RESEARCH ARTICLE

NEAR PRODUCT CORDIAL GRAPH-CYCLE

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ARTICLE INFO

ABSTRACT

In this paper we discuss about Near product cordial labeling graphs like \((C_4 \otimes C_4) n, Q_n, Parachute P_2, n-2\), Total graph. If the labeling in the graph satisfies the condition of Near product cordial then it is called Near product cordial graphs. In this paper we have proved that the above mentioned graphs except Total graph are Near product cordial graphs.

INTRODUCTION

The concept of cordial labeling was introduced by Cahit. The concept of product cordial labeling is introduced by M. Sundaram, R. Ponraj and S. Somasundaram. Motivated by the above definitions, Near Product cordial was defined.

**Theorem 1:** The concept of cordial labeling was introduced by Cahit. The concept of product cordial labeling is introduced by M. Sundaram, R. Ponraj and S. Somasundaram. Motivated by the above definitions, Near Product cordial was defined.

**Theorem 1:**

\((C_4 \otimes C_4) n\) is Near product cordial graph

**Proof:**

Let \(V(C_4 \otimes C_4) n = \{v^1_i: 1 \leq i \leq 4 \text{ and } 1 \leq j \leq n\}\) and 
\(E(C_4 \otimes C_4) n = \{v^1_i v^1_{i+1}: 1 \leq i \leq 3 \text{ and } 1 \leq j \leq n\} \cup \{v^1_i v^1_j: 1 \leq j \leq n\} \cup \{v^2_i v^2_{i+1}: 1 \leq j \leq n-1\}\).

**Case (i):**

\(n\) is even and let \(n = 2k\) (say)

Define \(f : V(C_4 \otimes C_4) 2k \to \{1,2,3,\ldots,8k-1, 8k+1\}\) by

\[f(v^1_i) = \begin{cases} 8(j-1) + 1, & 1 \leq j \leq k \\ 8(j-k-1) + 2, & k + 1 \leq j \leq 2k \end{cases}\]

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\[
\begin{align*}
f(v_j^1) &= \begin{cases} 
8(j-1) + 3, & 1 \leq j \leq k \\
8(j-k-1) + 4, & k+1 \leq j \leq 2k 
\end{cases} \\
f(v_j^2) &= \begin{cases} 
8(j-1) + 5, & 1 \leq j \leq k \\
8(j-k-1) + 6, & k+1 \leq j \leq 2k 
\end{cases} \\
f(v_j^3) &= \begin{cases} 
8(j-1) + 7, & 1 \leq j \leq k \\
8(j-k-1) + 8, & k+1 \leq j \leq 2k 
\end{cases}
\end{align*}
\]

**Edge Condition:**

\[e_f(0) = 5k \text{ and } e_f(1) = 5k-1 \text{, when } n = 2k\]

Then, \(|e_f(0) - e_f(1)| = 1\)

Hence, \((C_4 \otimes C_4)^{2k}\) is Near product cordial graph.

**Case (ii):**

Suppose \(n\) is odd and let \(n = 2k+1\) (say)

Define \(f: V((C_4 \otimes C_4)^{2k+1}) \rightarrow \{1, 2, 3, \ldots , 8k+3, 8k+5\}\) by

\[
\begin{align*}
f(v_j^1) &= \begin{cases} 
8(j-1) + 1, & 1 \leq j \leq k+1 \\
8(j-k-2) + 4, & k+2 \leq j \leq 2k+1 
\end{cases} \\
f(v_j^2) &= \begin{cases} 
8(j-1) + 3, & 1 \leq j \leq k+1 \\
8(j-k-2) + 6, & k+2 \leq j \leq 2k+1 
\end{cases} \\
f(v_j^3) &= \begin{cases} 
8(j-1) + 5, & 1 \leq j \leq k+1 \\
8(j-k-2) + 8, & k+2 \leq j \leq 2k+1 
\end{cases} \\
f(v_j^4) &= \begin{cases} 
8(j-1) + 7, & 1 \leq j \leq k \\
8(j-k-1) + 2, & k+1 \leq j \leq 2k+1 
\end{cases}
\end{align*}
\]

**Edge Condition:**

\[e_f(0) = 5k+2 \text{ and } e_f(1) = 5k+2, \text{ when } n = 2k+1\]

Then, \(|e_f(0) - e_f(1)| = 0\)

Hence, \((C_4 \otimes C_4)^{2k+1}\) is Near product cordial graph.

