Available online at http://www.journalcra.com

International Journal of Current Research
Vol. 11, Issue, 03, pp.1889-1893, March, 2019

DOI: https://doi.org/10.24941/ijcr.33334.03.2019

INTERNATIONAL JOURNAL OF CURRENT RESEARCH

ISSN: 0975-833X

RESEARCH ARTICLE

NEAR PRODUCT CORDIAL GRAPH-CYCLE

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ARTICLE INFO

Article History:
Received 26th December, 2018
Received in revised form
13th January, 2019
Accepted 17th February, 2019
Published online 31st March, 2019

Key Words:
Cordial labeling, Divisor cordial labeling and Near Product cordial labeling.

ABSTRACT

In this paper we discuss about Near product cordial labeling graphs like (C4 ⊗ C4) n,Qn, Parachute P2,n-2,Total graph. If the labeling in the graph satisfies the condition of Near product cordial then it is called Near product cordial graphs.In this paper we have proved that the above mentioned graphs except Total graph are Near product cordial graphs.

Citation: Davasuba, S. and Nagarajan, A. 2018. "Near product cordial graph-cycle", *International Journal of Current Research*, 11, (03), 1889-1893.

INTRODUCTION

The concept of cordial labeling was introduced by Cahit. The concept of product cordial labeling is introduced by M. Sundaram, R. Ponraj and S. Somasundaram. Motivated by the above definitions, Near Product cordial was defined.

Theorem 1: The concept of cordial labeling was introduced by Cahit. The concept of product cordial labeling is introduced by M. Sundaram, R. Ponraj and S. Somasundaram. Motivated by the above definitions, Near Product cordial was defined.

Theorem 1:

$(C_4 \otimes C_4)$ n is Near product cordial graph

Proof:

Let $V(C_4 \otimes C_4)$ n $= \{ v_i^j : 1 \le i \le 4$ and $1 \le j \le n\}$ and

$E(C_4 \otimes C_4)$ n $= \{v_i^j v_{i+1}^j : 1 \le i \le 3$ and $1 \le j \le n\} \cup \{ v_4^j v_1^j : 1 \le j \le n\} \cup \{ v_4^j v_2^{j+1} : 1 \le j \le n-1\}$.

Case (i):

n is even and let n = 2k (say)

Define $f : V(C_4 \otimes C_4)2k \rightarrow \{1,2,3,........,8k-1 , 8k+1\}$ by

$$f(v_1^j) = \begin{cases} 8(j-1)+1, 1 \le j \le k \\ 8(j-k-1)+2 , k+1 \le j \le 2k \end{cases}$$

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$f(v_2^j) = \begin{cases} 8(j-1)+3, 1 \le j \le k \\ 8(j-k-1)+4, k+1 \le j \le 2k \end{cases}$

$f(v_3^j) = \begin{cases} 8(j-1)+5, 1 \le j \le k \\ 8(j-k-1)+6, k+1 \le j \le 2k \end{cases}$

$f(v_4^j) = \begin{cases} 8(j-1)+7, 1 \le j \le k \\ 8(j-k-1)+8, k+1 \le j \le 2k \end{cases}$

Edge Condition:

$e_f(0) = 5k$ and $e_f(1) = 5k-1$, when $n = 2k$

Then, $|e_f(0)-e_f(1)| = 1$

Hence, $(C_4 \otimes C_4)2k$ is Near product cordial graph.

