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RESEARCH ARTICLE

A GENERAL METHOD FOR ESTIMATING OF POPULATION SIZE WORLWIDE WITH APPLICATIONS TO SELECTED INTERNATIONAL CENSUS DATA

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ABSTRACT

A mathematical model for estimating of the population size of West Bank and Gaza Strip is used. This model was first proposed in 1992. At that time there was no national, independent or reliable census there. Applying our (same) model showed a great deal of agreement, between our estimates for population size with the resulting figures of recent censuses, not only for these two areas, but for different countries worldwide. Testing and applying our model on accurate demographic data, from selected countries worldwide, with an emphasis to the Palestine case, shows its reliability and thus, its general validity. **Background:** so far, costly and lengthily census is the only way to estimate population size worldwide. So, there is a massive need to find a simpler and a more convenient alternative, a mathematical model; which saves time and reduces costs. **Objective:** To establish an independent source and framework for data on demography alternative workable, and practical mathematical model that can be generalized, to cater different cases globally, to come up with and to produce real and true data for policy planners, and decision makers anywhere in the World, and to save time and reduce costs, substantially. **Method:** Deriving a mathematical model, based on considering a population process as a stochastic one, finding the moment-generating function and the expected value of the proposed population size. **Results:** population size estimates were calculated, by using simple calculating mathematical formulae. **Conclusion:** our general mathematical model can be useful and reliable alternative for traditional censuses anywhere worldwide.

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INTRODUCTION

In concept Census is not an EROR-FREE method for estimating a population size. The alternative model-based methods are, also, approximating ones, by default and nature. So, the resulting figures are also approximate, i. e, we cannot expect highly accurate results. Therefore, the choice between the two methods depends on other considerations. Moreover, demographic data can easily be politicized. Assa'ad and Solqan (1998) have proposed a mathematical model (A-S population model), (see, Appendix), by which the population size of occupied West Bank (WB) and Gaza Strip (GS) was estimated. Actually we started our work early in 1992, i. e, before any Palestinian official comprehensive and reliable census was conducted in the WB and GS. The only census available, at that time, was that done by the military Israeli occupying authorities in 1967. In this article, the same A-S population model will be used to estimate population size of different countries. Comparisons of obtained results, from our model, with the Palestinian Central Bureau of Statistics (PCBS) census, and with censuses of the selected countries, will be made. In fact, since 1967 until early 1990's, the military Israeli occupying authorities and its affiliated departments and centers have had an absolute monopoly on figures and statistics of Palestinians and their life. So, those figures were politicized and employed according to the occupier power policies and propaganda. Almost all surveys, studies and reports which were done or issued during the same period, either by local or foreign organs, were affected, in one way or another, by that Israeli monopoly of figures concerning Palestinians. Of course each part worked to obtain and employ figures and collected data to serve his goals and strategies.

Institutions and organs which were active or involved in this regard, in the WB and GS: Israeli Institutions: Military departments, ICBS, MOI, and WBDP.

Palestinian Institutions: PLO, Arab Studies Society, Palestinian Universities, NGO'S, and Individuals. International Organs & Some Foreign Institutions**:** UNRWA, UNICEF, UNCTAD, UNDP, Other UN organs, World Bank, FAFO, and American Institutions. For example, Habitat (1989), UNICEF (1992), FAFO (1993), UNCTAD (1993), Benvenisti (1989), and Arab Studies Society (1987), are all based and made their estimates and projections relating to Palestinian population on Israeli "figures" and resources. With the emergence of PLO and the Palestinian national identity, both PLO and its counterparts showed more interest of Palestinian demographic data. The scene at that time was really deeply contradictory. So, beside these examples, Idris, A. (1993), has designed & supervised a secretly PLO-supported comprehensive survey for all Palestinian localities in both WB (including East Jerusalem) and GS, to serve as a basic date base for a "future" Palestinian state. Agricultural Relief Committee (ATRC) (1992), had conducted a" Population Agglomeration Survey", as well. The last mentioned surveys, had come up with pure Palestinian figures, though not so accurate, for the first time after the Israeli military occupation since 1967. Actually, there was a pushing intellectual and practical need and necessity:

