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REVIEW ARTICLE

MODELLING AND CONTROL DESIGN OF QUADRUPLE CONICAL TANK PROCESS WITH MINIMUM AND NON MINIMUM PHASE BEHAVIOUR

*Rajesh Kondareddy, Kalyanjee Barman, Sudhanshu Mishra and Santu Brahma

Department of IE, Central Institute of Technology, Kokrajhar, Assam, India

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ABSTRACT

In most of the process industries, the processes are highly non-linear and dynamic. Design of Multivariable control system is of great demand in the process industries. Systems with more than one actuating control input and more than one sensor output are considered as multivariable systems or Multi-Input-Multi-Output (MIMO*)* systems. These systems are one in which interactions are not negligible. This paper describes a multivariable process with the mathematical modeling of a laboratory process, the quadruple conical tank system using which the nonlinearities and uncertainties in industrial process can be analyzed. Here the mathematical modeling is done using the First principle. Here steady state analysis and linearization of the quadruple tank system in both minimum and non minimum phase is obtained (Process Control Modeling, Design, and Simulation by B.Wayne Bequette, Chapter 14; Modern Control Engineering, Fourth Edition by Katsuhiko Ogata; Anna Joseph and Samson Isaac 2013).

INTRODUCTION

Conical tank control of industrial process is a challenging task due to nonlinear behaviour. The control of conical tank liquid level is a major trouble in industrial process. The quadruple conical tank system is a benchmark system used to analyze the nonlinear effects in a multivariable process. This helps in realizing the multi loop systems in industries (Liu Jinkun 2006). The quadruple conical tank process is thus used to demonstrate coupling effects and performance limitations in multivariable control systems. The multivariable dynamic property in a quadruple tank system is the way in which each pump affects both the outputs of the system (Saju 2014). The quadruple tank system is widely used in visualizing the dynamic interactions and non-linarite exhibited in the operation of power plants, chemical industries and biotechnological fields.

These applications are Multi Input Multi Output (MIMO) systems. The control of such interacting multivariable processes is of great interest in process Industries. The constructional details of quadruple tank system and working are explained in the second section (Ge Lusheng Tao Yonghua and Yin Yixin 2000). The mathematical modeling of the real time system and its operating points are explained in the third section (Govinda Kumar *et al.,* 2014). The steady state equation, Relative gain array and the transfer function is described in the fourth section. The open loop response for both minimum and non-minimum phase is shown in simulation results section (Tang Xianlun *et al.,* 2005)

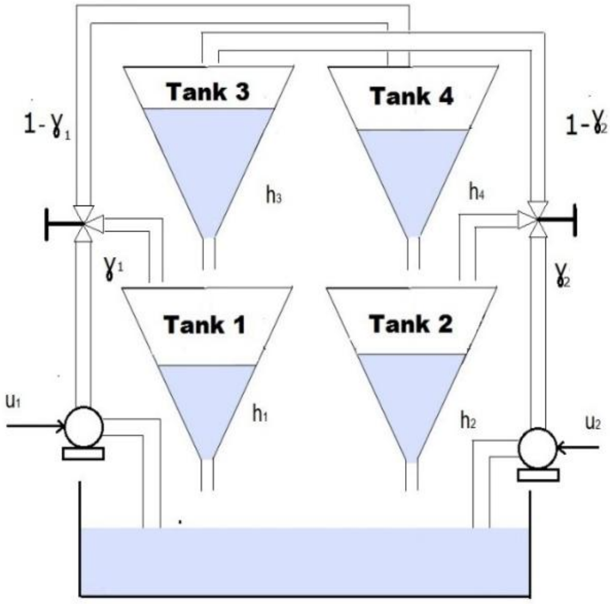
Quadruple tank system

The quadruple tank system is a multi input multi output system that could be used to analyze different control strategies. It is considered as a two double-tank process. The setup consists of four interacting tanks, two pumps and two valves. The two process inputs are the voltages v1 and v2 supplied to the two pumps. Tank 1 and tank 2 are placed below tank 3 and tank 4 to receive water flow by the action of gravity. To accumulate the outgoing water from tank 1 and tank 2 a reservoir is present in the bottom. Every tank has a valve fitted to its outlet. The action of pumps 1 and 2 is to suck water from the reservoir and deliver it to tanks based on the valve opening. Pump 1 delivers water to tank 2 and tank 3. Similarly the pump 2 delivers water to tank 1 and tank 4.Due to gravitational force the lower tanks receive water from their corresponding upper tanks. The system aims at controlling the liquid levels in the lower tanks. The controlled outputs are the liquid levels in the lower tanks (h1, h2). The valve positions are γ_1 and γ_2. These valve positions give the ratio in which the output from the pump is divided between the upper and lower tanks. The flow to the tanks can be adjusted by pump positions and flow rate can be monitored using the two rotameters. The valve position is fixed during the experiment and only the speed of pump is varied by changing the input voltage. The operation of quadruple tank

