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RESEARCH ARTICLE

ON τ^*-GENERALIZED β CONTINUOUS MULTIFUNCTIONS IN TOPOLOGICAL SPACES

[1]Ponsuryadevi, A. and *[2]Selvi, R.

[1]Research Scholar, Department of Mathematics, Sri Parasakthi College for Women, Courtallam, Affiliated to Manonmaniam Sundaranar University, Tirunelveli, Tamilnadu, India.
[2]Assistant Professor, Department of Mathematics, Sri Parasakthi College for Women, Courtallam, Affiliated to Manonmaniam Sundaranar University, Tirunelveli, Tamilnadu, India

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Corresponding author:
Selvi, R.

ABSTRACT

In this paper, we introduce the concept of τ^*-generalized β continuous multifunctions in topological spaces and study some of their properties where τ^* is defined by $\tau^* = \{G: cl^*(G^C) = G^C\}$.

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INTRODUCTION

In (7) C. Berge introduced the theory of Multifunctions. A multifunctions is a set-valued function. The concept of multifunctions has applications in functional analysis and fixed point theory. In 1970, Levine introduced the concepts of generalized closed set and discussed the properties of sets , closed and open maps, normal and separation axioms. Later in 1986 D. Andrijevic (2) gave a new type of generalized closed sets in topological space called semi pre open sets. In 1995, on generalizing semi pre open set is introduced by J. Doncthev. Dunham (9) introduced the concept of the closure operator Cl^* and a new topology τ^* where τ^*= τ^*={G:$Cl^*(G^c)$= G^c} and studied some of their properties. Pushpalatha *et al.,* (11) introduced and studied. τ^*-generalized closed sets, Eswaran and Pushpalatha (11) introduced and studied. τ^*-generalized continuous functions. Several authors have introduced and studied various function in topological spaces. For a multifunction F: X $\rightarrow$ Y, we shall denote the upper and lower inverse of a set G of Y by F^+(G) and F^-(G), respectively, that is F^+(G)={$x \in$ X:F(x)$\subset$G} and F^-(G)={$x \in$ X:F(x)$\cap$G$\neq \emptyset$}.. For a multifunction $F : X \rightarrow Y$, the graph multifunction $G_F : X \rightarrow X \times Y$ is defined as follows: $G_F (x) = \{x\} \times F (x)$ for every $x \in X$.

PREMILINARIES

Definition: 2.1 A subset A of a topological space X is called β-open (1) (or) semi pre open if A$\subset$ cl(int(cl (A))).(2)

Definition: 2.2 A subset A of a topological space X is called β-closed (or) semi pre closed if int(cl (int(A)))$\subseteq$ A.(2)

Definition: 2.3 A subset A of a topological space X is called a generalized β-closed (briefly, g β closed) if βcl(A) ⊆G whenever A⊆G and G is open in X.(8)

Definition: 2.4 For the subset A of a topological space X, the generalized closure operator Cl^* is defined by the intersection of all g-closed sets containing A. (9)

Definition: 2.5 For the subset A of a topological space X, the topology τ^* is defined by τ^*={G:$Cl^*(G^c)$= G^c}. (9)

Definition: 2.6 A subsets E of a topological space (X,τ^*)is called τ^*-generalized β closed set if Cl^*[(int(cl(int(E)))] ⊆W(briefly, Cl^*[(int(cl(int(E)))] denoted by Cl_β^*(E)) whenever E⊆W and W is τ^* open in X. The complement of τ^*-generalized β closed set is called the τ^*-generalized β open set. (4)

Lemma: 2.7

For a multifunction F: X→Y *, the following hold:*

(i) $G^+(A \times B) = A \cap F^+(B)$ *and*
(ii) $G^-(A \times B) = A \cap F^-(B)$ *for any subsets* $A \subset X$ *and* $B \subset Y$. (15)

Definition: 2.8: A subset A of a topological space X is nowhere dense if, for every nonempty open U□X, the intersection U∩A is *not* dense in U. Common equivalent definitions are:

(i) For every nonempty open set U⊂X, the interior of U\A is not empty.
(ii) The closure of A has empty interior.
(iii) The complement of the closure of A is dense.(3)

Definition: 2.9: A function f : X→Y from a topological space X into a topological space Y is called semi pre continuous if the inverse image of an open set in Y is semipreopen in X.(1)

Definition: 2.10: A function f : X → Y from a topological space X into a topological space Y is called g-continuous if the inverse image of a closed set in Y is g-closed in X.(5)

Definition: 2.11: A function f : X → Y from a topological space X into a topological space Y is called generalized semi pre-continuous (briefly gsp-continuous) if the inverse image of a closed set in Y is gsp-closed in X. (8)

Definition: 2.12 A function f : X → Y from a topological space X into a topological space Y is called τ^*-g continuous if the inverse image of a g-closed set in Y is τ^*-gclosed in X. (11)

Definition: 2.13 A function f : X → Y from a topological space X into a topological space Y is called τ^*-generalized β continuous function (briefly τ^*-gβ continuous) if the inverse image of every gβ open set in Y is τ^*-g open in X.