To establish an independent source and framework for data on demography to come up with an alternative workable, and practical mathematical model that can be generalized, to cater different cases globally, including Palestine case, to produce real and true data for policy planners, and decision makers anywhere in the World, and to save time and reduce costs, substantially. And therefore, with the above mentioned and other reasons, and to bridge the existing gap, Ass'ad and Solqan (1998) proposed a *mathematical model* (A-S Population Model), by which the population size of WB & GS was obtainable, and which substitute the absence of Palestinian census, and "pure Palestinian figures ", besides it is a generalizable model. Based on "solid", trusted figures, such as GSCE students numbers, Assa'd and Solqan (1998) had calculated the percentage annual growth (PAG) of population in the said areas. PAG is a main variable in the process of estimating any population size. From A-S Population model and after due derivation and simplifications, three simple "reduction" formulas were obtained which produced three estimate of the population size of WB and GS, i. e. , minimum, medium and maximum. Besides these three estimates, another formula based on geometric growth was added. In this article same approach and three of the reduction formulae are to be used to estimate population size globally, where PAG will be, alternatively obtained from two main sources; from official records of the selected countries, from the "global" statistics, regularly published by the World Bank, or otherwise, will be arithmetically calculated. The three reduction formulas are denoted, here, by $S1= i(e^{\beta t})$,(formula (1)), $S2 = 1+\beta t + \beta^2 t^2/2+ \beta^3 t^3/6+...+ + \beta^8 t^8/4032$, (formula (2)), (i.e, adding more terms to Taylor's expansion, different from previous A-S Model), and $S3=i(1+ \beta)^t$, (formula (2)). In literature, one can find a growing call for finding mathematical alternatives for costly and time consuming censuses, almost everywhere.

After exhibiting and discussing methods, including non-keyboard data entry and coding difficulties and problems in population censuses, Dekker, A.L. (1994), has concluded that: "a range of new technologies and methods is becoming available", this will assist statistical organizations in their efforts to provide increasingly timely, accurate and useful information while respecting individuals' right to privacy and keeping costs under control." Building on previous findings, Arbeev *et al.* (2005), exhibit different types of mathematical models with applications to the analysis of data on cancer incidences (at old ages). Recently, a growing interest and use of models in demographic studies, and in population estimates. Burch (2003), advocated model-based approach (in population research), and added that" from this perspective, much more formal demography can be seen as a collection of substantive models of population dynamics". Andreev (2004), proposed a method for "producing independent estimates of population aged 90^{+}". When this method has been applied to the evaluation of size of population 90^{+} in the census 2000 of the United States, the results show a high degree of agreement between the two estimates. Other researchers stressed that "the main source of variation in population estimates produced by the used method is likely due to variation in estimates of death ratios. This is clearly more important for countries will small population size rather than for large countries (Andreev, 2004)." Smith, T.M.F (1993) surveyed broadly, but partly, with philosophical approach, sample surveys for the interval (1970-1990). He compared between sample surveys and other techniques such as model- based methods, in terms of reliable (statistical) inference, noting that recent theoretical work has been model-based, seeing the population not as a series of unknown fixed entries but as a realization of an underlying stochastic process. So, he has finally called for a compromise between *randomizers and modelers.* He concluded that: "there is no single paradigm for statistical inference and that different classes of problems require different solutions". Bijak (2016) encouraged by recent work on "model-based "harmonization of migration flow estimates for Europe, describing this work among others, as a tool to "describe and deal with uncertainty".