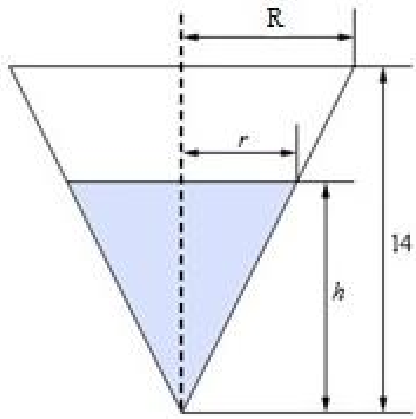
Corresponding author: Rajesh Kondareddy
Department of IE, Central Institute of Technology, Kokrajhar, Assam, India.

system can be comprehended in two phase's namely minimum phase and non- minimum phase.

Mathematical model



Conical Tank



Volume of the conical tank =1/3(Ah);

A=Cross-sectional area, and h=Height. $\frac{R}{H}=\frac{r}{h}=\tan\alpha$

The process inputs are V_1 and V_2. And the process outputs are h_1& h_2.

The output from level measurement devices
$Y_1=K_c h_1$ and $Y_2=K_c h_2$
(Where, K_c= Gain of sensor =1)

Minimum Phase: When the liquid enter the lower tanks is is less than that of upper tanks, then the system starts operating in minimum phase.

Non-Minimum phase: when fraction of liquid entering the upper tanks is less than that of lower tanks, then the system starts operating in non-minimum phase.

Variables and Constants

Variables	Description	units
H	Height of conical tank	cm
h_i	Height of liquid level	cm
h_1, h_2	Steady state height of liquid levels of tank 1&2.	cm
h_3, h_4	Steady state height of liquid levels of tank 1&2.	cm
		V
v_i	Voltage of the pumps(i=1&2)	
γ_1	Flow distribution to lower and diagonal upper tank of Valve 1.	
γ_2	Flow distribution to lower and diagonal upper tank of Valve 1.	cm^2
A		cm^3
V	Cross sectional Area of tank	cm^2
A	Volume of the conical tank	cm^3/sec
Q_i	Cross sectional Area of outlet	981 cm/sec^2
g	Pump's flow	
K_1 &K_2	Acceleration due to gravity	
	Pump flow constants	

Flow from pumps to Tanks

	Tank 1	Tank 2	Tank 3	Tank 4
Pump 1	$\gamma_1K_1V_1$	-	-	$(1-\gamma_1)K_1V_1$
Pump 2	-	$\gamma_2K_2V_2$	$(1-\gamma_2)K_2V_2$	-

For derivation mathematical model of quadruple conical tank, we setup the basic equations that hold for each of the tanks and for two pumps. They are put together to obtain the model of whole system.

Mass Balance equation

$V=Ah=q_{in}-q_{out}$

Where, V= volume of the tank=1/3(Ah);
Again, $q_{out}=av_w=a\sqrt{2gh}$
A= cross sectional area of the tank;
a= cross sectional area at outlet;
h= water level
h= water level,
q_{in}= in flow,
q_{out}= outflow.
v_w= speed of water at outlet;
g= acceleration due to gravity.

Pump Generation Flows

$q_{pumpj}= K_p v_j$

K_p= pump flow constant; v_i= input voltage to the pump

The system equations

$\frac{dh1}{dt}=\frac{3}{A1}\{a\sqrt{(2gh_3)}+\gamma_1K_1v_1-a\sqrt{(2gh_1)}\}$;
$\frac{dh2}{dt}=\frac{3}{A2}\{a\sqrt{(2gh_4)}+\gamma_2K_2v_2-a\sqrt{(2gh_2)}\}$;
$\frac{dh3}{dt}=\frac{3}{A3}\{(1-\gamma_2)K_2v_2-a\sqrt{(2gh_1)}\}$;
$\frac{dh4}{dt}=\frac{3}{A4}\{(1-\gamma_1)K_1v_1-a\sqrt{(2gh_4)}\}$;
$1/T_1=\frac{3\,a\sqrt{(2g)}}{A1(2\sqrt{h1})}$; $1/T_1^{/}=\frac{3\,a\sqrt{(2g)}}{A1(2\sqrt{h3})}$; $1/T_2^{/}=\frac{3\,a\sqrt{(2g)}}{A2(2\sqrt{h4})}$; $1/T_2=\frac{3\,a\sqrt{(2g)}}{A2(2\sqrt{h2})}$;
$1/T_3=\frac{3\,a\sqrt{(2g)}}{A3(2\sqrt{h3})}$; $1/T_4=\frac{3\,a\sqrt{(2g)}}{A4(2\sqrt{h4})}$;

