



COMPARISON OF TRAIN VALUES OF PGT FOR DIFFERENT STRUCTURAL ASPECTS LIKE SYMMETRY, COMPACTNESS AND DRIVER SPEED

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ABSTRACT

Automatic transmission is used to maintain the equilibrium between power and torque produced by the engine. The power produced by the engine is transmitted to drive wheel through transmission unit, reduction unit and differential. A desired speed ratio, kinematic and dynamic requirements can be fulfilled by the equilibrium of torque and power. Accurate speed ratio's can be achieved by using Planetary Gear Trains which are used to transmit motion between two or more shafts.

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INTRODUCTION

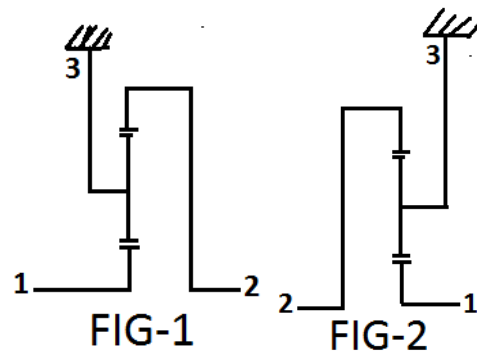
A rigid body rotates about its own center or axis and at the same time it also revolves about another axis then the mechanism is known as planetary mechanism. A mechanism which consists of one central gear, one arm and one or two planet gears then the mechanism is called basic PGT. A mechanism which consists of two central gears and one or more planet gears and one arm carrying the planet gears then the mechanism is known as simple PGT. If the planet gears are directly in mesh with two central gears on different pitch circles at different planes then the mechanism is known as compound PGT. There are several reasons for using different notation for a Planetary Gear Train (PGT), the main reason is complicated arrangements can be simplified by using the notation that completely defines the configuration of the PGT. In a simple PGT central gear or carrier can be used as input or output. A PGT is a two Degree Of Freedom (DOF) chain however one of the gear elements is kept stationary, thus it is reduced to one DOF. There are many methods to determine the kinematic equation of motion such as relative velocity, superposition, train value, vector loop method and graphical method.

VELOCITY RATIO

Various researchers [1-11] are working to achieve a desired set of input-output ratio by conceptualizing a configuration of planetary gear sets, fixed interaction between gear members and the input output connections. To design or investigate a PGT, analytical and graphical methods are used. By using the basic ratio, the characterization of a simple PGT is possible, this is according to Willis's [6]. Ratio of two central gears angular velocities with respect to the relative arm is known as basic ratio(B), this ratio for fig(1) is given as,

$$B_{12} = \left(\frac{w_2 - w_3}{w_1 - w_3} \right)$$

In a PGT, Velocity ratio or train value is different from the basic ratio. For a given PGT always the basic ratio is fixed and is characterized uniquely whereas train value or velocity ratio of a PGT is varying depending on the arrangement of PGT. $Bw_1-w_2-(b-1)w_3=0$, this equation is known as kinematic equation of a PGT.



Consider a two DOF geared kinematic chain keeping input to link 3 fixed and input given to element 1 or element 2. The velocity ratio is not same when the case is taken as the elements 3,1 are input elements compared to the case where input is given to the elements 3 and 2. One can achieve different velocity ratios by varying the fixed element and the input element. In the above case the velocity ratio is similar to the basic ratio. In Fig (1) the input elements are link1 and link3, and in Fig (2) the input elements are link 2 and link 3, the variation in the velocity ratio is inverse to the first case, it is happened because of changing the arrangement of PGT. Functional representation of a basic PGT is shown in Fig (3) and the graphical and rotation graph representations of same PGTs are shown in figures (4) & (5) respectively. In the basic PGT two meshing gears 2 and 3 rotate about the two parallel shafts that are supported by a carrier or arm. In graphical representation links are denoted by vertices and joints by edges.

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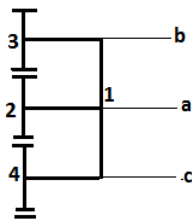


FIG-3: SCHEMATIC REPRESENTATION

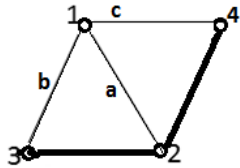


FIG-4 GRAPH REPRESENTATION

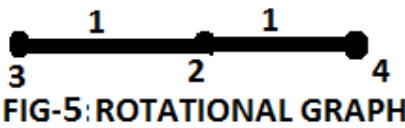


FIG-5: ROTATIONAL GRAPH

Graph consists of turning pair edges and gear pair edges. In a graph turning pair edges and gear pair edges are represented by a thin or single line and thick or double line respectively. Representing thin and thick lines in a graph is known as a labelled graph, otherwise the graph is known as unlabeled graph [4, 5, 7]. Removing the turning pairs in a graph and marking the transfer vertex on the gear pair is known as the rotational graph shown in fig(5). Transfer vertex separates all the edges at same level on one side and all the remaining edges on the other side in a fundamental circuit.