EMIGRATION

Emigration rate is an essential component (variable) in estimating population size, as well as, births and death rates, and immigration. In last decade, migration was estimated to be about 244 million in 2015 or 3.4 % of world population worldwide, (Juran, 2016). A great deal of specialized studies and state laws tackled this important blowing serious problem, in terms of its human implications on stability, political conflicts and sustainable development. But the existing statistics on international migration do not capture the migration processes well; detailed information is often unavailable, and the quality of the data can be very problematic, (Bijak, 2016). In Palestine, this problem has greater implications and effects, taking political struggle in consideration. A few specialized surveys and researches were conducted here, despite existing shocking figures. For example, Maswadeh (1979) estimated emigration rate from Palestinian occupied territories (WB and GS) from1967-1977 to be 0.226, or 75,000 emigrant annually. Benvenisti (1989) estimated it from 1967-1987 to be 328,000. While Sayegh (1994), quoting from ESCWA records, from 1967-1990 estimated it to be 761,000. FAFO (1993) estimated it to be between 0.005 to 0.009. Recently, PCBS, (Palestinian Central Bureau of Statistics, 2011) estimated the number of (outside) Palestinian emigrants, from 2005-2009, to be 6570 emigrant persons each year, and from 2007, the number was 22,000. On surveying Palestinian emigration PCBS found that 5.9 % of all Palestinians are holding another "nationality "(i. e, passport). And, also, found that, 6.7 % of all Palestinian families (estimated to be about 800,000 families) have at least one outside emigrant, and 17.590 Palestinians were born outside Palestine and stayed outside. In 1997, the number of Palestinians born outside according to PCBS was estimated to be more than

129,000. Recent statistics issued by Palestinian police in Karama Crossing point (from Palestine to Jordan and beyond), as follows:

Table 1. Palestinian Emigrants by selected years

Year	Palestinian Emigrants*
2009	72, 397
2011	30,803
2012	32,770
2014	26,823

*Calculated by Subtracting numbers of arrivals from departures passengers.
Sources: Palestine Dialogue Network (PALDF. NET).

It should be noticed that the number of Palestinian emigrants is increasing year after year. On the other hand, to obtain exact estimates for population size, these figures should be subtracted from PCBS final estimates, which means that final figures, issued by PCBS in its census data (see, Table 3), are greater than the actual number of resident Palestinians in WB and GS .

RESULTS AND COMPARISON

Applying A-S population model with its three reduction calculating formulae, with due consideration to calculating or testing accuracy for PAG, We found a great deal agreement between our calculated estimates for population size, and those officially reported from some selected countries worldwide. In fact, we find our model with its easy-use reduction formulae, is applicable and valid alternative for costly and time-consuming censuses conducted anywhere worldwide. In the following two tables population size has been calculated for five countries; two of them with relatively small sizes (Portugal and Palestine), and of two contradictory PAG's. The other two countries are of big population sizes; USA and China. In all of these five cases when the data taken from its official resources were accurate, calculations were performed and done easier, than the opposite cases, i. e, when data is not well-organized. In Tables 3, our population estimates is calculated by what is referred to as A-S Formula (1) which gives a minimum estimate, A-S Formula (2) which gives a medium estimate, and A-S Formula (3) which gives a maximum estimate. In Table 2, only two of these estimates were calculated, namely, the minimum and the maximum. That is because no arithmetic difference was found between estimates using formula (1) and those using formula (2). Comparisons between our estimates and official figures reported from the appropriate executed census for each country is shown on the same table. In order to execute calculations rapidly, a little MATHEMATICA has been used.

Table 2. Population Size of selected countries worldwide using different base years in millions

		Egypt			USA			China			Portugal		
	Year	2006	2016	2017	2014	2016	2017	2005	2010	2017	2001	2011	2017
	PGA	0.0194[1]	0.0225[1]	0.0225[1]	0.0078[2]	.0078[2a]	.0078[2a]	0.0059[3]	0.0048[3]	.0059[3a]	0.0071[4]	-0.0015[4]	-0.0041[4]
	State Official figures based On Census*	72.6[5]	NA	94.8[5]	318.6[6]	323.4[6]	325.7[6]	1307.6[3]	1340.9	1387.9‡	10.4[7]	10.6[7]	10.3[8]
Year Base Population													
1996 (61M**)	(1) A-S Formula S1	74.1	93.4	97.8									
	(3) A-S Formula S3	73.9	95.2	97.3									
2000 (1.3 B††)	(1) A-S Formula S1							1308	1332	1385† 1695.5			
	(3) A-S Formula S3							1307.9	1332.3	13847† 1691.4			
2001 (10.4 M)	(1) A-S Formula S1											10.2	9.7
	(3) A-S Formula S3											10.2	9.7
2010 (308.7M)	(1) A-S Formula S1				318.5	323.5	326.1						
	(3) A-S Formula S3				318.5	323.5	326						

*In Millions. †considering PAG to be 0.0051 according to World Bank's 2015 figures.
** Millions. ††Billions, ‡According to population pyramid (4. below), this figure is estimated as 1388.2 millions.