State space equations

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} (-1/T_1) & 0 & 1/T_1' & 0 \\ 0 & (-1/T_2) & 0 & 1/T_2' \\ 0 & 0 & 1/T_3 & 0 \\ 0 & 0 & 0 & 1/T_4 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} + \begin{pmatrix} 3\gamma_1 K_{1/A1} & 0 \\ 0 & 3\gamma_2 K_{2/A2} \\ 0 & (1-\gamma_2)K_{2/A2} \\ (1-\gamma_1)K_{1/A2} & 0 \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \end{pmatrix}$$

$$\begin{pmatrix} Y_1 \\ Y_2 \end{pmatrix} = \begin{pmatrix} K_c & 0 & 0 & 0 \\ 0 & K_c & 0 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix}$$

$$G(s) = \begin{pmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \end{pmatrix}$$

Structure of PI controller design

The structure of MIMO control system using PI controller for minimum and no minimum phase shown figure bellow:-

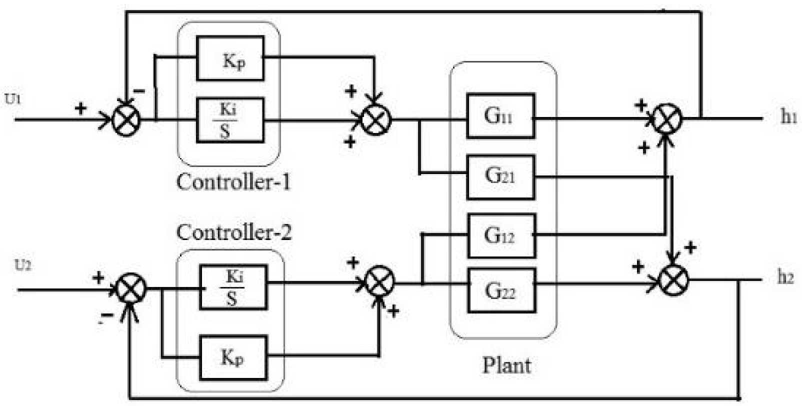


fig-(a) Stracture of minimum phase control system with PI Controller

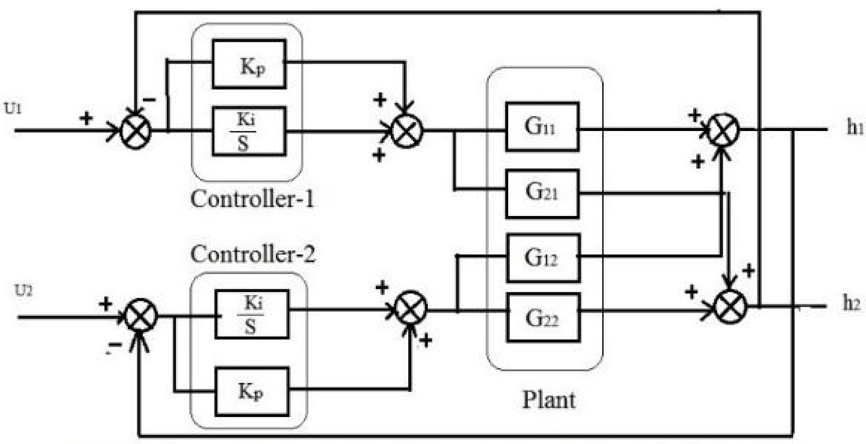


fig-(b) Stracture of nonminimum Phase control system with PI controller

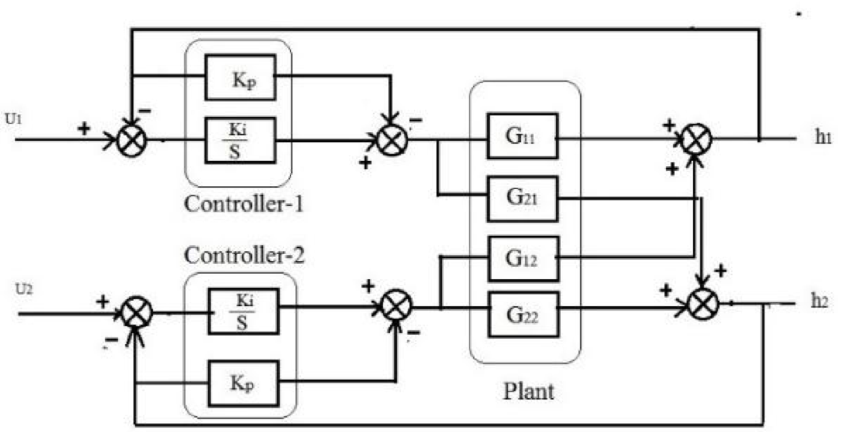


fig-(c) Stracture of minimum phase control system with modified PI Controller

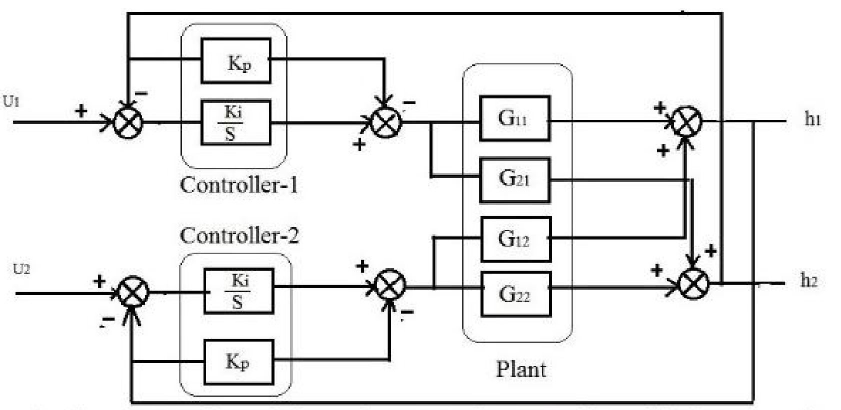


fig-(d) Stracture of nonminimum phase control system with modified PI Controller

System analysis

The rate of change of liquid height (h1, h2, h3, h4) and relates the difference between inlet flow and outlet flow. Common Parameter:- H= Height of conical tank. R= Outer radius of Conical tank.
r=Radius of the outlet =1cm & a=cross sectional area of the outlet = $3.14x1^2$=3.14cm^2

Parameter	Operating Point-1	Operating Point-2