Definition: 2.14: A subset A of a topological space X is said to be

(i)α-paracompact if every cover of A by open sets of X is refined bya cover of A which consists of open sets of X and is locally finite in X; (18)

(ii) α-regular if for each a ∈ A and each open set U of X containing a there exists an open set G of x such

that a ∈ G ⊂ Cl (G) ⊂ U. (13)

Definition: 2.15: A rare set is a set S such that $int(S) = \emptyset$, and a dense set is a set *S* such that c*l(S)=X.* (14)

Definition: 2.16 A multifunction F : X → Y is said to be upper rarely continuous at a point x of X if for each open set G of Y containing F (x), there exists a rare set R_G with C*l*(R_G) ∩G = ∅ and an open set U containing x such that F(U) ⊂ G∪R_G. A multifunction is said to be upper rarely continuous if it has the property at each point of X. (16)

ON -GENERALIZED CONTINUOUS MULTIFUNCTIONS

- upper τ^*-generalized β-continuous (briefly *u.* τ^**-g.β.c.)* at a point x∈X if for each open set V containing F (x), there exists

$U \in \tau^*$-$g\,\beta(X, x)$ such that $F(U) \subset V$;

- lower τ^*-generalized β-continuous (briefly *l.* τ^*-*g.β.c.*) at a point $x \in X$ if for each open set V such that $F(x) \cap V \neq \emptyset$ there exists $U \in \tau^*$-$g\,\beta(X, x)$ such that $U \subset F^-(V)$;
- upper (lower) τ^*-$g\,\beta$-continuous if F has this property at every point of X.

Theorem: 3.2 : The following are equivalent for a multifunction $F: (X, \tau^*) \to (Y, \sigma)$:

- F is upper τ^*-$g\,\beta$-continuous at a point $x \in X$;
- for each open neighborhood U of x and each open set V of Y with $x \in X\, F^+(V)$, $F^+(V) \cap U$ is not nowhere dense;
- for each open neighborhood U of x and each open set V of Y with $x \in X\, F^+(V)$, *there* exists an open set G of X such that $\emptyset \neq G \subset U$ and $G \subset Cl(F^+(V))$;
- for each open set V of Y with $x \in F^+(V)$, there exists $U \in \tau^*$-$gsO(X,x)$ such that $U \subset cl(F^+(V))$;
- $x \in Cl(int(Cl(F^+(V))))$ for every open set V of Y with $x \in F^+(V)$.

Proof:

(i) ⇒ (ii) and (ii) ⇒ (iii): The proofs are obvious .

(iii)⇒ (iv): Let V be an open set of Y containing $F(x)$. By $U(x)$ we denote the family of all open neighbourhoods of x. For each $U \in U(x)$, there exists an open set G_U of X such that $G_U \subset U \neq \emptyset$ and $G_U \subset Cl(F^+(V))$. Put $W = \cup\{G_U : U \in U(x)\}$. Then W is an open set of X, $x \in Cl(W)$ and $W \subset Cl(F^+(V))$. Then, we put $U_o = W \cup \{x\}$. Then $W \subset U_o \subset Cl(W)$ and $U_o \in \tau^*$-$gsO(X,x)$ and also $U_o \subset Cl(F^+(V))$.

(iv)⇒ (v): Let V be an open set of Y containing $F(x)$. There exists $U \in \tau^*$-$gsO(X,x)$ such that $U \subset Cl(F^+(V))$. So, we have $x \in U \subset Cl(\mathrm{int}(U)) \subset Cl(\mathrm{int}(Cl(F^+(V))))$.

