtained from the generalized polynomial equation (7), for each mode and each data point, in STRAIN matrix of size  $m \times n$ ; where  $n$  = number of data points and  $m$  = number of modes.
3. Create TRACE matrix of size  $1 \times n$  for creating the outline profile of the intersecting odd mode shapes.
4. The algorithm follows three simple steps; {For each value of  $i = 1$  to  $n$ },
  - a. If the  $i^{\text{th}}$  value of magnitude of strain of the first mode in greater than or equal to the magnitude of strain of third and fifth mode, then store the value of strain of first mode in TRACE matrix.
  - b. If the  $i^{\text{th}}$  value of magnitude of strain of the third mode in greater than or equal to the magnitude of strain of first and fifth mode, then store the value of strain of first mode in TRACE matrix.
  - c. If both the above conditions are not satisfied then store the value of strain of fifth mode in TRACE matrix.
5. To identify the peak locations, every value of TRACE matrix is evaluated for the condition that;

If the  $i^{\text{th}}$  value of the strain is greater than the  $(i-1)^{\text{th}}$  value and also greater than  $(i+1)^{\text{th}}$  value, then store that value of strain.

**RESULTS AND DISCUSSION**

The proposed methodology involves the use of a generalized polynomial as presented in equation (7), which is in the form of a dimensionless parameter ( $x/l$ ). This generalized polynomial is used directly in the presented algorithm for tracing the maxima of strain regions along the length of the beam. The trace of the intersecting profile of the odd modal strain pattern generated using the proposed algorithm is as shown in Fig. 8. It is seen that the trace provides a clear profile of available maximas. These maximas are the locations for placement of sensor/actuator pair. Once the profile is generated, the algorithm gives the coordinates of the maximum strain locations, shown in Table 2.