h_1, h_2, h_3, h_4 γ_1, γ_2 K_1 and K_2	10 cm, 8 cm ,8 cm,6 cm 0.7 , 0.8 3.32 and 3.3	20 cm, 15 cm , 9 cm , 10 cm 0.85 , 0.75 3.32 and 3.12
A_1, A_2	44.156 cm^2 , 28.26 cm^2	176.625 cm^2 , 99.35 cm^2
A_3, A_4	28.26 cm^2 15.896 cm^2	35.766 cm^2 , 44.156cm^2
$1/T_1, 1/T_2, 1/T_3, 1/T_4$	1.494, 2.61, 2.61,5.35	0.2641,0.542,1.944,1.494
$1/T_1' \cdot 1/T_2'$	1.67 , 3.013	0.394, 0.664

Parameter	Operating Point-1	Operating Point-2
$h_1, h_2, h_3\ h_4$ γ_1, γ_2 K_1 and K_2	8.5 cm, 12.5 cm, 6.5 cm, 7.9 cm .25, 0.30 3.3 and 3.15	13cm, 8cm, .06cm, 4cm 0.35, 0.31 3.32 and 3.12
A_1, A_2	31.902 cm^2, 68.994 cm^2	74.624 cm^2, 28.26 cm^2
A_3, A_4	18.656 cm^2, 27.57 cm^2	15.896 cm^2, 7.065 cm^2
$1/T_1, 1/T_2, 1/T_3, 1/T_4$	2.243, 0.855, 4.386, 2.698	0.753, 2.610, 5.358, 14.764
$1/T_1' \cdot 1/T_2'$	2.565, 1.0758	1.1413, 3.691

Transfer Function

Non Minimum Phase

Operating Point 1

$$G(s)=\begin{bmatrix} \frac{0.0775}{(S+2.243)} & \frac{0.90929}{(s+2.243)(S+4.386)} \\ \frac{0.29035}{(S+0.855)(S+2.698)} & \frac{0.04109}{(S+0.855)} \end{bmatrix}$$

Operating Point 2

$$G(s)=\begin{bmatrix} \frac{0.0467}{(S+0.77538)} & \frac{0.4370}{(S+0.77538)(S+5.358)} \\ \frac{3.590}{(S+2.610)(S+14.764)} & \frac{0.1026}{(S+2.61)} \end{bmatrix}$$

Minimum Phase

Operating Point 1

$$G(s)=\begin{bmatrix} \frac{0.1569}{(S+1.494)} & \frac{0.1177}{(S+1.494)(S+2.61)} \\ \frac{0.5613}{(S+2.61)(S+5.357)} & \frac{0.281}{(S+2.61)} \end{bmatrix}$$

Operating Point 2

$$G(s)=\begin{bmatrix} \frac{0.04793}{S+0.2641} & \frac{0.01645}{(S+0.2641)(S+1.944)} \\ \frac{0.03518}{(S+0.542)(S+1.494)} & \frac{0.07066}{(S+0.542)} \end{bmatrix}$$

