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RESEARCH ARTICLE

GENERALIZED NELSON-AALEN ESTIMATOR FOR FUZZY SURVIVAL TIMES

*Jaisankar, R., Parvatha Varshini, K.S. and Narayamoorthy, S.

Department of Statistics, Bharathiar University, Coimbatore, Tamil Nadu, India

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ABSTRACT

The survival data are often censored in nature and hence semi-parametric or nonparametric methods are mostly used for estimating the survival or hazard functions and also for making other types of analysis. It is to be noted that the life-times of several natural phenomena are continuous in nature, not the precise numbers and hence denoting the life-times as precise may be absurd. Viertl (2009) has shown that life times are more or less fuzzy. In this research paper, a procedure for Generalized Nelson-Aalen Estimator is introduced on the basis of fuzzy survival times. Examples are given to substantiate the proposed method.

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INTRODUCTION

Survival Analysis is a branch of Biostatistics which encompasses a huge variety of methods for analyzing the time for an event to happen relating to living organisms. The survival time may be defined in years, months, weeks or days from the initial follow-up of an individual till the occurrence of the event of interest in a study, which may be called as lifetime, failure time, or survival data. The event may be defined as death, disease incidence, relapse from the disease, divorce, unemployment which may happen for any individual. The main goals of survival analysis are to estimate and to interpret the survivor function or the hazard function from the survival data, to compare the survivor and the hazard function of different groups and to ascertain the relationship of explanatory variables to the survival time. Due to the censored nature of survival data non-parametric procedures are used to estimate the survival function or hazard function as and it does not require any mathematical assumptions, either about the underlying hazard function or about proportional hazards, uses simple, the empirical probability of surviving past certain times in the sample by taking into account of censoring. The Kaplan Meier estimator (1958) is one of the familiar methods in a non-parametric procedure often used to estimate the survivor function. In this study, another method called the Nelson-Aalen estimator is taken-up for formulating the Generalized fuzzy Nelson-Aalen estimator.

Corresponding author: **Jaisankar, R.,**
Department of Statistics, Bharathiar University, Coimbatore, Tamil Nadu, India.

THE NELSON-AALEN ESTIMATOR

The Nelson-Aalen estimator (1970) is a non-parametric estimator used to estimate the cumulative hazard rate from censored and non censored survival data. This estimator is highly useful to check graphically the fit of parametric models. Assuming right censoring, it is further assumed they are independent in the sense that the additional knowledge of censorings before any time t does not alter the risk of failure t. Let $t_1, t_2, \ldots$ are the times when deaths are observed and let d_j be the number of individuals who die at t_j The Nelson Aalen estimator for the cumulative hazard rate function is given by the following formula.

$$\hat{H}(t) = \begin{cases} 0 & if \ \ t < t_1 \\ \sum_{t_i \le t} \frac{d_i}{Y_i} & if \ t_1 < t. \end{cases}$$

where, Y_i is the number of individuals at risk (alive and not censored) just prior to time t_j.

Fuzzy information: In statistics, the sample observations which are precise numbers are used for the estimation of parameters and for the testing of the hypotheses. Because of the fact that exact measurements of real continuous variables are not possible, it is unrealistic to represent them as precise numbers or vectors. In survival analysis, life times are continuous and not precise and which more or less fuzzy. Hence, instead of classical statistical tools for the analysis of survival times, one may expect that the fuzzy number approaches as more suitable and realistic. Hence for the fuzzy lifetime observations, a Generalized Nelson-Aalen estimator is proposed in this paper.

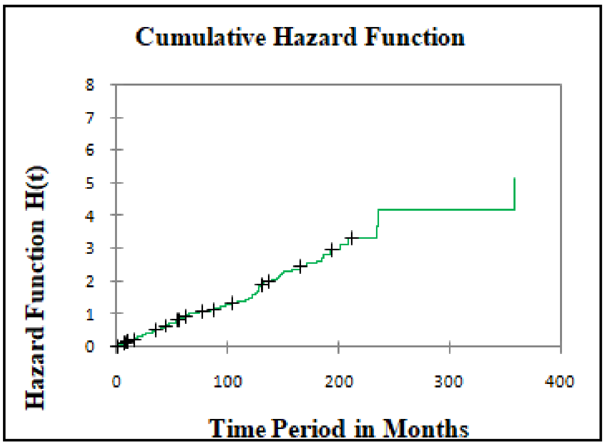
Fuzzy Numbers

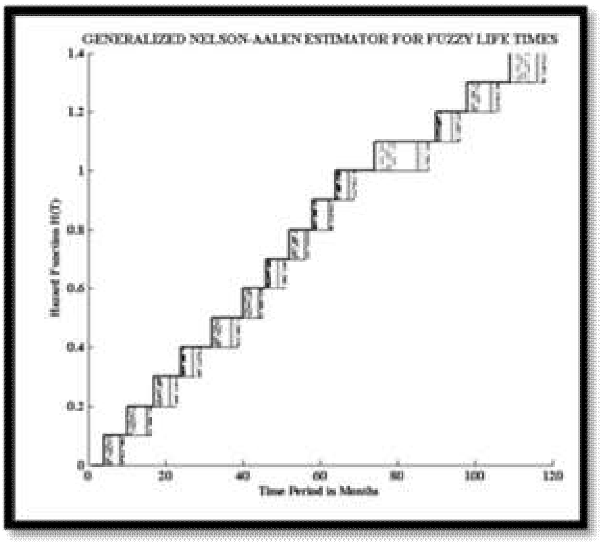
Let t^* be a fuzzy number with their characterizing function $\xi(\cdot)$ which is a function of a real variable obeying the following:

$$\xi : \mathbb{R} \to [0;1]$$

The δ is defined as δ-cut for all $\delta \in (0;1]$ $C_\delta(t^*) = \{t \in \mathbb{R} : \xi(t) \geq \delta\}$ is a finite union of compact intervals $[a_{\delta,j}; b_{\delta,j}]$ i.e. $C_\delta(t^*) = \bigcup_{j=1}^{k_\delta} [a_{\delta,j}; b_{\delta,j}] = \phi$

The support of a characterizing function $\xi(\cdot)$ is bounded, i.e. $sup[\xi(\cdot)] := \{t \in \mathbb{R} : \xi(t) > 0\} \subseteq [a;b]$. The symbol $\mathcal{F}(\mathbb{R})$ denotes the set of all fuzzy numbers. If all δ-cuts of a fuzzy number are non-empty closed bounded intervals, the corresponding fuzzy number is called a fuzzy interval.





FUZZY VECTORS

An n-dimensional fuzzy vector $\underline{t}^*$ is determined by its so-called vector characterizing function $\zeta(.,...,)$ which is a real function of n real variables $t_1, t_2, ..., t_n$ obeying the following three conditions:

$$\zeta : \mathbb{R}_n \to [0;1]$$

For all $\delta \in (0;1]$ the so-called δ-*cut* $C(\underline{t}^*) := \{\underline{t} \in \mathbb{R}n : \zeta(\underline{t}) \geq \delta\}$ are non-empty, bounded, and a finite union of simply connected and closed sets. The support of $\zeta(.,...,)$ defined by $sup[\zeta(.,...,)] := \{\underline{t} \in \mathbb{R} : \zeta(\underline{t}) > 0\}$ is a bounded set. $F(\mathbb{R}_n)$ is the set of n-dimensional fuzzy vectors. Let the stochastic quantity be T having their observation space as $M_T \subseteq [0;\infty)$, and a sample of size n i.e. $t_1, t_2, ..., t_n$ is considered from it. Each t_i is an element of the observation space and $(t_1, t_2, ..., t_n)$ is an element of the sample space from M_T^n which is the Cartesian product of n copies of M_T, that is $M_T^n = M_T \times M_T \times ... \times M_T$. Hence, were in the case of fuzzy observations, each fuzzy observations of $t_i^* = 1(1)$ with their characterizing function $\xi_i(.)$ is a fuzzy element of M_T then $(t_1^*, t_2^*, t_3^*, ..., t_n^*)$ is not a fuzzy element of M_T^n.

Algorithm for fuzzy-nelson aalen estimator: For the development of generalized Nelson-Aalen estimator the $H^*(t)$, upper and lower δ-level curves are obtained with the help of δ-cuts in the following way:

$$C_\delta(H^*(t)) = \left[minH(t)_{\underline{t} \in X^n_{1=i} C_{\delta t_i^*}} ; maxH(t)_{\underline{t} \in X^n_{1=i} C_{\delta t_i^*}} \right]$$

With $\underline{t} = (t_1, t_2, ... , t_n) \in [0;\infty)\ \forall \delta \in (0;1]$

where, $minH(t)_{\underline{t} \in X^n_{1=i} C_{\delta t_i^*}}$ is the lower end of the δ-cut the lower δ-level curve and $maxH(t)_{\underline{t} \in X^n_{1=i} C_{\delta t_i^*}}$ is the upper end of the δ-cut which defines the upper δ-level curve. The calculations are done through the following algorithm:

Step 1: The values for δ are taken from 0 to 1 with an increment $\Delta \in (0; 1)$.

Step 2: For a given value of δ the δ-cut off the fuzzy combined sample t^* is calculated.

Step 3: The minimum and maximum values from the δ-cuts used to generate hypothetical classical samples.

Step 4: The Nelson-Aalen cumulative hazard probabilities are calculated

Step 5: Steps 2-4 are repeated for each $\delta = 0\ (\Delta)\ 1$ and the Nelson-Aalen hazard curves are drawn for each fixed δ -level.

Illustration: For the purpose of illustration two hundred random numbers are generated from an exponential population with the mean life time 60. From the generated samples, 10% of observations are censored at random. Using this data the hazard function is calculated using the classical Nelson-Aalen method. The details are given below. From the hazard function *H(t)* is calculated and the curve has been drawn as above and the median survival time is observed as 47 months. It is observed that the median survival time for the Fuzzy-Nelson Aalen estimator is 40-45 time months. Hence, the fuzzy Nelson-Aalen estimator provides a more flexible estimate of the median survival time when compared to the classical Nelson-Aalen estimator.

Conclusion

The present work utilizes the concept of fuzzy set theory in survival analysis with reference to the Nelson-Aalen estimating for Hazard functions. As it would be more realistic to consider the non-negative life times as fuzzy numbers, this approach may yield better than that of the classical approach. However, the proper choice of the characteristic function is to be considered as important in this aspect.

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