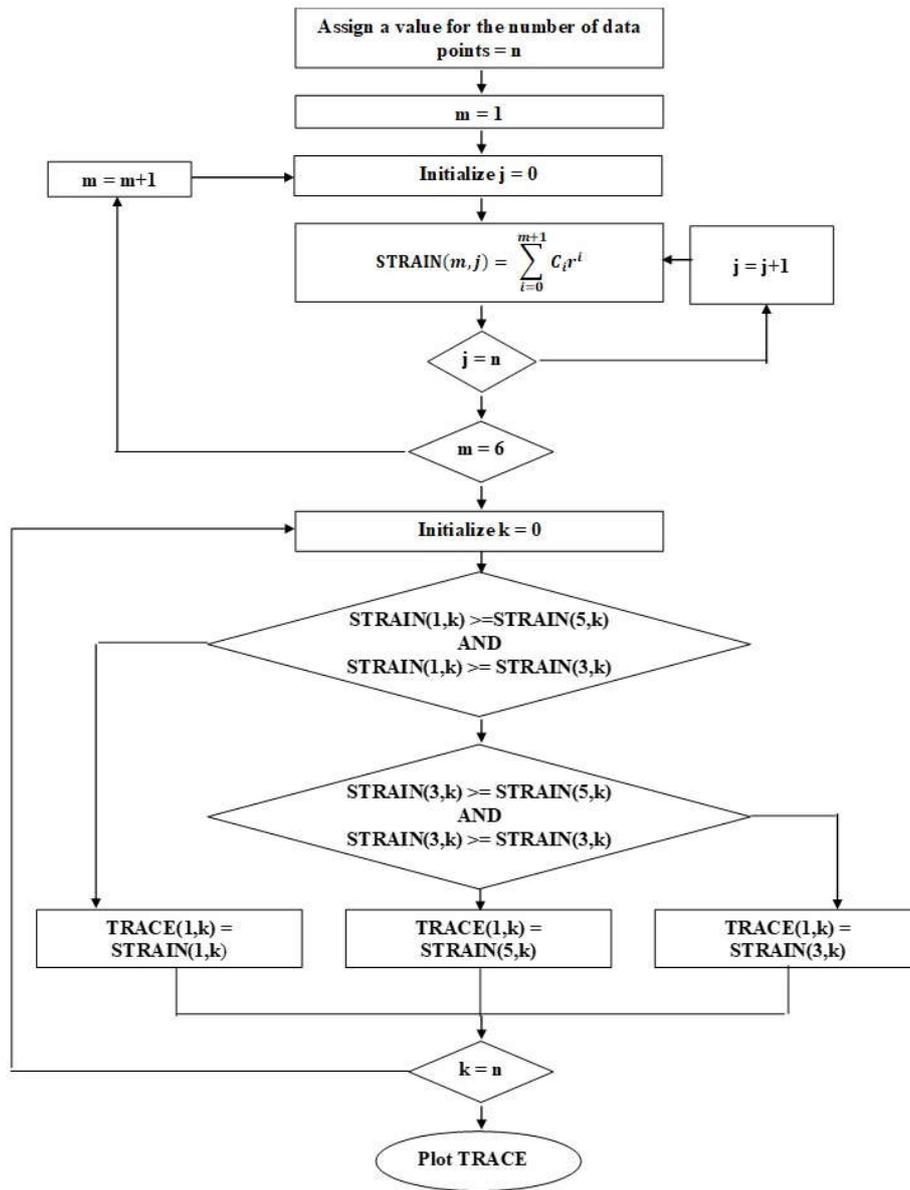


Fig. 7. Flow chart of the proposed algorithm

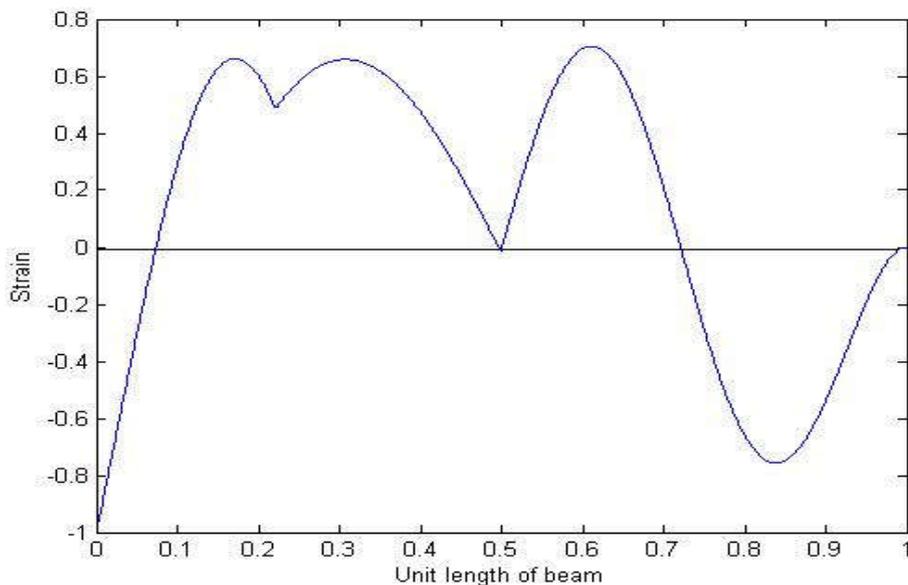


Fig. 8. Combined strain pattern for odd modes

**Table 2. Location for patches (present study)**

Location	1	2	3	4	5
$x/l$ ratio	0.14	0.22	0.50	0.78	0.86

The patch locations identified from the present methodology in the form of a dimensionless parameter can be used for any beam of length ' $l$ ', with the patches being located at the given  $x/l$  ratio distance from the edge of the beam.

The results obtained from the present methodology is compared to a study conducted by earlier research work [10]. The authors had used LQR performance based controller dependent objective function for identifying optimal locations for controlling multimode vibration for a 500mm fixed beam and the result is illustrated in Table 3.

**Table 3. Location for patches (Kumar and Narayanan)**

Location	1	2	3	4	5
Distance (m)	0.05	0.085	0.25	0.41	0.45

Based on the given  $x/l$  ratios for placement of patches in the present study, the location of patches obtained for a beam of length 500mm is shown in Table 4.

**Table 4. Location for patches (0.5m beam)**

Location	1	2	3	4	5
Distance (m)	0.07	0.11	0.25	0.39	0.43

It is observed that the results obtained from the present methodology are in close agreement with the results presented by (Ramesh Kumar *et al.*, 2008). Hence, the present methodology, which involves the direct use of the generalized polynomial equation in the algorithm, can be used for identification of coordinates of patch location in a fixed-fixed beam.

## Conclusion

A methodology for identification of the multiple patch locations for control of first six modes of vibration in a fixed-fixed beam is proposed in the present study. A generalized polynomial equation for modal strain pattern, based on the modal strain fit function for individual modes, is presented along with the coefficients for the first six modes. The generalized polynomial is in the form of dimensionless parameter, which can be used for beam of any length. This generalized polynomial can be directly used with the presented algorithm for tracing the maxima of strain regions along the length of the beam. The effectiveness of the methodology proposed is compared with the existing methodology in the literature. The results are found to be in close agreement with each other. Hence, the present methodology can be used effectively for finding out the optimal location for placement of piezopatches to control multimode vibration of clamped-clamped beam, in a simplified manner.

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