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RESEARCH ARTICLE

MATHEMATICAL MODELS OF AIR VESSELS FOR PRESSURE TRANSIENT CONTROL IN WATER PIPELINES - A REVIEW

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ABSTRACT

Air vessels are used to mitigate both up and down surge events in pipeline systems. When the air vessel swings into action in the event of a pressure surge, the compressed air pressure changes accordingly to suppress the adverse pressure. The rate at which the pressure changes in the air cushion occur is very important in determining the effectiveness of air vessels as surge control devices. This rate is dependent partly on the initial air volume contained in the vessel (hence its size) and the thermodynamic process that the air undergoes. Consequently, the dynamic behavior of the entrapped air in these vessels forms an integral part of the overall scheme and needs to be well understood for a proper transient analysis and sizing of air vessels. The Polytrophic model and the Rational Heat Transfer (RHT) model which happen to be the only two existing models for describing the dynamic behavior of entrapped air in air vessels, have been covered in this review. The limitations of these models have been mentioned and the need for an improved model has been highlighted.

INTRODUCTION

Pressure transients in pipeline systems are unsteady flow conditions which are usually experienced when there is a sudden change in flow velocity. They are sometimes referred to as "hydraulic transient", "surge pressure" or in water applications "water hammer". These transient conditions sometimes lead to unacceptable conditions that require strict attention during the design phase of pipeline systems. In his work, Thorley (2004) listed the unacceptable conditions as: extremely high pressures causing deformation or crack in pipeline and components; extremely low pressures causing caving in of pipeline; reverse flow damaging hydraulic components like pumps, pipeline movement and vibration and low velocity of flow. The causes of most of these transient conditions have been linked to either one or more of these; the starting and stopping of pumps (whether controlled or uncontrolled); the sudden closure of valves; check valve slam, excitation of resonant vibrations, improper filling practices and air in lines— in pumped system. (Stephenson, 2002; Thorley, 2004; Stone, 2005; Wylie and Streeter, 1978; Lee *et al.*, 2010 and Ishikawa *et al*., 2008). Pipelines often see their first surge during filling when the air being expelled from a pipeline rapidly escapes through a manual vent or a throttled valve followed by the system fluid (e.g. water). In the case of water, being many times denser than air, it follows the air to the outlet at a high velocity, but its velocity is restricted at the outlet thereby causing a surge.

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Some of the known methods for controlling/suppressing surge pressures are (but not limited to) the utilization of air vessels, surge tanks, stronger pipes, vacuum and relief valves, reduction in flow velocity, increased pump inertia and selecting the right check valves. All the adverse effects associated with transient conditions can be avoided if the engineers involved in the planning of pumping systems adopt the right surge control devices. Many hydraulic systems make use of air vessels in surge pressure (especially water hammer) control (Izquierdo *et al.,* 2006). The most common and important example of this is represented by a rising main supplying water to a storage reservoir as shown in Fig.1. The rising main may represent a large economic investment and it is built to meet some water supply demand, hence its safety must be guaranteed. Air vessels are one the methods used especially in water applications. The thermodynamic behavior of the entrapped air in the vessel affects the sizing of the vessel. The Polytrophic model and the Rational Heat Transfer (RHT) model which happen to be the only two existing models for describing the thermodynamic behavior of entrapped air in air vessels, have been covered in this review. The aim of this paper is to review the existing methods of modeling air vessels, their short comings and highlight possibleareas of improvements.

Hydraulic transient analysis in pipelines

The study of true transient flows must include fluid inertia and may also include the elasticity or compressibility of the fluid and the conduit. The rigid column theory and water hammer theory are the two approaches adopted in pressure transient analysis. In fluid transients, there are some flow situations where it may be possible to ignore the elastic effects of the fluid and conduit material without compromising the accuracy of the analysis. These form the foundation for the rigid column analysis. When the velocities in a pipe system change so rapidly that the elastic properties of the pipe and liquid must be considered in an analysis, then an elastic theory of hydraulic transient, commonly known as water hammer analysis must be applied. In both rigid column and water hammer analysis, the application of Newton's second law which leads to the Euler equation (Larock *et al.,* 2000) is required.