Theorem: 3.3: The following are equivalent for a multifunction $F: (X, \tau^*) \to (Y, \sigma)$:

(i) F is lower τ^*-g β -continuous at a point $x \in$ X;

(ii) for any open neighborhood U of x and any open set V *of* Y *with* $x \in F^-(V)$, $F^-(V) \cap U$ is not nowhere dense;

(iii) for any open neighborhood U of x and any open set V *of* Y with $x \in F^-(V)$, there exists an open set G *of* X such *that* $G \subset U \neq \emptyset$ *and* $G \subset cl(F^-(V))$;

(iv) for any open set V *of* Y *with* $x \in F^-(V)$, *there exists* $U \in \tau^*$-$gsO(X,x)$ *such that* $U \subset cl(F^-(V))$;

(v) $x \in cl(int(cl(F^-(V))))$ *for every open set* V *of* Y *with* $x \in F^-(V)$.

Proof:

The proof is similar to the theorem 3.2.

- **Theorem: 3.4** The following are equivalent for a multifunction F: $(X, \tau^*) \to (Y, \sigma)$:

- *F is upper* τ^*-g β -continuous;
- $F^+(V) \in \tau^*$-$g\,\beta(X)$ for every open set V *of* Y;
- $F^-(K)$ is τ^*-$g\,\beta$ *closed in* X for every closed set K *of* Y;
- τ^*-$g\,\beta cl(F^-(B)) \subset F^-(cl(B))$ *for* every subset B *of* Y;
- $int(cl(int(F^-(B)))) \subset F^-(Cl(B))$ for every subset B *of* Y.

Proof:

(i) ⇒ (ii): Let V be any open set of Y and $x \in F^+(V)$. There exists $U \in \tau^*$-$g\,\beta(X, x)$ such that $F(U) \subset V$. Here, we obtain $x \in U \subset Cl\,\mathrm{int}(Cl(U)) \subset Cl(\mathrm{int}(Cl(F^+(V))))$. Then $F^+(V) \subset Cl(\mathrm{int}(Cl(F^+(V))))$ and hence $F^+(V) \in \tau^*$-$g\,\beta(X)$.

(ii)⇒ (iii): In fact that $F^+(Y \setminus B) = X \setminus F^-(B)$ for every subset B of Y.

(iii)⇒ (iv): For any subset B of Y, $Cl(B)$ is closed in Y and $F^-(Cl(B))$ is $\boldsymbol{\tau}^*$-g β closed in X. Here, we obtain $\boldsymbol{\tau}^*$-g $\beta Cl(F^-(B)) \subset F^-(B)$.

(iv)⇒ (v): Let B be any subset of Y. Then $\mathrm{int}(Cl(\mathrm{int}(F^-(B)))) \subset \tau^*$-$g\,\beta Cl(F^-(B)) \subset F^-(Cl(B))$.

(v)⇒ (ii): Let V be any open set of . Then $Y \setminus V$ is closed in Y and we have $X \setminus F^+(V) = F^-(Y \setminus V) \supset \mathrm{int}(Cl(\mathrm{int}(F^-(Y \setminus V)))) = \mathrm{int}(Cl(\mathrm{int}(X \setminus F^+(V)))) = X \setminus Cl(\mathrm{int}(F^+(V))))$. Then $F^+(V) \subset Cl(\mathrm{int}(Cl(F^+(V))))$ and hence $F^+(V) \in \tau^*$-$g\,\beta(V)$.

(ii) ⇒ (i): Let $x \in X$ and V be an open set of Y containing $F(x)$. By (ii), we have $x \in F^+(V) \in \tau^*$-$g\,\beta(X)$. Put $U = F^+(V)$. Then we obtain $U \in \tau^*$-$g\,\beta(X, x)$ and $F(U) \subset V$. Hence F is upper τ^*-g β continuous.

Theorem: 3.5 The following are equivalent for a multifunction F: $(X, \tau^*) \to (Y, \sigma)$:

(i) F is lower τ^*-g β -continuous;
(ii) $F^-(V) \in \tau^*$-g β(X) for every open set V of Y;
(iii) $F^+(K)$ is τ^*-g β closed in X for every closed set K of Y;
(iv) τ^*-g β$cl(F^+(B)) \subset F^+(cl(B))$ for every subset B of Y;
(v) $int(cl(int(F^+(B)))) \subset F^+(cl(B))$ for every subset B of Y.
(vi) $F(int(cl(int(A)))) \subset cl(F(A))$ for every subset A of X;
(vii) $F(\tau^*$-g β$cl(A)) \subset cl(F(A))$ for every subset A of X.