1. https://www.capmas.gov.eg/Pages/IndicatorsPage.aspx?page_id=6135&ind_id=1097.
2. https://www.populationpyramid.net/hnp/population-growth/2014/united-states-of-america/.

2a. PAG is calculated from Estimates of the Components of Resident Population Change by Race and Hispanic Origin for the United States: April 1, 2010 to July 1, 2017. (Source: U.S. Census Bureau, Population Division).

3. http://www.stats.gov.cn/tjsj/ndsj/2006/indexeh.htm.

3a.http://www.stats.gov.cn/tjsj/ndsj/2006/indexeh.htm., for the year 2016.

4. https://www.populationpyramid.net/hnp/population-growth/2015/.
5. http://www.capmas.gov.eg/Pages/ShowPDF.aspx?page_id=/Admin/Pages%20Files/Presentation.pdf.
6. https://factfinder.census.gov/faces/tableservices/jsf/pages/productview.xhtml?src=bkmk.
7. http://censos.ine.pt/xportal/xmain?xpgid=censos2011_apresentacao&xpid=CENSOS.
8. https://www.populationpyramid.net/portugal/2017/.

Table 3. Calculated Population Size of WB & GS from different PAG resources for different years using two base years

				MOH*	UNICEF**	UNCTAD‡	Pupils' numbers†	PCBS figures††
Year	1997	PAG		0.0211[1]	.0325[2]	.032[3]	.0507[4]	0.028[5]
		Base year	A-S formula estimates					
		1967	(1) minimum	1,972,520	2,211,881	2,781,990	1,443,846	
			(2) medium	1,972,520	2,211,881	2,781,990	1,443,846	2,895,683
			(3) maximum	1,959,570	2,734,099	2,740,460	1,441,387	
	2007	1967	(1) minimum	2,435,890	3,843,221		1,606,906	
			(2) medium	2,435,890	3,843,221		1,606,906	
			(3) maximum	2,414,590	3,764,566		1,603,257	3,767,549
		1997	(1) minimum	3,575,910	3,662,775		3,222,700	
			(2) medium	3,575,910	3,662,775		3,222,700	
			(3) maximum	3,568,070	3,987,049		3,220,870	
	2017	1967	(1) minimum	3,008,110	5,319,136	5,275,990	1,788,381	
			(2) medium	3,008,110	5,319,136	5,275,750	1,788,381	
			(3) maximum	2,975,270	5,183,410	5,145,380	1,783,310	4,780,978 to
		1997	(1) minimum	4,415,940	5,546,799		3,586,660	4,900,000
			(2) medium	4,415,940	5,546,799		3,586,660	
			(3) maximum	4,396,600	5,489,745		3,582,580	

‡Population size, of WB & GS, according to UNCTAD in 1967 was estimated at 1,065,204. This explains higher estimates here. In our calculations we adapt the figure 1,047,400, when 1967 is used as a base year.
‡UNCTAD, 1989.
* Ministry of Health, PHIC, Health Status, Palestine, 2017, July 2018.
**UNICEF, 1992.
† http://www.pcbs.gov.ps/Portals/_Rainbow/Documents/Education-1994-2016-02A.htm.
†† Palestinian Central Bureau of Statistics (PCBS) (2017) Statistical Yearbook of Palestine, 2017, N0. 18, Ramallah, Palestine. (See it also at, http://www.pcbs.gov.ps/census2017/).
1. https://www.site.moh.ps/index/Books/BookType/2/Language/ar.PAG is calculated from annual pure population increase divided by population midyear estimates, then weight mean is calculated for the interval of years in consideration.
2. UNICEF, 1992.
3. UNCTAD, 1989.
4. PAG is calculated annually then the weighted average is calculated for the whole interval of years i. e, from 1994-2017, as taken from, http://www.pcbs.gov.ps/Portals/_Rainbow/Documents/Education-1994-2016-02A.htm.
5. http://www.pcbs.gov.ps/Indicators. When it was calculated as a weighted average for different years, results were almost equal.