**Theorem 2:**

\(Q_n\) is Near product cordial graph if and only if \(n\) is odd.

**Proof:**

Let \(V(Q_n) = \{v_1^i : 1 \leq i \leq n, v_2^i : 1 \leq i \leq n \}

and \(v_3^i : 1 \leq i \leq n+1\} \) and

\[
E(Q_n) = \{(v_1^i v_3^i : 1 \leq i \leq n) \cup (v_2^i v_3^i : 1 \leq i \leq n) \cup (v_1^i v_3^{i+1} : 1 \leq i \leq n) \cup (v_2^i v_3^{i+1} : 1 \leq i \leq n) \}
\]

Define \(f: V(Q_n) \rightarrow \{1, 2, 3, \ldots , 3n, 3n+2\}\) as follows

When \(n\) is odd,

\[
\begin{align*}
\tilde{f}(v_1^i) &= \begin{cases} 
3 + 6(i-1), & 1 \leq i \leq \frac{n+1}{2} \\
4 + 6(i - \frac{n+3}{2}), & \frac{n+3}{2} \leq i \leq n 
\end{cases} \\
\tilde{f}(v_2^i) &= \begin{cases} 
5 + 6(i-1), & 1 \leq i \leq \frac{n+1}{2} \\
6 + 6(i - \frac{n+3}{2}), & \frac{n+3}{2} \leq i \leq n 
\end{cases}
\end{align*}
\]

\textit{Davasuba and Nagarajan, Near product cordial graph}

\textit{cycle}
\[ f(v_3^i) = \begin{cases} 
1 + 6(i - 1), & 1 \leq i \leq \frac{n+1}{2} \\
2 + 6\left(i - \frac{n+3}{2}\right), & \frac{n+3}{2} \leq i \leq n + 1 
\end{cases} \]

**Edge Condition:**

\[ e_f(0) = 2n \text{ and } e_f(1) = 2n \]

Then, \[ |e_f(0) - e_f(1)| = 0 \]

Hence, \( Q_n \) is Near product cordial graph.

Conversely, Suppose \( n \) is even

For any labeling \( f: (G) \rightarrow \{1, 2, \ldots, 3n, 3n+2\}, \) It is observed that there are \( \frac{3n}{2} \) odd numbers and \( \frac{3n}{2} + 1 \) even numbers in \( f(V(Q_n)) \). To get more edge as 1, a maximal connected sub graph of \( Q_n \) on \( \frac{3n}{2} \) vertices should be labeled with odd numbers. So label induced subgraph of \( Q_n \) with the vertex set \( \{v_1^{(i)} v_2^{(i)} v_3^{(i)}: 1 \leq i \leq \frac{n}{2}\} \) by odd numbers to get maximum number of 1 as edge label.

Then \( e_f(1) \leq 2n-2 \) and \( e_f(0) \geq 2n+2 \)

Therefore, \( e_f(0) - e_f(1) \geq 4 \)

Hence, \( Q_n \) is not near product cordial graph.

**Theorem 3:**

Parachute \( P_{2,n-2} \) is a near product cordial when \( n \geq 3 \).

**Proof:**

Let \( V(P_{2,n-2}) = \{ u, u_i : 1 \leq i \leq n \} \) and

\[ E(P_{2,n-2}) = \{u_iu_{i+1} : 1 \leq i \leq n\} \cup \{u_nu_1\} \cup \{uu_1\} \cup \{u_nu\} \]

Define \( f: V(P_{2,n-2}) = \{1, 2, 3, \ldots, n, n+2\} \) as follows

**Case (i): When \( n \) is odd**

When \( n = 3 \)

\[ f(u_i) = 2i-1, \quad 1 \leq i \leq 3 \]

\[ f(u) = 2 \]

**Edge Condition:**

\[ e_f(0) = 2 \text{ and } e_f(1) = 3 \]

Then, \[ |e_f(0) - e_f(1)| = 1 \]

When \( n \) is odd and \( n \geq 5 \)

\[ f(u_i) = 2i - 1, \quad 1 \leq i \leq \frac{n+3}{2} \]

\[ f(u_i) = 2 + 2\left(i - \frac{n+5}{2}\right), \quad \frac{n+5}{2} \leq i \leq n \]

\[ f(u) = n-1 \]
**Edge Condition:**

\[ e_f(0) = \frac{n+3}{2} \text{ and } e_f(1) = \frac{n+1}{2} \]

Then, \( |e_f(0) - e_f(1)| = 1 \)

Hence, Parachute \( P_{2,n-2} \) is Near product cordial graph when \( n \) is odd.