Proof:

$(i) \Rightarrow$ (ii): Let V be any open set of Y and $x \in F^-(V)$. There exists $U \in \tau^*$-$g\,\beta\,(X, x)$ such that $F(U) \cap V \neq \emptyset$. Then $x \in U \subset Cl(\text{int}(Cl(U))) \subset Cl(\text{int}(Cl(F^+(V))))$. Then $F^-(V) \subset Cl\,(\text{int}(Cl\,(F^-(V))))$ and hence $F^-(V) \in \tau^*$-$g\,\beta\,(X)$.
(ii)⇒ (iii): I n fact that $F^-(Y \setminus B) = X \setminus \boldsymbol{F}^+(B)$ for every subset B of Y.
(iii)⇒ (iv): For any subset B of Y, $Cl\,(B)$ is closed in Y and $F^+(Cl(B))$ is $\boldsymbol{\tau}^*$-g β closed in X. So that $\boldsymbol{\tau}^*$-g β$Cl\,(\boldsymbol{F}^+(B)) \subset \boldsymbol{F}^+(B)$.
(iv)⇒ (v): Let B be any subset of Y. Then $\text{int}(Cl(\text{int}(F^+(B)))) \subset \tau^*$-$g\,\beta Cl(F^+(B)) \subset F^+(Cl(B))$.
(v)⇒(vi): Let A be any subset of X. Then we have $\text{int}(Cl(\text{int}(A))) \subset \text{int}(Cl(\text{int}(F^+(F(A))))) \subset F^+(Cl(F(A)))$. So that $F\,(\text{int}(Cl(\text{int}(A)))) \subset Cl(F(A))$.
(vi)⇒(vii): Let A be any subset of X. Wehave $F\,(\tau^*$-g β$Cl(A)) = F(A \cup \text{int}(Cl(\text{int}(A)))) = F(A) \cup F(\text{int}(Cl(\text{int}(A)))) \subset (F(A))$.
(vii)⇒(iii): Let K be any closed set of Y. Then we have $F(\boldsymbol{\tau}^*$-g β$Cl\,(F^+(K))) \subset Cl(F(\boldsymbol{F}^+\,(K))) \subset Cl(K) = K$. Hence, $\boldsymbol{\tau}^*$-g β$Cl\,(\boldsymbol{F}^+(K)) \subset \boldsymbol{F}^+(K)$, and hence $\boldsymbol{F}^+(K)$ is β-closed in X.

Theorem: 3.6

Let F: $(X,\tau^*) \rightarrow (Y,\sigma)$ be a multifunction such that F (x) is compact for each $x \in X$. Then F is upper τ^*-g β continuous if and only if $G_F : X \rightarrow X \times Y$ is upper τ^*-g β continuous.

Proof:

Suppose that F: $(X,\tau^*) \rightarrow (Y,\sigma)$ is upper τ^*-g β continuous . Let $x \in X$ and W be any open set of $X \times Y$ containing $G_F(X)$. For each $y \in F(x)$, there exist open sets $U(y) \subset X$ and $V(y) \subset Y$ such that $(x, y) \in U(y) \times V(y) \subset W$. The family $\{V(y) : y \in F(x)\}$ is an open cover of $F(x)$, and $F(x)$ is compact. Therefore, there exist a finite number of points $y_1, y_2, \ldots, y_n$ in $F(x)$ such that $F(x) \subset \cup\{V(y_i) : 1 \leq i \leq n\}$. Set $U = \cap\{U(y_i) : 1 \leq i \leq n\}$ and $V = \cup\{V(y_i) : 1 \leq i \leq n\}$. Then U and V are open in X and Y, respectively, and $\{x\} \times F(x) \subset U \times V \subset W$. Since F is upper τ^*-g β continuous, there exists $U_o \in \tau^*$-g $\beta(X, x)$ such that $F(U_o) \subset U$. By Lemma 2.7(i), we have $U \cap U_o \subset U \cap F^+(V) = G^+(U \times V) \subset G^+(W)$. So that $U \cap U_o = \tau^*$-g β(X, x) and $G_F(U \cap U_o) \subset W$. Then G_F is upper τ^*-g β continuous. Conversely assume that $G_F : X \rightarrow X \times Y$ is upper τ^*-g β continuous. . Let $x \in X$ and V be any open set of Y containing $F(x)$. Since $X \times V$ is open in $X \times Y$ and $G_F(x) \subset X \times V$, there exists $U \in \tau^*$-g β(X, x) such that $G_F(U) \subset X \times V$. By Lemma 2.7(i), we have $U \subset G^+(X \times V) = F^+(V)$ and $F(U) \subset V$. Hence F is upper τ^*-g β continuous.