DISCUSSION

The contribution of migration, next to fertility mortality, is too important to be ignored; no population forecasts can be considered reliable if they fail to incorporate human mobility, Bijak (2016). It is important to advocate for the greater release of population and census data, and to exploit migration data to their fullest use (Juran, S., Snow, R., 2016). Perhaps by then, there will be some constructive further thinking about this topic if not even progress towards some pilot experiments of A World Migration Survey, Bilsborrow (2016). Finally, one can call for a decade of using of mathematical model-based surveys, to estimate migration, annual growth rate and therefore, population size. There is an encouraging evidence to start this process.

APPENDIX

- We start by considering a population process $\{W_t, t \in (0,\infty)\}$ where W_t is the size of the population at time t. Let *i* be the birth rate, andμ be the death and emigration rate. If we consider the state of population process to be characterized by the size of population, then the *transition diagram* of the process could be given by:

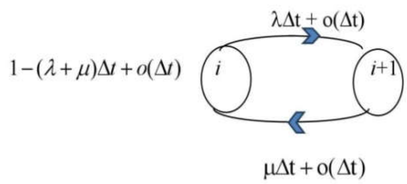

Figure 1. Transition diagram of population process characterized by the size of population

Let W_{to} be the initial size population and equal to *i* members.

Consider the process in the time interval (t, t + Δt) then

$W_{t+\Delta t} = W_t + X_1 + X_2 + \ldots + X_{to} + Z$, Where:

$$Z = \begin{cases} 1 & \text{if there is an immigrant joining the population in } (t, t+\Delta t) \\ & \text{with probablity } \alpha\Delta t + o(\Delta t) \\ 0 & \text{otherwise with probability } 1 - \alpha\Delta t + o(\Delta t) \end{cases}$$

$$\text{and } X_l = \begin{cases} 1 & \text{if the } (l\,th) \text{ member gives a birth in } (t, t+\Delta t) \text{ with probability } l\Delta t + o\Delta t \\ -1 & \text{if the } (l\,th) \text{ member dies or emigrates in } (t, t+\Delta t) \text{ with probability } \mu\Delta t + o(\Delta t) \\ 0 & \text{otherwise, i.e with probability } 1-(\lambda+\mu)\Delta t + o\Delta t \end{cases}$$

It can be shown that,

$$\Pr(W_{t+\Delta t} = j \mid W_t = i) = \Pr(X_1 + X_2 + \ldots + X_i + Z = j - i)$$
$$= \begin{cases} (i\lambda + \alpha)\Delta t + o(\Delta t) & j = i+1 \\ 1 - (i\lambda + i\mu + \alpha)\Delta t + o(\Delta t) & j = i \\ i\mu\Delta t + o(\Delta t) & j = i-1 \\ o(\Delta t) & \text{otherwise} \end{cases}$$

, (see, Coleman, 1974).

Define $P_{ij}(t)$: to be the probability that the system would be in state j when it was in state i before t time units, and $P(t) = (P_{ij}(t))$ to be the *transition matrix.*

$$P(t) = \begin{array}{c|ccccccc} & 0 & 1 & 2 & 3 & 4 & 5 & \ldots \\ \hline 0 & 1-(0+0+\alpha)1+0 & (0\lambda+\alpha)1+0 & 0 & 0 & 0 & 0 & \ldots \\ 1 & 1\mu.1+0 & 1-(\lambda+\mu+\alpha)1+0 & (\lambda+\alpha)1+0 & 0 & 0 & 0 & \ldots \\ 2 & 0 & 2\mu.1+0 & 1-(2\lambda+2\mu+\alpha) & 2\lambda+\alpha & 0 & 0 & \ldots \\ 3 & 0 & 0 & 3\mu & 1-(3\lambda+3\mu+\alpha) & 3\lambda+\alpha & 0 & \ldots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \end{array}$$