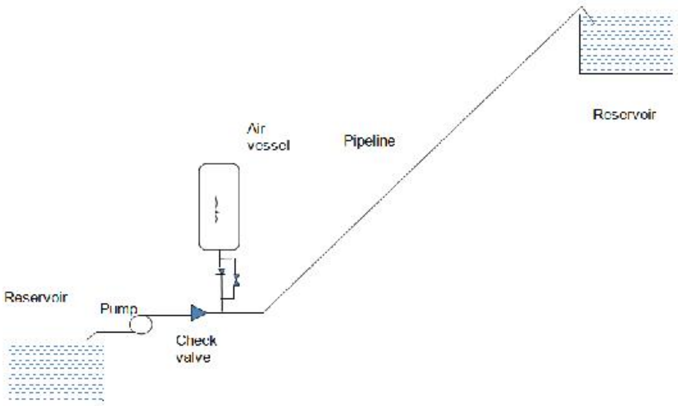

Fig. 1. A Typical rising main installation protected by air vessel from the consequences of pump trip

The Euler equation (Eq.1) can be written in the alternative form as:

$$\frac{1}{g}\frac{\partial v}{\partial t} + \frac{\partial}{\partial s}\left(H + \frac{v^2}{2g}\right) + \frac{f}{D}\frac{v|v|}{2g} = 0 \qquad (1)$$

Where f =Darcy friction factor, H =Piezometric pressure head in meter of water, s= displacement, v=flow velocity, D =pipeline diameter, g=acceleration due to gravity and t=time

Air Vessels

One of the diversionary devices used in the suppression of fluid transients in pipeline systems, particularly to reduce the adverse effects of the complete pump stoppage is the air vessel (Thorley, 2004). It is usually installed on the pumping station main, downstream of a pump to prevent negative pressure and column separation in the pipeline downstream of the vessel. It is a sealed vessel, particularly filled with the system fluid and topped with pressurized air or gas. The gas may be in contact with the liquid in which case an air compressor or gas supply is used to maintain the proper mass of air or gas (e.g compressor type air vessel) or the gas may be separated from the liquid by containing it in a flexible bladder (e.g. bladder type air vessel) (Lee 1998; Wylie and Streeter, 1978 and Thorley, 2004). In the bladder type there is no direct contact between the compressed air and the water, hence, there is no dissolution of gases in water. This singular difference makes it suitable for very remote applications where less maintenance is required. The major advantages of the bladder system include the need for minimum maintenance, absence/extremely low possibility of having air entrainment in the pipe system, minimal rate of internal corrosion of the walls of the cylinder (Charlatte, 2010). Despite its advantages over the compressor type, the compressor type is still widely used for drinking water applications. The reason is more psychological than technical.

The basic working principle of an air vessel (Fig. 2) as a surge control device is to prevent the dangerous conversion of the steady state kinetic energy into elastic deformation energy. The pressurized air cushion in the chamber stores potential energy during upsurge. If there was no air vessel, the dreaded conversion of fluid kinetic energy into elastic deformation energy following a pump trip would take place at the pump outlet which would cause the liquid column to separate. With an air vessel installed, however the stored energy of the compressed air takes over the work of the pump, discharging water into the pipeline. This act would prevent rapid changes in the flow velocity in the pipeline, hence causing the water level in the vessel and un-deformed liquid column in the pipeline to rise and fall over a long period of time. This process is kept in motion by the energy discharged by the air cushion each time fluid flows out of the vessel and by the energy absorbed again by the air cushion on the fluid's return. The energy in the air cushion is only gradually dissipated. This process of gradual energy dissipation accounts for why it takes many minutes for air vessel oscillations to die away especially in long pipelines. (Lüdecke and Kothe, 2010; Thorley, 2004 and Lee, 1998).