Adjacency matrix: J. J. Uicker and Raicu [3] first defined the adjacency matrix for planar linkages and later the definition is modified to include linkages with gear elements. The vertices of a graph is numbered from 1 to n, adjacency matrix, A, is a square and symmetric matrix of order n with its elements a (i, j), given by [9, 10]

$$\text{Adjacency matrix, } A = \begin{cases} 1 & \text{if link } i \text{ is connected to link } j \text{ by a turning pair} \\ 2 & \text{if link } i \text{ is connected to link } j \text{ by a gear pair} \\ 0 & \text{if link } i \text{ is not connected to link } j \text{ or } i = j \end{cases}$$

Hamming Matrix: Hamming matrix of the adjacency matrix is given by following rules:

$$h_{ij} = \sum_{k=1}^n a_{ik} + a_{jk} \quad \text{If } a_{ik} \neq a_{jk}$$

$$h_{ij} = 0 \text{ for } a_{ik}=a_{jk}, h_{ii}=0$$

Train value: transforming the energy from the plane of generation to the work location is called transmission. This transforming is done by driver and driven shafts. The ratio of the speed of the driven shaft to the speed of the driver shaft is known as train value or velocity ratio or kinematic coefficient [2, 9].

$$\frac{N_i}{N_j} = \frac{-Z_j}{Z_i}$$

Where N represents the speed of the shaft in rpm and Z represents the number of the teeth on gear. Power is transmitted through a speed reduction unit the speed ratio of the last pair of meshing gears is larger than that of the first gear pair because the torque is greater at the low speed end. For an available amount of space, higher number of teeth can be used on gears with less pitch modules. Higher speed reductions are possible at the high speed end. In such applications as speed reducers, clocks where in hour hand is connected to minute hand, machine tools etc. Compactness is one of the main structural aspects. If the center distance between a pair of gears is fixed by the design considerations or special constraints of the machine. In such cases modifications cannot be made by varying the center distance. When center distance is increased, the base circles of the two gears do not change. Larger center distance results in an increase in pressure angle and large pitch circles passing through a new adjusted pitch point and shortening the path of contact, which in turn reduces the contact ratio. When the contact ratio is less than 1 then no teeth can be in contact at all. The center distance must not be higher than that corresponding to a contact ratio of unity. Gear ratio or Train value influences either the output torque or output speed of the PGT but not both. For the given power input either the higher velocity ratio or higher torque forces are achieved. If the center distance between the gears is very small than the gear system then it is said to be compact. Smaller center distance will leads to the higher gear ratio, this is illustrated by the following examples:

Example 1:

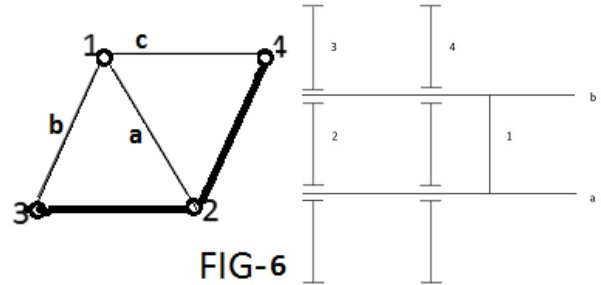


FIG-6

The given two examples conclude that higher the compactness results in o select one in this high gear ratio or high train value and varying the input speed has no effect on train value. For the above two examples the input speed as taken as 100rpm if it increased from 100rpm to 150rpm then there is no effect on the train value.

Table 1. Comparison of GR at different center distance

S.No	No. of teeth on wheel-2	a+b=150; a+c=100		a+b=100;a+c=60;	
		G3	G4	G3	G4
1	30	1.25	1.25	1.42	1.5
2	35	1.3	1.33	1.53	1.71
3	40	1.36	1.42	1.66	2
4	45	1.42	1.53	1.81	2.4
5	50	1.5	1.66	2	3
6	55	1.57	1.81	2.22	4

Example 2:

Consider another five link PGT and compare the changed gear ratio values which are occurred for changing the center distance of the meshing gears. The results obtained are shown in Table-2.

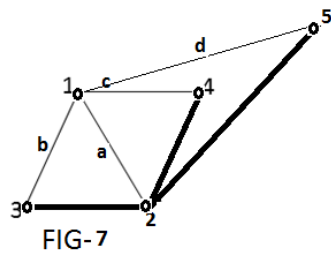


Table 2. comparison of Gear Ratio values at different center distances

S.No.	No. of teeth on wheel-2	a+b=100; a+c=80; a+d=60;			a+b=140; a+c=120; a+d=100;		
		G3	G4	G5	G3	G4	G5
1	30	1.42	1.45	1.5	1.27	1.26	1.25
2	35	1.53	1.6	1.71	1.33	1.33	1.33
3	40	1.66	1.77	2	1.4	1.41	1.42
4	45	1.81	2	2.4	1.47	1.5	1.53
5	50	2	2.2	3	1.55	1.6	1.66

2. SYMMETRY

Symmetry in PGT is defined as identical location of identical elements in a PGT. The generation of gear trains is useful for the designer to know other structural characteristics. Symmetry is one such characteristic. Symmetry gives better balancing of elements, forces, power distributions and leads to reduction in generation effort. For better balance, two or more planet gears are added to increase the number of forces to the existing gear trains as shown in Fig-(8). But the added planet gears do not affect the kinematic performance. Hence these additional planet gears are referred as idler gears. Further aesthetics also improved by structural symmetry. Instead of random selection of a PGT from a set of isomorphic PGTs for a given number of elements and DOF, it is advised to select one with higher symmetry.