MATLAB Analysis

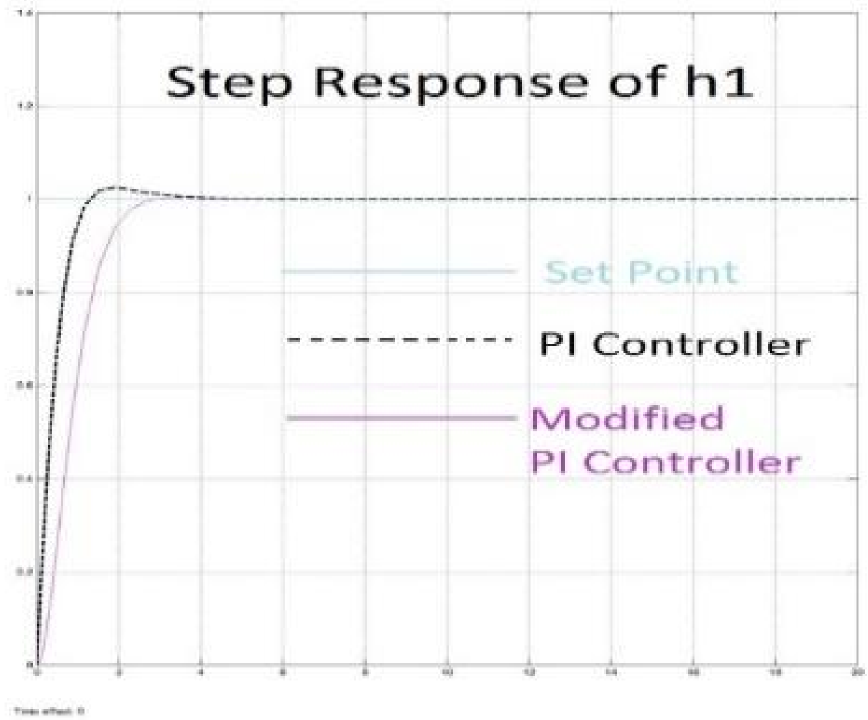
The objective of the work is to design and observed the step response of the PI controller and modified PI controller for multivariable 4-conical tank level control process. The designs are demonstrated on a quadruple tank level control process with 2 inputs and 2 outputs.

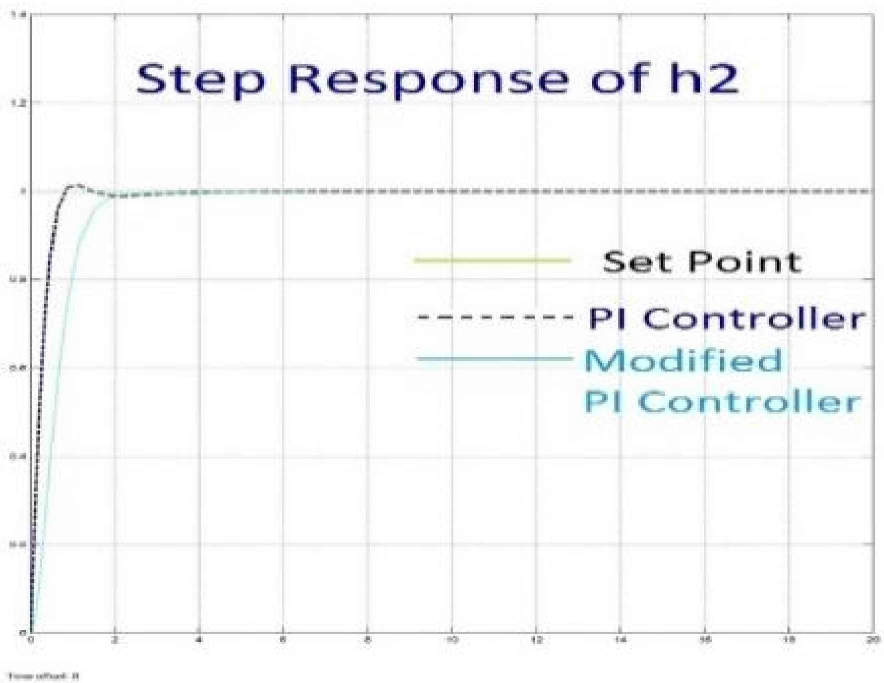
The different forms of the controller structures are used on the 4 conical tank system and the step response is compared .All the simulation results are simulated using MATLAB software and comparing the performance of different controller.