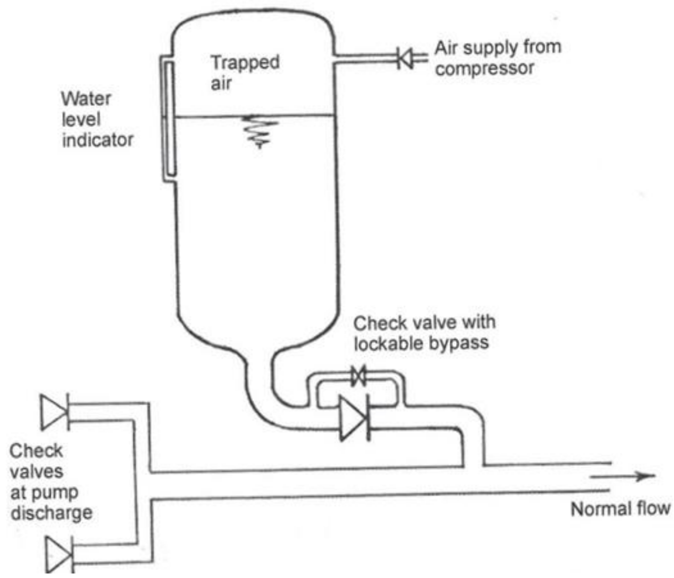

Fig. 2. A Schematic diagram of a vertically positioned air vessel

Thermodynamic behavior of entrapped air in an air vessel

The entrapped air in the air vessel is bounded by the walls of the cylinder at the top and vertical walls while the bottom is covered by water (the system fluid) (see Fig.3). A closed system is fully defined when the knowledge of enclosed fluid, boundary between the fluid under consideration and its surrounding; mass of the fluid in the boundary is available (Rogers and Mayhew, 1992). Hence, the entrapped air, in the air vessel could be treated as a 'closed system'. Since only heat and work, 'but not mass' may be transferred across the boundaries of the system, the process undergone by the air in the 'closed system' is a non-flow process.

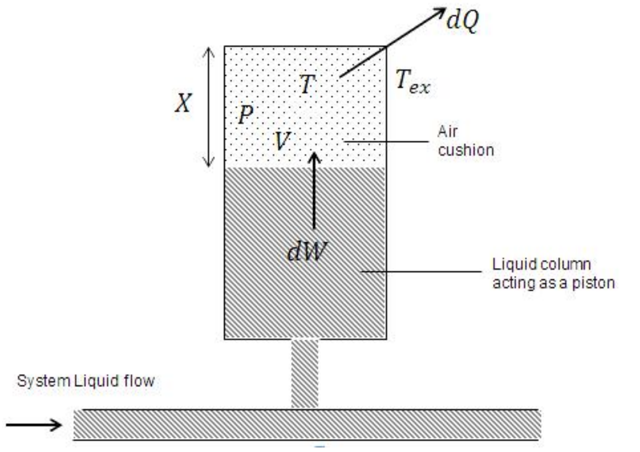

Fig. 3. Schematic diagram of a vessel used in showing the thermodynamic effect of heat and work on the air cushion

The state of most closed systems is only changed when one or both of the following occurs (Rogers and Mayhew 1992, Graze, 1967).

(i) Mechanical work is done as a consequence of boundary displacement, and
(ii) Heat transfer occurs across the boundaries of the system

The first law of thermodynamics clearly gives a relationship between the heat outflow, dQ, defined as positive for convenience, work ,dW , and the internal energy change, dU, of the system in Eq. (3).

$$dU = -dQ - dW \tag{3}$$

It can be shown that Eq. 4 is an alternative statement of Eq. 3 (Akpan, 2011). Equation 4 shows the relationship between pressures, volume and heat transfer rate for quasi-steady conditions. The LHS term is the rate of change of absolute pressure in Pascal, while the terms V, γ, $\frac{dQ}{dt}$ and t represent volume of air [m^3], ratio of specific heats of air, rate of heat outflow [W] and time [s] respectively. The last term on the right hand side of Eq.4 determines how far the air behavior deviates from an adiabatic process. This energy flow is an integral part of the dynamic behavior of the air (Graze, 1967).

$$\frac{dP}{dt} = -\gamma\frac{P}{V}\frac{dV}{dt} - \frac{(\gamma-1)}{V}\frac{dQ}{dt} \tag{4}$$

Given that $H = P/\rho_w g$, Eq. 4 could be conveniently written as:

$$\frac{dH}{dt} = -\gamma\frac{H}{V}\frac{dV}{dt} - \frac{(\gamma-1)}{\rho_w gV}\frac{dQ}{dt} \tag{5}$$

H is absolute pressure head of the air volume in meter of water [m] and ρ_w is the density of water [kg/ m^3]. At present, two models exist for describing the heat transfer term, $(\frac{dQ}{dt})$, namely; the empirical Polytropic model and the rational heat transfer (RHT) model. Each model is discussed in sections 4.1.1 and 4.2.1 respectively.