Theorem: 3.7

A multifunction F: $(X,\tau^*) \rightarrow (Y,\sigma)$ is lower τ^*-g β continuous if and only if $G_F : X \rightarrow X \times Y$ is lower τ^*-g β continuous .

Proof:

Suppose that F is lower τ^*-g β continuous . Let $x \in X$ and W be any open set of $X \times Y$ such that $x \in G_F(W)$. Since $W \cap (\{x\} \times F(x)) \neq \emptyset$, there exists $y \in F(x)$ such that $(x, y) \in W$, and hence $(x, y) \in U \times V \subset W$ for some open sets $U \subset X$ and $V \subset Y$. Since $F(x) \cap V \neq \emptyset$, there exists $G \in \tau^*$- g β(X, x) such that $G \subset F^-(V)$. By Lemma 2.7(ii), we have $U \cap G \subset U \cap F^-(V) = G_F(U \times V) \subset G_F(W)$. Moreover, $x \in U \cap G \subset \tau^*$-g β$(X)$ and hence G_F is lower τ^*-g β continuous.

Assume that G_F is lower τ^*-g β continuous. Let $x \in X$ and V be an open set of Y such that $x \in F^-(V)$. Then $X \times V$ is open in $X \times Y$ and $G_F(x) \cap (X \times V) = (\{x\} \times F(x)) \cap (X \times V) = \{x\} \times (F(x) \cap V) \neq \emptyset$. Since G_F is τ^*-g β continuous, there exists $U \in \tau^*$-g β(X, x)

Lemma: 3.8

If A is an α-regular α-paracompact set of a topological space X and U is an open neighborhood of A, then there exists an open set G of X such that $A \subset G \subset cl(G) \subset U$.(13)

For a multifunction $F : X \rightarrow Y$, by clF $: X \rightarrow Y$ we denote a multifunction defined as follows: $(cl\text{F})(x) = cl(F(x))$ for each $x \in X$. Similarly, we can define βclF $: X \rightarrow Y$, sclF $: X \rightarrow Y$, pclF $: X \rightarrow Y$ or $\alpha\, cl$F$: X \rightarrow Y$. (6)

Lemma: 3.9

If F: $(X,\tau^*)\rightarrow(Y,\sigma)$ is a multifunction such that F (x) is α-paracompact α-regular for each $x\in X$, then for each open set V of Y , $G^+(V) = F^+(V)$, where G denotes βclF, sclF, pclF, αclF or clF.

Proof: Let V be any open set of Y. Let $x \in G^+(V)$. Then $G(x)\subset V$ and $F(x)\subset G(x) \subset V$. We have $x \in F^+(V)$, and hence $G^+(V)\subset F^+(V)$. Conversely, let $x \in F^+(V)$, then $F(x) \subset V$. By Lemma 3.8, there exists an open set H of Y such that $F(x) \subset H \subset cl(H) \subset V$; hence $G(x) \subset cl(H) \subset V$. Then, we have $x \in G^+(V)$ and $F^+(V) \subset G^+(V)$.

Theorem: 3.10

Let F: $(X,\tau^*)\rightarrow(Y,\sigma)$ be a multifunction such that F (x) is α- paracompact and α-regular for each $x \in X$. Then the following are equivalent:

(i) F is upper τ^*-g β continuous ;
(ii) βclF is upper τ^*-g β continuous ;
(iii) sclF is upper τ^*-g β continuous ;
(iv) pclF is upper τ^*-g β continuous ;
(v) clF is upper τ^*-g β continuous;
(vi) αclF is upper τ^*-g β continuous.

By Lemma 3.9, we put $G = \beta cl$ F, scl F, pcl F, or clF. Suppose that F is upper τ^*-g β *continuous* . Let $x\in X$ and V be any open set of Y containing $G(x)$. By Lemma 3.9, $x\in G^+(V) = F^+(V)$ and there exists $U\in \tau^*$-g $\beta(X, x)$ such that $F(U)\subset V$. Since $F(u)$ is α-paracompact and α-regular for each $u\in U$, by Lemma 3.9, there exists an open set H such that $F(u) \subset H \subset cl(H) \subset V$; hence $G(u) \subset cl(H) \subset V$ for each $u \in U$. So that $G(U) \subset V$. Hence G is upper τ^*-g β continuous. Conversely, suppose that G is upper τ^*-g β continuous . Let $x \in X$ and V be any open set of Y containing $F(x)$. By Lemma 3.9, $x \in F^+(V) = G^+(V)$ and hence $G(x) \subset V$. There exists $U \in \tau^*$-g $\beta(X, x)$ such that $G(U) \subset V$. Thus $U \subset G^+(V) = F^+(V)$, and hence $F(U) \subset V$. So that F is *upper* τ^*-g β continuous.