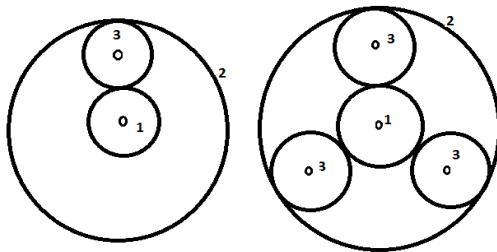


FIG-8

Symmetrical placement of members in the structure i.e. identical location of identical members with reference to another member indicates symmetry about the member. Hamming values of the links in a PGT are useful in identifying the symmetry of the links. For example consider two 4-link PGTs shown in Fig-(6) and Fig-(9). Fig-(6) is having high symmetry whereas Fig-(9) is having no identical links and which leads to no symmetry.

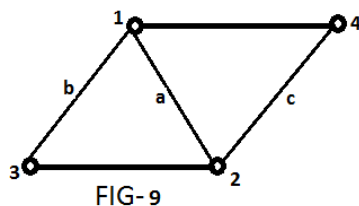


FIG-9

Comparing the G4 values in Table (1) and Table (3), it is clearly known that the values in Table (3) is very less i.e. when the PGT having no symmetry then the gear ratio is very less. It is also observed that when there is no symmetry then the number of possible structures is also less. PGT shown in Fig (6) is having high symmetry, the number of possible of graphs are two, they are (12),(13)(14) and (12),(13,14) whereas the PGT shown in Fig (9) has no symmetry so the number of possible structures is only one i.e. (12),(13),(14).

$$A = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 2 & 0 \\ 1 & 2 & 0 & 2 \\ 1 & 0 & 2 & 0 \end{bmatrix} \quad hij = \begin{array}{c|c} \begin{matrix} 0 & 6 & 8 & 6 \\ 6 & 0 & 6 & 0 \\ 8 & 6 & 0 & 6 \\ 6 & 0 & 6 & 0 \end{matrix} & \begin{matrix} 20 \\ 12 \\ 20 \\ 12 \end{matrix} \\ \hline 64 \end{array}$$

Adjacency and Hamming matrix for graph in fig 6

Hamming string is 64[20,20,12,12]

$$A = \begin{bmatrix} 0 & 1 & 1 & 2 \\ 1 & 0 & 2 & 0 \\ 1 & 2 & 0 & 1 \\ 2 & 0 & 1 & 0 \end{bmatrix} \quad hij = \begin{array}{c|c} \begin{matrix} 0 & 7 & 8 & 5 \\ 7 & 0 & 5 & 6 \\ 8 & 5 & 0 & 7 \\ 5 & 6 & 7 & 0 \end{matrix} & \begin{matrix} 20 \\ 18 \\ 20 \\ 18 \end{matrix} \\ \hline 76 \end{array}$$

Adjacency and Hamming matrix for graph in fig9

Hamming string is 76[20, 20, 18, 18]

Table 3. Gear ratios for the PGT in Fig (9) – a 4-link PGT

S. No	Number of teeth on wheel-2	a+b=100; a+c=140;	
		G3	G4
1	30	1.42	-0.33
2	35	1.53	-0.4
3	40	1.66	-0.47
4	45	1.81	-0.55
5	50	2	-0.64

It means symmetry effects the gear ratio as well as number of possible structures of a PGT. Consider another graph shown in fig (7) which is having high symmetry and the number of possible structures are given below i) (12), (13), (14), (15), ii) (12), (13, 14, 15), iii) (12), (13), (14,15) and iv) (12), (13,14), (15). As the number of links increases it also affects the gear ratio. Compare the G4 values of the graphs shown in fig (6) and fig (7). Table 4 conclude that gear ratio is affected with the number of links in a PGT. More the number of links then the gear ratio is the gear ratio for five link structure is less when compared to the gear ratio of four link structures.

Table 4. Comparison of G4 values of FIG (6&7)

S.No	Number of teeth on wheel	G4 value for four link	G4 value for five link
1	30	1.5	1.26
2	35	1.71	1.33
3	40	2	1.41
4	45	2.4	1.5
5	50	3	1.6

Conclusion

For an automotive transmission it is required to maintain the equilibrium between torque produced by the engine and power. A PGT is used to achieve the equilibrium between these two. Desired gear ratios or train values are chosen to get the different kinematic requirements. In this work kinematic structural characteristics of PGTs are identified and compared with some factors like speed of the wheel, symmetry and center distance between wheels in a PGT. As the center distance is decreased then the gear ratio is high, when a PGT with high symmetry leads to high in gear ratio and change of speed of the gear wheel is not affecting the gear ratio.

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