Polytropic Model

The Polytropic model assumes that the state of the entrapped air cushion within the air vessel can be approximated to a Polytropic process as its pressure, temperature and volume changes during surge events. The Polytropic relation is given by Eq. 6.

$$PV^n = C \tag{6}$$

On differentiating Eq.6, it becomes

$$\frac{dP}{P} = -n\frac{dV}{V} \tag{7}$$

Substituting Eq.7 into Eq.4 we get Eq. 8

$$\frac{dQ}{dt} = -P\frac{(\gamma-n)}{(\gamma-1)}\frac{dV}{dt} \tag{8}$$

hence,

$$\Delta Q_{1,2} = \left(\frac{C}{n-1}\right)\frac{(\gamma-n)}{(\gamma-1)}\left(\frac{1}{V_2^{n-1}} - \frac{1}{V_1^{n-1}}\right) n \neq 1 \tag{9a}$$

$$\Delta Q_{1,2} = C\ln\frac{V_1}{V_2}\ n = 1 \tag{9b}$$

The net heat outflow, $\Delta Q_{1,2}$, between the states 1 and 2 is a function of the constants, n,γ, the end states V_1, V_2 or P_1, P_2 respectively only. If in a particular Polytropic process, P_1, V_1 represents the initial state, then the net heat outflow of the system is a fixed value depending on the final state P_2, V_2 only. Work done by or on the system is given in Eqs. 10a and 10b.

$$\Delta W_{1,2} = \frac{C}{n-1}\left(\frac{1}{V_2^{n-1}} - \frac{1}{V_1^{n-1}}\right)\ n \neq 1 \tag{10a}$$

$$\Delta W_{1,2} = C\ln(\frac{V_2}{V_1})\ n = 1 \tag{10b}$$

Designers of air vessels based on this model, have often used different values of n ranging from n = 1.0 – 1.4 for their design. Some adopted a value of n = 1.4, assuming that the air transients were very rapid and that no heat energy entered or left the system, i.e. adiabatic (Thorley, 2004). Others have adopted a more conservative value of n =1.0, implying that the changes in the air volume were relatively slow and that the behavior of the air could be considered isothermal (Graze, 1967; Graze, 1977 and Graze and Forest 1974). The general trend currently is to use an intermediate value of$n$ =1.2 (Thorley, 2004).

Limitations of the Polytropic model

The thermodynamic validity of using the Polytropic process in describing the behavior of an enclosed air in an air vessel came under scrutiny when Graze (1967) observed the theoretical results of temperature and rate of heat outflow against time for a Polytropic process having n= 1.2. He observed that the calculated temperature of the air is always in excess of the ambient temperature. In such a situation, the second law of thermodynamics clearly states that there can only be heat outflow, and yet during certain time intervals, the Polytropic process postulates large rate of heat inflow which negates the second law of thermodynamics. Previous works done by (Graze *et al.*, 1977) have shown that it is incorrect to assume a constant value of Polytropic index, n, as no one single value can satisfactorily describe the thermodynamic process from beginning to the end. For an actual process a variable value of index (n_0) shown in Eq.11 was observed by Graze (1967) experimentally for the instantaneous value of pressure (P) and volume (V) conditions with reference to an initial pressure and volume P_1 and V_1 respectively.

$$n_0 = -\frac{\log {}^{P}/_{P_1}}{\log {}^{V}/_{V_1}} \tag{11}$$

Furthermore, extreme values of pressure and volume of the air are not necessarily provided for by the Polytropic process of $n = 1.0$ and $n = 1.4$ because of the release of energy in the form of latent heat having values in excess of those predicted by the aforementioned Polytropic process. This additional energy is not incorporated in the Polytropic process, It has been shown that due to the large heat sink of the air chamber itself and the importance of the operating temperature factor in preventing the release or the absorption of additional energy which could lead to greater pressure extremes, the application of the Polytropic relationship in these hydraulic systems is fundamentally wrong (Graze, 1967; Graze, 1972 and Graze and Horlacher, 1992).

Rational Heat Transfer Model (RHT)

Graze (1967) proposed a Rational Heat Transfer (RHT) model for describing the thermodynamic behavior of the entrapped air in the air vessel. This model obtains the heat transfer term,dQ/dt, in Eq. 5 by carrying out a rational heat transfer (RHT) analysis of a typical air vessel system shown in Fig. 3.