Lemma: 3.11: If F:$(X,\tau^*)\rightarrow(Y,\sigma)$ is a multifunction, then for each open set V of Y , $G^-(V) = F^-(V)$, where G denotes βclF, sclF, pclF , αclF or clF.

Proof: Let V be any open set of Y and $x \in G^-(V)$. Then $G(x) \cap V \neq \emptyset$, and hence $F(x)\cap V\neq \emptyset$. *S*ince V is open. Thus, $x \in F^-(V)$ and hence $G^-(V)\subset F^-(V)$. Conversely, assume that $x \in F^-(V)$. Then we have $\emptyset \neq F(x) \cap V \subset G(x) \cap V$ and hence $x \in G^-(V)$. Thus, we have $F^-(V)\subset G^-(V)$. Then $G^-(V) = F^-(V)$.

Theorem: 3.12

Let F: $(X,\tau^*)\rightarrow(Y,\sigma)$ be a multifunction then the following are equivalent:
(i) F is lower τ^*-g β continuous ;
(ii) βclF is lower τ^*-g β continuous ;
(iii) sclF is lower τ^*-g β continuous ;
(iv) pclF is lower τ^*-g β continuous ;
(v) clF is lower τ^*-g β continuous ;

Theorem: 3.13: Let F: $(X,\tau^*)\rightarrow(Y,\sigma)$ be a multifunction and $\{U_\lambda : \lambda\in \Delta\}$ be an open cover of X. If the restriction functions F/U_λ is upper τ^*-g β continuous for each $\lambda\in \Delta$, then F is upper τ^*-g β continuous.

Proof: Let V be any open subset of Y. Since F/U_λ is upper τ^*-g β continuous for each $\lambda\in \Delta$, hence $F/U_\lambda{}^+(V)=U_\lambda \cap F^+(V)$ is τ^*-g β *open set. Then* $\bigcup_{\lambda\in\Delta}(U_\lambda) \cap F^+(V) = X \cap F^+(V) = F^+(V)$ is τ^*-g β open set. Hence F is upper τ^*-g β *continuous.*

Theorem: 3.14 If F: $(X,\tau^*)\rightarrow(Y,\sigma)$ is upper τ^*-g β continuous and *F(X)* is a subspace of Y, then *F:X→F(X) is* upper τ^*-g β continuous.

Proof: Since F: $(X,\tau^*)\rightarrow(Y,\sigma)$ is upper τ^*-g β continuous, for every open subset *V of Y* ,$F^+(V\cap F(X))=F^+(V)\cap F^+F(X)=F^+(V)$ *is* τ^*-g β open. Hence *F:X→F(X) is* upper τ^*-g β continuous.

Theorem: 3.15

If F: $(X,\tau^*)\to(Y,\sigma)$ is lower τ^*-g β *continuous and F(X) is a subspace of Y, then F:X→F(X) is lower* τ^*-g β continuous.

Proof: Since F: $(X,\tau^*)\to(Y,\sigma)$ is lower τ^*-g β continuous, for every open subset *V of Y*, $F^-(V\cap F(X))= F^-(V)\cap F^-F(X)= F^-(V)$ is τ^*-g β open. Hence *F: X →F(X)* is lower τ^*-g β continuous.

SOME PROPERTIES

Theorem 4.1: A multifunction F: $(X,\tau^*)\to(Y,\sigma)$ is upper τ^*-generalized rarely continuous at each point $x \in X$ and for each open set G containing F (x), $F^-(Cl(\tau^*\text{-}gR_G))$ is a β-closed set of X, where $\tau^*\text{-}gR_G$ is the rare set, then F is upper τ^*-g β continuous.