Response of Minimum Phase

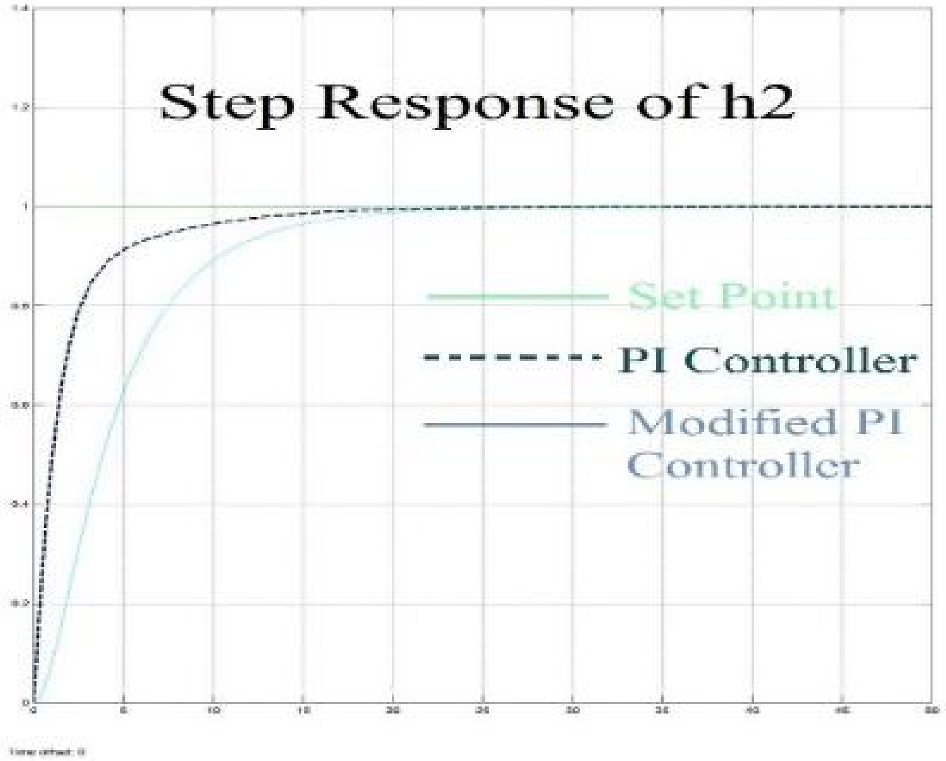
Comparison between PI controller and Modified PI Controller