The three modes of heat transfer (conduction, convection and radiation) across the walls of the air vessel (see Fig.3) were considered. The steady state outflow rate of heat transfer $\frac{dQ}{dt}$ is expressed in Eq. 12

$$\frac{dQ}{dt} = U'.(T - T_{ex}) \times A \tag{12}$$

Where surface area (A) is expressed as $2\pi r_2(X + r_2)$; X =Length of the air column in the vessel (m)and

$$\frac{1}{U'} = \frac{1}{2\pi}\left[\frac{1}{r_2(h_{c_2} + h_{r_2})} + \frac{\ln({}^{r_3}/_{r_2})}{K_t} + \frac{1}{r_3(h_{c_3} + h_{r_3})}\right] \tag{13}$$

Where U' = overall heat transfer co-efficient per unit length of tube [W/m^{2}K], T, T_{ex} =internal, external quiescent fluid temperature [K] respectively, r_2, r_3 = internal, external wall radius [m], h_{c_2}, h_{c_3} = internal, external convection heat transfer co-efficient [W/m^{2}K], h_{r_2}, h_{r_3} = Internal, external radiative heat transfer co-efficient [W/m^{2}K], K_t =Thermal conductivity of the vessel wall, assumed constant [W/mK]. Certain simplifications were made in determining a suitable expression for the overall heat transfer co-efficient per unit length of the vessel wall.

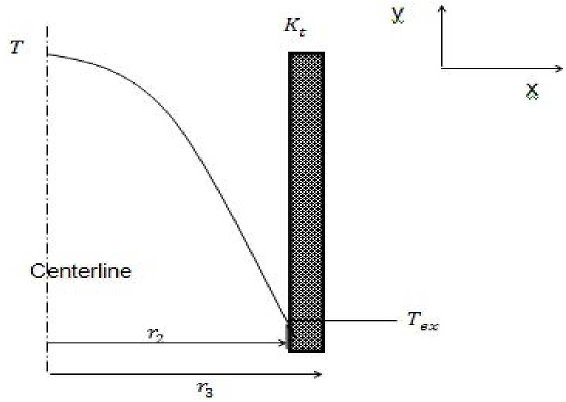

Fig. 3. Vertical wall of an air vessel

(i) Air is a perfect gas
(ii) The steady state heat transfer out of a pipe or cylinder filled with 'dry air only' (although the actual heat transfer is quasi-steady)
(iii) The value of h_{r_2} is essentially zero, since Nitrogen and Oxygen gas present in air are transparent to radiation and the surfaces of the inside walls of the wall are at the same temperature at any instant of time
(iv) The radiative effect of any CO_2 and water vapour in the gas/air is negligible
(v) A thin –walled tube $r_2 \approx r_3$ implying that the conductive term is negligible when compared to the convective terms
(vi) Convective and radiative resistance on the outside wall surface is negligible ($h_{c_3} \approx 0$; $h_{r_3} \approx 0$) considering the thermal capacity of the chamber

Equation13 issimplified by Graze (1967) toobtain a suitable expression for the overall heat transfer co-efficient per unit length of tube as shown in Eq. 14.

$$U' = h_{c_2} \times A \tag{14}$$

Due to the complexities involved in the actual heat transfer process such as: changing boundaries of confined air; varying velocities of the air column with unit length as well as time and periodic reversal of flow, which together with the developing heat boundary layer could create instability of the flow further simplifications were made in determining a suitable value for the co-efficient of convective heat transfer,h_{c_2};A turbulent free convective heat transfer coefficient on a semi-infinite vertical plate proposed by McAdams (1954) was adopted (Eq.15).The water bounding the air at the bottom surface is assumed to act like a piston and its temperature is approximated to the ambient temperature.

$$h_{c_2} = 1.079\ |\Delta T|^{1/3} \quad [\frac{W}{m^2}K] \tag{15}$$

Where:

$$\Delta T = (T - T_{ex}) \qquad [K] \tag{16}$$

The summary of the RHT model for the air vessel is embodied in Equations (5), (12), (14) and (15).