Proof: Let $x \in X$ and G be an open set such that $F(x) \subset G$. Since F is upper τ^*-*g* rarely continuous, there exist an open set V of X containing x and a τ^*-*g* rare set $\tau^*\text{-}gR_G$ with $cl(\tau^*\text{-}gR_G)\cap G = \emptyset$ such that $F(V)\subset G \cup \tau^*\text{-}gR_G$. Let $U = V \cap (X \setminus F^-(cl(\tau^*\text{-}gR_G)))$. Then we have $U \in \tau^*\text{-}g\,\beta(X)$ and $x \in U$, since $x \in V$ and $x \in X \setminus F^-(cl(\tau^*\text{-}gR_G))$. We have suppose that $x \in F^-(cl(\tau^*\text{-}gR_G))$ then $F(x)\cap cl(\tau^*\text{-}gR_G) \neq \emptyset$, $G\cap cl(\tau^*\text{-}gR_G) = \emptyset$. Let $s \in U$. Then $F(s) \subset G \cup (\tau^*\text{-}gR_G)$ and $F(s) \cap cl(\tau^*\text{-}gR_G) = \emptyset$. Then, we have, $F(s)$ $\tau^*\text{-}gR_G = \emptyset$, then hence $F(s)\subset G$. Since U is a τ^*-*g β* -open set containing x, then F is upper τ^*-*g* β continuous.

Definition 4.2 A multifunction F: $(X,\tau^*)\to(Y,\sigma)$ *is said to be upper* τ^*-*gα continuous* if for each $x \in X$ and each open set V of Y containing F (x), there exists an τ^*-*gα* open set U containing x such that $F(U) \subset V$.

Theorem 4.3 If F, G : $(X,\tau^*)\to(Y,\sigma)$ are multifunctions and Y is a normal space such that

(i) F and *G* are punctually closed;
(ii) F is upper τ^*-*g β* continuous;
(iii) *G is upper* τ^*-*gα* continuous, then the set $\{x \in X : F(x) \cap G(x) \neq \emptyset\}$ is τ^*-*g β* closed in *X*.

Proof: Put $A = \{x \in X : F(x) \cap G(x) \neq \emptyset\}$ and let $x \in X \setminus A$. Then $F(x) \cap G(x) = \emptyset$. Since Y is normal, there exist disjoint open sets V and W such that $F(x) \subset V$ and $G(x) \subset W$. Since F is upper τ^*-*g β* continuous, there exists $U_1 \in \tau^*\text{-}g\,\beta(X, x)$ such that $F(U_1) \subset V$. Since G is upper τ^*-*gα* continuous, there exists an τ^*-*gα* open set U_2 containing x such that $G(U_2) \subset W$. Put $U = U_1 \cap U_2$. Then $U \in \tau^*\text{-}g\,\beta(X, x)$ and $F(U) \cap G(U) = \emptyset$. So that $U \cap A = \emptyset$ and hence A is τ^*-*g β*-closed in *X*.

Definition: 4.4: The τ^*-*g β*-frontier of a subset *A of X,* denoted by $\tau^*\text{-}g\,\beta Fr(A)$, is defined by $\tau^*\text{-}g\,\beta Fr(A) = \tau^*\text{-}g\,\beta cl(A) \cap \tau^*\text{-}g\,\beta cl(X \setminus A) = \tau^*\text{-}g\,\beta Cl(A) - \tau^*\text{-}g\,\beta int(A)$.

Theorem: 4.5: The set of all points x of X at which is a multifunction F: $(X,\tau^*)\to(Y,\sigma)$ is not upper τ^*- g β continuous is identical with the union of τ^*-g β frontier of the upper inverse images of open sets containing F (x).

Proof: Let x be a point of X at which F is not upper τ^*-*g β* continuous. Then there exists an open set V of Y containing $F(x)$ such that $U \cap (X\setminus F^+(V)) \neq \emptyset$ for every $U \in \tau^*\text{-}g\,\beta(X, x)$. Therefore, $x \in \tau^*\text{-}g\,\beta Cl(X \setminus F^+(V)) = X\setminus \tau^*\text{-}g\,\beta\mathrm{int}(F^+(V))$ and $x \in F^+(V)$. Then, $x \in \tau^*\text{-}g\,\beta\mathrm{Fr}(F^+(V))$. Conversely, suppose that V is an open set containing $F(x)$ and that $x\in \tau^*\text{-}g\,\beta\mathrm{Fr}(F^+(V))$. If F is upper τ^*-*g β* continuous at x, then there exists $U \in \tau^*\text{-}g\,\beta(X, x)$ such that $U\subset F^+(V)$; hence $x \in \tau^*\text{-}g\,\beta\,\mathrm{int}(F^+(V))$. Which is a contradiction, hence F is not upper τ^*-*g β* continuous at x.