The *Kolmogorov* differential equations relate P(t) to Q, (Cinlar,1975), where Q is the rate matrix which is equal to

$$Q = \begin{array}{c|ccccccc} & 0 & 1 & 2 & 3 & 4 & 5 & \ldots \\ \hline 0 & -\alpha & \alpha & 0 & 0 & 0 & 0 & \ldots \\ 1 & \mu & -\mu-\lambda-\alpha & \lambda+\alpha & 0 & 0 & 0 & \ldots \\ 2 & 0 & 2\mu & -2\lambda-2\mu-\alpha & 2\lambda+\alpha & 0 & 0 & \ldots \\ 3 & 0 & 0 & 3\mu & -3\mu-3\lambda-\alpha & 3\lambda+\alpha & 0 & \ldots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \end{array}$$

Then from the above we get the *transition matrix* P(t):

The *forward equations* are given by:

$P'_{ij}(t) = \frac{d}{dt} P_{ij}(t)$, then the following are true; where $W_{to} = i$

$P'_{i0} = -\alpha P_{i0} + \mu P_i$

$$P'_{ij} = ((j-1)\lambda+\alpha)P_{i,j-1} - (j(\lambda+\mu)+\alpha)P_{ij} + (j+1)\mu P_{i,j+1}, \quad j = 1,2 \quad \text{, (Coleman,1974).} \tag{1}$$

$P_{ij} = 0, \forall j < 0$

Multiply (1) by S^j to get the *z-transformation* of these functions, and define π to be

$\pi_t(S) = \sum_{j=-\infty}^{\infty} p_{ij}(t) s^j$, the *probability generation function*.

To get the moment generating function (m. g. f) of $\pi_t(s)$ we find, and obtain:

$$\frac{\partial \pi_t(S)}{\partial t} + (\lambda S - \mu)(1-S)\frac{\partial \pi_t(S)}{\partial S} = \alpha(S-1)\pi_t(S) \tag{2}$$

Solve this partial differential equation; to get:

$$\frac{\lambda S - \mu}{1-S} e^{-(\lambda-\mu)t} = \text{constant}, \ \lambda \neq \mu \tag{3}$$

and, $\frac{\alpha}{\lambda}\left[\frac{\lambda dS}{\lambda S - \mu}\right] = -\frac{d\pi_t(S)}{\pi_t(S)}$, therefore :

$$(\lambda S - \mu)^{\alpha/\lambda} \pi_t(S) = \text{constant} \tag{4}$$

From (3), (4) and the fact that (2) is a first order partial differential equation; it can have only one arbitrary constant, we conclude that:

$$(\lambda S - \mu)^{\alpha/\lambda} \pi_t(S) = f\left\{\frac{\lambda S - \mu}{1-S} e^{-(\lambda-\mu)t}\right\} \tag{5}$$

Using the initial condition $W_{to} = i$, i.e. $\pi_o(S) = S^i$, we obtain

$$(\lambda S - \mu)^{\alpha/\lambda} (S^i) = f\left\{\frac{\lambda S - \mu}{1-S}\right\} \tag{6}$$

Differentiating equation (2) w. r. t. S, we get:

$$\frac{\partial^2 \pi_t(S)}{\partial S \partial t} + (\lambda S - \mu)(1-S)\frac{\partial^2 \pi_t(S)}{\partial S^2}\{(\lambda S-\mu)(-1) + \lambda(1-S) - \alpha(S-1)\}\frac{\partial \pi_t(S)}{\partial S} = \alpha \pi_t(S) \tag{7}$$

But $\pi_t(1) = 1$, $\left.\frac{\partial \pi_t(S)}{\partial S}\right|_{S=1} = M_t = E(W_t)$,

and $\left.\frac{\partial^2 \pi_t(S)}{\partial S \partial t}\right|_{S=1} = \frac{\partial}{\partial t}\left.\left(\frac{\partial \pi_t}{\partial S}\right)\right|_{S=1} M'_t$

Let $S \to 1$ in (7), we get a first order linear differential equation:

$$\frac{dM_t}{dt} + (\mu - \lambda)M_t = \alpha \tag{8}$$

Where,

$M_t = E(W_t) = \left.\frac{\partial \pi_t}{\partial S}\right|_{S=1}$ is the *expected value* of the population process .