Limitations of existing RHT Model

The RHT model models the air behavior perfectly when the surge event causes an initial increase in pressure like in a sudden valve closure. However, its failure to predict perfectly the pressure, volume and temperature time history of the air cushion for surge events like pump trips that cause an initial decrease in pressure down-surge has made it practically impossible for designers to use. This limitation is due to the difficulty associated with determining, the right overall heat transfer coefficient occasioned by the complex nature of the heat transfer process. The heat transfer involves both free and forced convection and the latent heat (which was ignored) (Graze and Horlacher, 1992). Graze *et al*. (1977) has shown that the latent heat involvement of the gas/liquid transformation (condensation) of the water vapour in the air cushion is generally dwarfed in air chamber installations by the kinetic energy of the moving column of water in the pipeline. However, the latent heat involvement of the liquid/solid transformation of the water vapour appears to markedly affect the overall dynamics of the hydraulic system, hence the need for further research. Modifications have been made to the overall heat transfer coefficient (h_{c_2}) in Eq. 15 which is a steady state free convective heat transfer coefficient for a turbulent flow over a semi-infinite vertical flat plate. The most recent modification by Graze and Horlacher (1992) adopts an empirical relation (see Eq.17) which is approximately one order of magnitude higher than the steady state value represented in Eq. 15.

$$h_{c_2} = 15|\Delta T|^{1/3} \qquad (17)$$

Graze and Horlacher(1992) hoped that in due course, a better analysis can be done that accommodates the individual components (especially free and forced convection and latent heats) rather than the overall heat transfer coefficient, h_{c_2}, utilized so far.

Conclusion

A rational heat transfer (RHT) model was proposed by Graze (1967), in order to accurately describe the dynamic behavior of the entrapped air. This expression involves a rational inclusion of the heat transfer parameter. This model perfectly describes the time-history of pressure, volume and temperature of the entrapped air when the initial event leading to the surge causes the air pressure to rise (e.g. sudden valve closure). However, it fails to predict perfectly the time-history of the pressure, volume and temperature of the entrapped air when the initial event leading to the surge causes the air pressure to drop (e.g. pump trip). This limitation is associated with the difficulties involved in determining the overall heat transfer coefficient – caused by the complex nature of the heat transfer involving both forced and free convection and the involvement of latent heats (Graze and Horlacher 1992; Graze, 1972; Graze, 1968 and Graze, 1967). This setback has made it practically impossible for designers to use it in air vessel design. However, Graze and Horlacher (1992) and Graze *et al.* (1987) hoped that in due course that a more accurate model is possible if:

- The time sequence of the acceleration/deceleration of the air volume and the time over which this takes place is taken into account in the heat transfer co-efficient
- The heat transfer involved in air vessels installations are accommodated by its individual components (especially free and forced convection and latent heats), rather than having the overall heat transfer co-efficient utilized so far.

Hence, the need has arisen for a more accurate thermodynamic model that would predict the behavior of the air cushion.

NOMENCLATURES

Symbol	Definition	Unit
[illegible]	transient propagation speed	[m/s]
[illegible]	pipeline diameter	[m]
[illegible]	rate of heat outflow	[W]
[illegible]	work done by/on the system	[J]
[illegible]	internal energy change	[J]
[illegible]	friction factor in the pipe	[m]
[illegible]	acceleration due to gravity	[m/s^2]
h_{c2}, h_{c3}	internal, external convection heat transfer coefficient	[W/m^{2}K]
h_{r2}, h_{r3}	internal, external radiative heat transfer coefficient	[W/m^{2}K]
[illegible]	absolute pressure head of the air (in meter of water)	[m]
[illegible]	thermal conductivity of tube, assumed constant	[W/mK]
[illegible]	mean Polytropic index	[-]
[illegible]	variable value of Polytropic index	[-]
[illegible]	an outer normal unit vector	[-]
[illegible]	absolute pressure of the air	[Pa]
[illegible]	heat outflow	[J]
[illegible]	Internal, external vessel wall radius	[m]
[illegible]	universal gas constant	
[illegible]	time	[s]
[illegible]	internal, external quiescent fluid temperature	[K]
[illegible]	overall heat transfer co-efficient per unit length of tube	[W/m^{2}K]
[illegible]	flow velocity	[m/s]
[illegible]	volume of air	[m^3]
[illegible]	length of air pocket measured from upper end of plastic pipe	[m]

Greek

Symbol	Definition	Unit
[illegible]	density of water	[kg/m^3]
[illegible]	ratio of specific heats of air	

Subscript

[illegible]	initial condition
1	upstream condition
2	downstream condition

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