Theorem: 4.6: The set of all points x of X at which is a multifunction F: $(X,\tau^*)\to(Y,\sigma)$ is not lower τ^*- g β continuous is identical with the union of τ^*-g β frontier of the lower inverse images of open sets containing F(x).

Proof: Let x be a point of X at which F is not lower τ^*-*g β* continuous. Then there exists an open set V of Y containing $F(x)$ such that $U \cap (X\setminus F^-(V)) \neq \emptyset$ for every $U \in \tau^*\text{-}g\,\beta(X, x)$. Therefore, we have $x \in \tau^*\text{-}g\,\beta Cl(X\setminus F^-(V)) = X\setminus \tau^*\text{-}g\,\beta\mathrm{int}(F^-(V))$ and $x\in F^-(V)$. Then, $x \in \tau^*\text{-}g\,\beta\mathrm{Fr}(F^-(V))$. Conversely, suppose that V is an open set containing $F(x)$ and that $x \in \tau^*\text{-}g\,\beta\mathrm{Fr}(F^-(V))$. If F is lower τ^*-*g β* continuous, at x, then there exists $U \in \tau^*\text{-}g\,\beta(X, x)$ such that $U\subset F^-(V)$; hence $x \in \tau^*\text{-}g\,\beta\,\mathrm{int}(F^-(V))$. Which is a contradiction, hence F is not lower τ^*-*g β* continuous at x.

REFERENCES

(1) M.E.Abd El-Mosef, S.N. El-Deeb, and R.A.Mahmoud, β -open sets and β- continuous mapping , *Bull.Fac.Sci.Assiut.Univ*.12 (1983), 77-90.

(2) D. Andrijevic, Semi-Pre open Sets, *Mat.Vesnik*, 38 (1986), 24-32.

(3) A.V. Arkhangelskii, V.I. Ponomarev, "Fundamentals of general topology: problems and exercises" , Reidel (1984)

(4) C. Aruna and R.Selvi, on τ^* -generalized β closed sets in topological spaces, *International Journal of Advances in Mathematics,* Vol. 2019, No.3, pp.24-33.

(5) K. Balachandran, P.Sundaram and H.Maki, On generalized continuous functions in topological spaces, *Mem. Fac. Sci. Kochi Univ*. (Math) 12 (1991), 5-13. Monthly, 70 (1963), 36-41.

(6) T. Banzaru, Multifunctions and M-Product spaces, Bul.St.Tehn.Inst.Politeh. "T.Vuia" Timisoara Mat.Fiz.Mec.Teor.Apl.17 (31) (1972), 17-23, in Romanian.

(7) C. Berge, Topological spaces , *Macmillian, New York*, (1963).

(8) J. Dontchev, On generalizing semi pre open sets, *MEM, Fac. Sci.Kochi Uni.Ser A.Math.,* 16 (1995), 3-48.

(9) W. Dunham, A New closure Operator for Non-T_1 Topologies, *Kyungpook Math*. J.22 (1982),55-60.

(10) S. Eswaran and N.Nagaveni, , τ^*-generalized semi continuous functions in Topological spaces, *Int. Journal of Math. Analysis*, Vol.7, 2013 No.42, 2091-2100.

(11)S. Eswaran and A.Pushpalatha, τ^*-generalized continuous functions in topological spaces, international *J. of Math Sci & Engg. Appls*. (IJM-SEA) ISSN 0973-9424, Vol. 3, No.IV, (2009), pp. 67-76

(12) J. Ewert, Almost quasicontinuity of multivalued maps, Math. Slovaka (1996).

(13) I. Kovacevic, Subsets and paracompactness, Univ.u Novom Sadu, ZB, Rad. Prirod, Mat. Fac. Ser. Mat. 14 (1984), 79-87.

(14) Long. P. E. and Herrington, L. L. Properties of rarely continuous functions, Glasnik Mat.17 (37), 229–236, 1982.

(15) T. Noiri and V.Popa, Almost weakly continuous multifunctions, Demonstratio Math. 26 (1993), 363-380.

(16) V. Popa, Some properties of rarely continuous multifunction, Conf.Nat.Geom. Topoligie, 1988, Univ. Al. I. Cuza Iasi (1989), 269-274.

(17) A. Pushpalatha, S.Eswaran and P.Rajarubi, τ^* -generalized closed sets in topological spaces, *Proceeding of World Congress on Engineering* 2009 Vol II WCE 2009, July 1-3, 2009, London, UK., 1115-1117.

(18) Wine, D. 1975. Locally paracompact spaces, Glasnik Mat. 10 (30) (1975), 351-357.

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