Case (ii):

Suppose n is odd and let $n = 2k+1$ (say)

Define $f: V(C_4 \otimes C_4)2k+1 \rightarrow \{1,2,3, \ldots ,8k+3, 8k+5\}$ by

$f(v_1^j) = \begin{cases} 8(j-1)+1, 1 \le j \le k+1 \\ 8(j-k-2)+4, k+2 \le j \le 2k+1 \end{cases}$

$f(v_2^j) = \begin{cases} 8(j-1)+3, 1 \le j \le k+1 \\ 8(j-k-2)+6, k+2 \le j \le 2k+1 \end{cases}$

$f(v_3^j) = \begin{cases} 8(j-1)+5, 1 \le j \le k+1 \\ 8(j-k-2)+8, k+2 \le j \le 2k+1 \end{cases}$

$f(v_4^j) = \begin{cases} 8(j-1)+7, 1 \le j \le k \\ 8(j-k-1)+2, k+1 \le j \le 2k+1 \end{cases}$

Edge Condition:

$e_f(0) = 5k+2$ and $e_f(1) = 5k+2$, when $n = 2k+1$

Then, $|e_f(0)-e_f(1)| = 0$

Hence, $(C_4 \otimes C_4)2k+1$ is Near product cordial graph.

Theorem 2:

Q_n is Near product cordial graph if and only if n is odd.

Proof:

Let $V(Q_n) = \{ v_1^i : 1 \le i \le n, v_2^i : 1 \le i \le n$ and $v_3^i : 1 \le i \le n+1\}$ and

$E(Q_n) = \{(v_1^i v_3^i : 1 \le i \le n) \cup (v_2^i v_3^i : 1 \le i \le n) \cup (v_1^i v_3^{i+1} : 1 \le i \le n) \cup (v_2^i v_3^{i+1} : 1 \le i \le n\}$

Define $f: V(Qn) \rightarrow \{1, 2, 3, \ldots ,3n, 3n+2\}$ as follows

When n is odd.

$f(v_1^i) = \begin{cases} 3+6(i-1), 1 \le i \le \frac{n+1}{2} \\ 4+6(i-\frac{n+3}{2}), \frac{n+3}{2} \le i \le n \end{cases}$

$(v_2^i) = \begin{cases} 5+6(i-1), 1 \le i \le \frac{n+1}{2} \\ 6+6(i-\frac{n+3}{2}), \frac{n+3}{2} \le i \le n \end{cases}$

$$f(v_3^i) = \begin{cases} 1+6(i-1), 1 \le i \le \frac{n+1}{2} \\ 2+6\left(i-\frac{n+3}{2}\right), \frac{n+3}{2} \le i \le n+1 \end{cases}$$

Edge Condition:

$e_f(0) = 2n$ and $e_f(1) = 2n$

Then, $|e_f(0) - e_f(1)| = 0$

Hence, Q_n is Near product cordial graph.

Conversely, Suppose n is even

For any labeling f: (G)→{1, 2, . . . ,3n,3n+2},It is observed that there are $\frac{3n}{2}$ odd numbers and $\frac{3n}{2}+1$ even numbers in $f(V(Q_n))$.To get more edge as 1, a maximal connected sub graph of Q_n on $\frac{3n}{2}$ vertices should be labeled with odd numbers.So label induced subgraph of Q_n with the vertex set $\{v_1^{(i)}\ v_2^{(i)}\ v_3^{(i)}: 1 \le i \le \frac{n}{2}\}$ by odd numbers to get maximum number of 1 as edge label.

Then $e_f(1) \le 2n-2$ and $e_f(0) \ge 2n+2$
Therefore, $e_f(0) - e_f(1) \ge 4$
Hence, Q_n is not near product cordial graph.

Theorem 3:

Parachute $P_{2,n-2}$ is a near product cordial when $n \ge 3$.

Proof:

Let $V(P_{2,n-2}) = \{ u, u_i : 1 \le i \le n\}$ and

$E(P_{2,n-2}) = \{u_iu_{i+1} : 1 \le i \le n\} \cup \{u_nu_1\} \cup \{uu_1\} \cup \{u_nu\}$

Define $f : V(P_{2,n-2}) = \{1, 2, 3, \ldots, n, n+2\}$ as follows

Case (i): When n is odd

When n = 3

$f(u_i) = 2i-1,\ 1 \le i \le 3$

$f(u) = 2$

Edge Condition:

$e_f(0) = 2$ and $e_f(1) = 3$

Then, $|e_f(0) - e_f(1)| = 1$

When n is odd and n ≥ 5

$f(u_i) = 2i-1,\ 1 \le i \le \frac{n+3}{2}$

$f(u_i) = 2+2(i-\frac{n+5}{2}),\ \frac{n+5}{2} \le i \le n$

$f(u) = n-1$

Edge Condition:

$e_f(0) = \frac{n+3}{2}$ and $e_f(1) = \frac{n+1}{2}$

Then, $|e_f(0) - e_f(1)| = 1$

Hence, Parachute $P_{2,n-2}$ is Near product cordial graph when n is odd.

Case (ii):

When n is even

When n is even there are $\frac{n}{2}$ odd numbers and $\frac{n+2}{2}$ even numbers in f(V(G)).

In order to get maximum number of 1, we should label maximal connected sub graph on $\frac{n}{2}$ vertices of G with odd numbers. Note that maximal connected sub graph on $\frac{n}{2}$ vertices should contain the cycle (uu_1u_n) as a sub graph.It should be a unicyclic graph and hence its number of edges also $\frac{n}{2}$.

Then, $e_f(0) \geq \frac{n}{2}+2$ and $e_f(1) \leq \frac{n}{2}$

Hence $|e_f(0) - e_f(1)| \geq 2$. It is not a Near product cordial graph.

But it is a weak near product cordial graph

Now label ($P_{2,n-2}$) as follows

$f(u) = 3$
$f(u_n) = 1$
$f(u_i) = 5+2(i-1)$, $1 \leq i \leq \frac{n-4}{2}$
$f(u_i) = 2+2(i-\frac{n-2}{2})$, $\frac{n-2}{2} \leq i \leq n-1$

Edge Condition:

$e_f(0) = \frac{n+4}{2}$ and $e_f(1) = \frac{n}{2}$

Then, $|e_f(0) - e_f(1)| = 2$

Hence, Parachute $P_{2,n-2}$ is Weak Near product cordial graph when n is even and n > 4.

Case (iii):

When n = 4

In this case, we have 5 vertices and 6 edges exactly and there are 2 odd numbers and 3 even numbers in f(V(G)).

Clearly, $e_f(0) = 5$ and $e_f(1) = 1$

Hence, Parachute $P_{2,n-2}$ is not a Near product cordial graph, when n = 4.

Theorem 4:

The Total graph $T(P_n)$ is not Near product cordial

Proof:

Let $V(T(P_n)) = \{u_i : 1 \leq i \leq n\}$ and
$\{v_i : 1 \leq i \leq n-1\}$ and
$E(T(P_n)) = \{(u_i u_{i+1}) \cup (u_i v_i) \cup$
$(u_{i+1} v_i) : 1 \leq i \leq n-1\} \cup$
$(v_i v_{i+1}) : 1 \leq i \leq n-2\}$

For any labeling $f: V(T(P_n)) \rightarrow \{1, 2, 3, \ldots, 2n-2, 2n\}$ there are n-1 odd numbers and n is even numbers. In order to get maximum edge label 1, a maximal connected subgraph of $T(P_n)$ on n-1 vertices should be labeled with odd numbers. This can be done as follows. If n is odd, the induced sub graph with vertex set $\{u_i, v_i : 1 \leq i \leq \frac{n-1}{2}\}$ of $T(P_n)$ should be labeled with odd numbers and if n is even then the induced sub graph with vertex set $\{u_i : 1 \leq i \leq \frac{n}{2}\}\{v_i : 1 \leq i \leq \frac{n-2}{2}\}$ of $T(P_n)$ should be labeled with odd numbers.

Edge Condition:

In both the cases we have,

$e_f(1) \leq 2n-5$ and $e_f(0) \geq 2n$

Then, $e_f(0) - e_f(1) \geq 2n - (2n-5) = 5$

Hence, $T(P_n)$ is not Near product cordial graph.

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