Using the fact that $M_0 = E(W_{t0})$=i in (8), we get:

$$M_t = E(W_t) = \begin{cases} \frac{\alpha}{\mu-\lambda} + (i - \frac{\alpha}{\mu-\lambda})^{e^{-(\mu-\lambda)t}} & \text{if } \lambda \neq \mu \\ \alpha t + i & \text{if } \lambda = \mu \end{cases} \quad (9)$$

Let $\beta = \lambda - \mu$, where β is the *annual rate* of growth in the population size, then (9), becomes

$$E(W_t) = \begin{cases} (i + \frac{\alpha}{\beta})e^{\beta t} - \frac{\alpha}{\beta} & \text{if } \lambda \neq \mu \\ \alpha t + i & \text{if } \lambda = \mu \end{cases} \quad (10)$$

Equation (10) can be approximated if we consider the approximate case,

$$\alpha \to 0, \text{ then } \lim E(W_t) = \begin{cases} ie^{\beta t} & \text{if } \lambda \neq \mu \\ i & \text{if } \lambda = \mu, \text{ equilibrium state} \end{cases} \quad (11)$$

$E(W_t) = ie\beta^t$ exhibiting *exponential growth* or *decay* according as $\lambda > \mu$ or $\lambda < \mu$ where i is the size of population at time t = 0.

It is interesting to note that $\lim M_t \to \infty$ where,

$t \to \infty$

$\lambda \geq \mu$

$$\lim M_t \to \frac{\alpha}{\mu - \lambda} = -\frac{\alpha}{\beta} > 0$$

$t \to \infty$

$\lambda < \mu$

These results suggest that in the second case, where $\lambda < \mu$, the population size stabilizes in the long run in some form of *statistical equilibrium.* Sensitivity was studied as well. In order to get an idea about the effect of change of β and t on the size of population we can determine the first partial derivative of $E(W_t)$ with respect to β and to t, using equation (11).

$\frac{\partial E(W_t)}{\partial t} = i\beta e^{\beta t}$, which yields that $E(W_t)$ varies exponentially with respect to the time .

And $\frac{\partial E(W_t)}{\partial \beta} = i\beta e^{\beta t} = ite^{\beta t}$, which shows again that $E(W_t)$ varies exponentially with respect to the annual growth rate β

We know that the *elasticity* "$\in$" is a measure of population of relative change between two quantities, which is defined mathematically as

$$\in_{f,x} = \frac{\frac{\partial f}{f}}{\frac{\partial x}{x}} = \frac{\partial \ln f}{\partial \ln x}, \qquad \text{(Bartman, } et\ al\text{, 1989)}$$

Using this definition of elasticity and Equation (11), we get

$$\in_{E(W_t),t} = \beta t \quad (12)$$

$$\in_{E(W_t),\beta} = \beta t \quad (13)$$

From (12), we conclude that if the value of β increased by r%, then the size of population will increase by r βt%.Same conclusion can be reached from (13). Equations (12) and (13) imply that the elasticity over β and over t is linearly dependent on β and on t respectively.

ABBREVIATIONS

ATRC: (Palestinian) Arab Thought Forum and Agricultural Relief Committee.
A-S Population model: Assa'ad and Solqan model.

ESCWA: Economic and Social Commission for West Asia
FAFO: Fafo Research Foundation (Norway).
GS: Gaza Strip
HABITAT: United Nations Human Settlements Programme.
ICBS: Israel Central Bureau of Statistics.
MOI: (Israel) Ministry of Interior.
NGO's: Non-Governmental Organizations.
PAG: the percentage annual growth. (For a population).
PCBS: Palestinian Central Bureau of Statistics.
PLO: Palestine Liberation Organization.
WBDP: West Bank Data Base Project.
UN: United Nations.
UNCTAD: United Nations for Commerce on Trade and Development.
UNDP: United Nations Development Programme.
UNICEF: United Nations Children's Fund.
UNRWA: United Nations Relief and Works for Palestine Refugees in the Near East.

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