**Case (ii):**

**When \( n \) is even**

When \( n \) is even there are \( \frac{n}{2} \) odd numbers and \( \frac{n+2}{2} \) even numbers in \( f(V(G)) \).

In order to get maximum number of 1, we should label maximal connected sub graph on \( \frac{n}{2} \) vertices of \( G \) with odd numbers. Note that maximal connected sub graph on \( \frac{n}{2} \) vertices should contain the cycle \((u_u u_n)\) as a sub graph. It should be a unicyclic graph and hence its number of edges also \( \frac{n}{2} \).

Then, \( e_f(0) \geq \frac{n}{2} + 2 \) and \( e_f(1) \leq \frac{n}{2} \)

Hence \( |e_f(0) - e_f(1)| \geq 2 \). It is not a Near product cordial graph.

But it is a weak near product cordial graph

Now label \( (P_{2,n-2}) \) as follows

\[
\begin{align*}
    f(u) &= 3 \\
    f(u_n) &= 1 \\
    f(u_i) &= 5 + 2(i-1) \quad 1 \leq i \leq \frac{n-4}{2} \\
    f(u_i) &= 2 + 2(i - \frac{n-2}{2}) \quad \frac{n-2}{2} \leq i \leq n - 1
\end{align*}
\]

**Edge Condition:**

\[ e_f(0) = \frac{n+4}{2} \text{ and } e_f(1) = \frac{n}{2} \]

Then, \( |e_f(0) - e_f(1)| = 2 \)

Hence, Parachute \( P_{2,n-2} \) is Weak Near product cordial graph when \( n \) is even and \( n \geq 4 \).

**Case (iii):**

**When \( n = 4 \)**

In this case, we have 5 vertices and 6 edges exactly and there are 2 odd numbers and 3 even numbers in \( f(V(G)) \).

Clearly, \( e_f(0) = 5 \) and \( e_f(1) = 1 \)

Hence, Parachute \( P_{2,n-2} \) is not a Near product cordial graph, when \( n = 4 \).

**Theorem 4:**

The Total graph \( T(P_n) \) is not Near product cordial
Proof:

Let $V(T(P_n)) = \{u_i: 1 \leq i \leq n \}$ and $\{v_i: 1 \leq i \leq n-1 \}$ and

$E(T(P_n)) = \{(u_iu_{i+1}) \cup (u_iv_i) \cup (u_{i+1}v_i): 1 \leq i \leq n-1 \} \cup (v_iv_{i+1}): 1 \leq i \leq n-2 \}$

For any labeling $f:V(T(P_n)) \rightarrow \{1, 2, 3, \ldots, 2n-2, 2n\}$ there are $n-1$ odd numbers and $n$ is even numbers. In order to get maximum edge label 1, a maximal connected subgraph of $T(P_n)$ on $n-1$ vertices should be labeled with odd numbers. This can be done as follows. If $n$ is odd, the induced sub graph with vertex set \{u_i ,v_i: 1 \leq i \leq \frac{n-1}{2}\} of $T(P_n)$ should be labeled with odd numbers and if $n$ is even then the induced sub graph with vertex set \{u_i: 1 \leq i \leq \frac{n}{2}\} \{v_i: 1 \leq i \leq \frac{n-2}{2}\}$ of $T(P_n)$ should be labeled with odd numbers.

Edge Condition:

In both the cases we have,$$e_{f}(1) \leq 2n-5 \text{ and } e_{f}(0) \geq 2n$$

Then, $e_{f}(0) - e_{f}(1) \geq 2n - (2n-5) = 5$

Hence, $T(P_n)$ is not Near product cordial graph.

REFERENCES


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