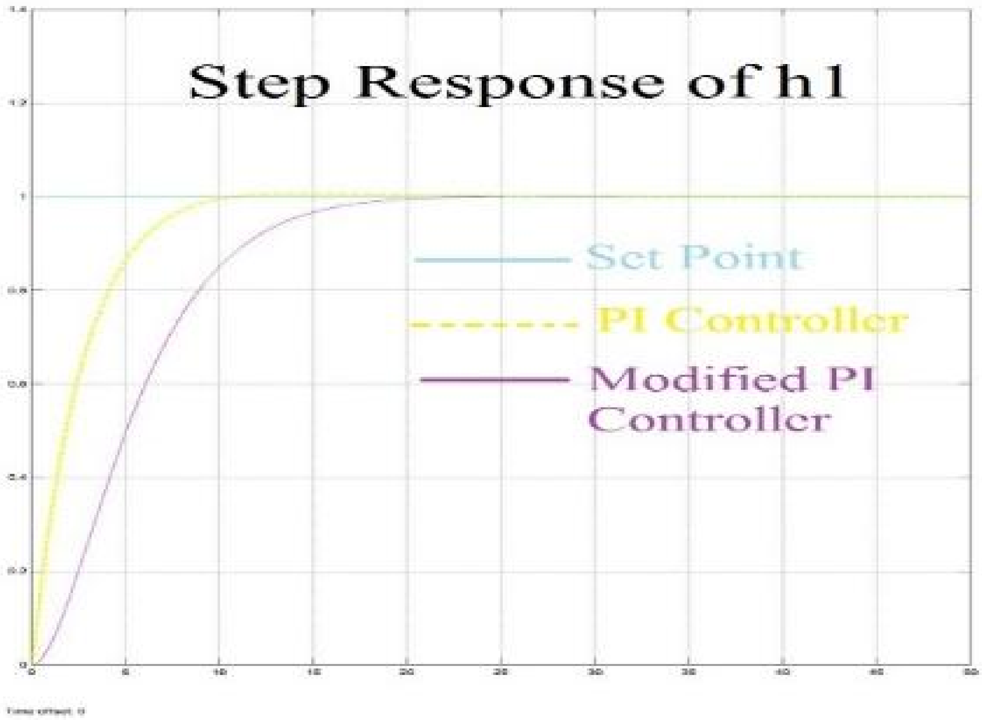
Operating Point 1





Operating Point 2

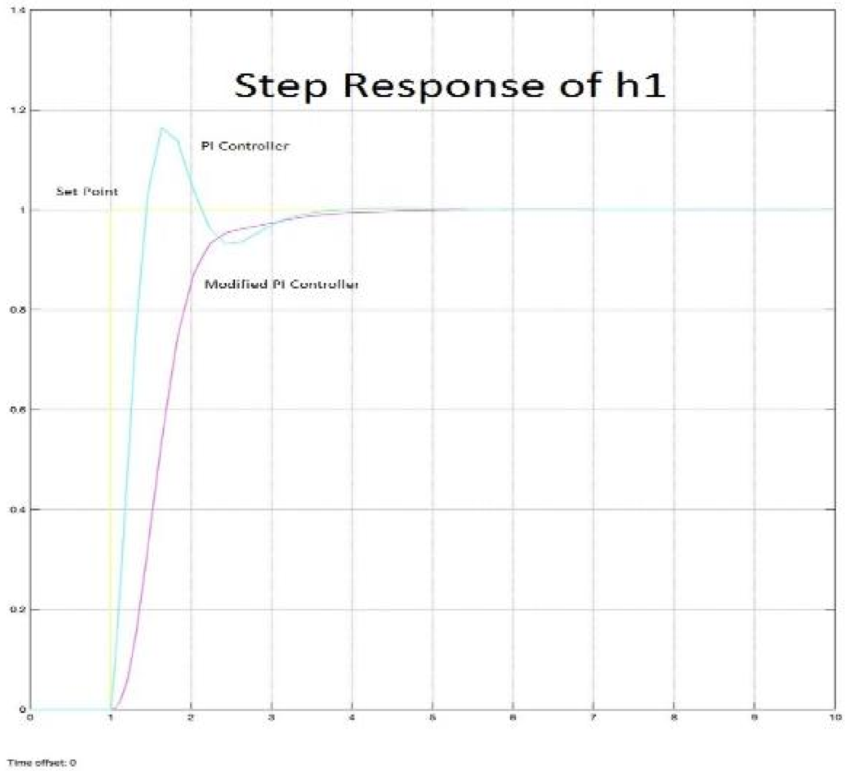


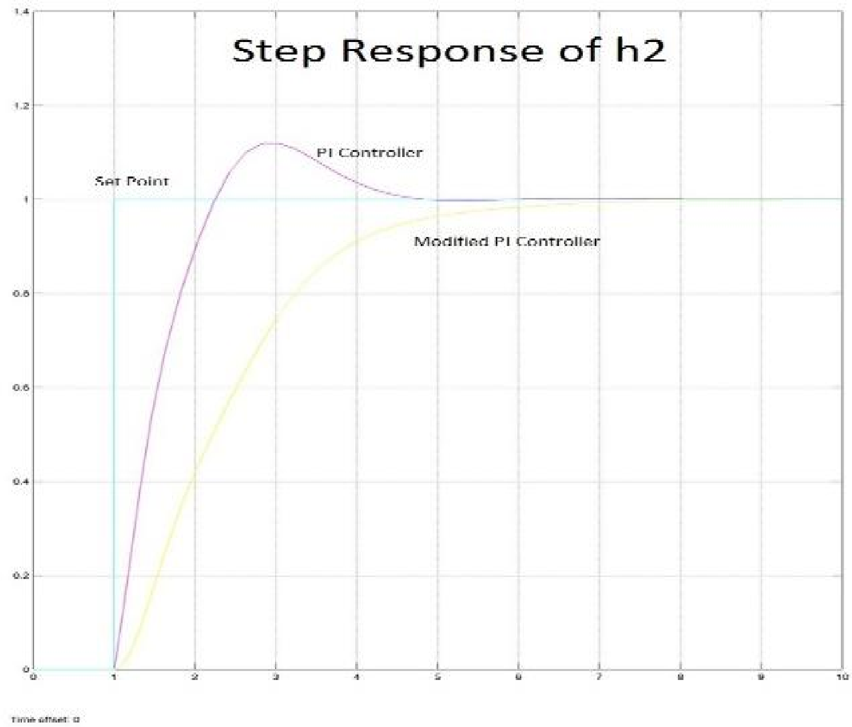


Response of NonMinimum Phase

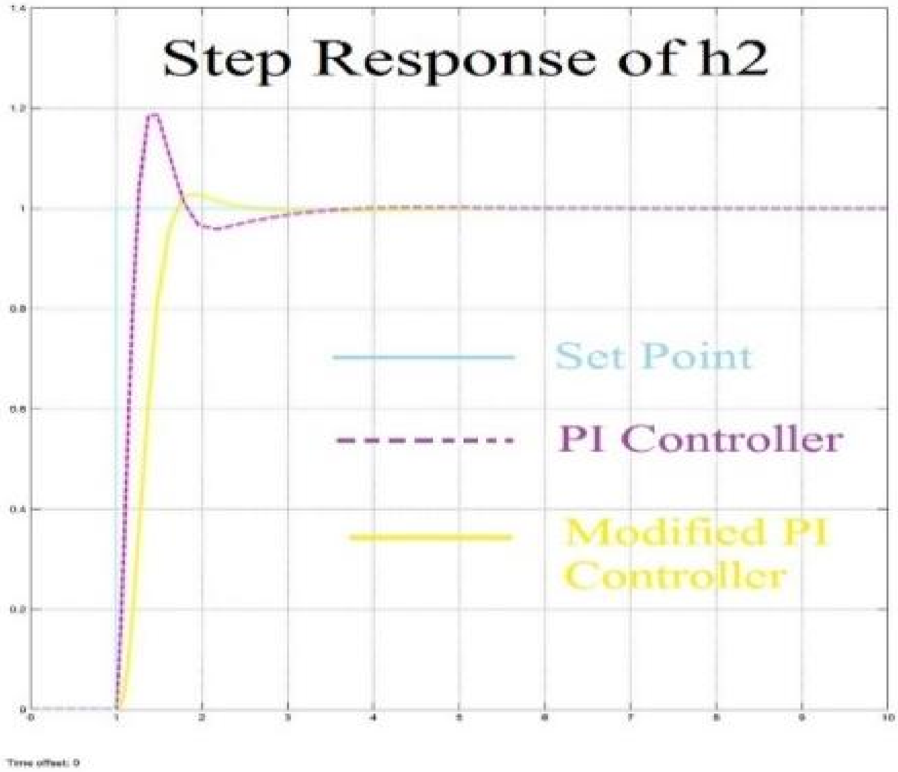
Comparison between PI controller and Modified PI Controller

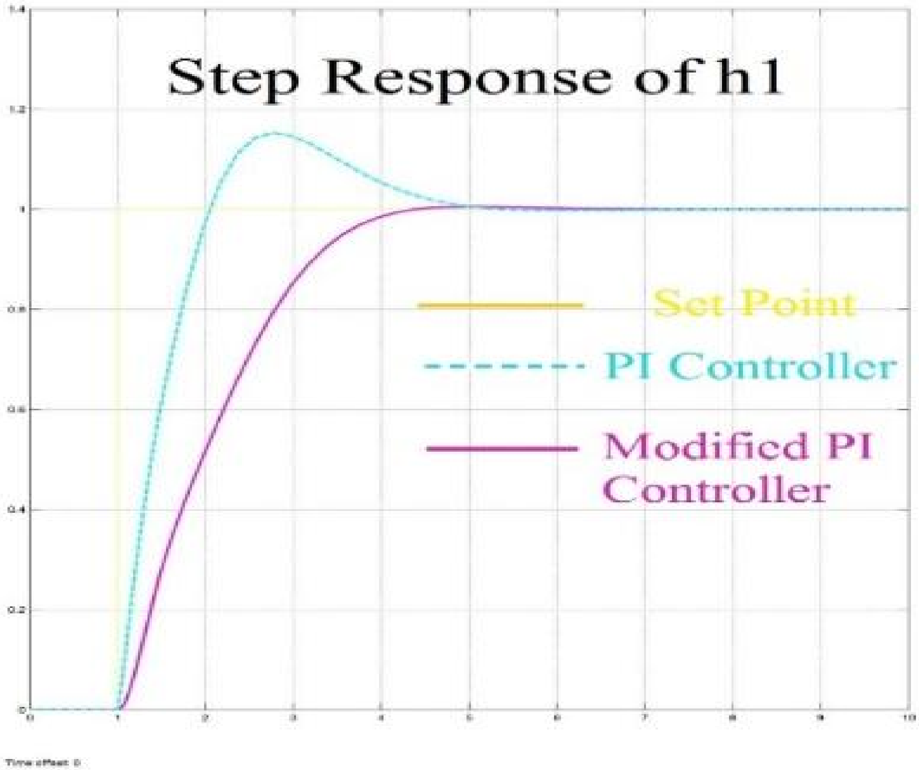
Operating Point 1





Operating Point 2





PI Controller

	Operating Point 1			Operating Point 2	
	Controller 1	Controller 2		Controller 1	Controller 2
Minimum Phase	Kp=12; Ki= 22	Kp=10; K_i= 25	Minimum Phase	K_p=7; K_i=2	Kp=9; Ki= 3
Nonminimum Phase	Kp=15.5; K_i=13.5	Kp=22.9; K_i=46.9	Nonminimum Phase	K_p=23; Ki=99.2	$K_{p=}$17.61; K_i= 15.89

Controller Tuning Methods

The ultimate cycle/ ultimate gain/ cyclic oscillations methods are the simple and more effective ways for setting up the PID controller gains. Basically, these methods are of three types, mentioned as follows.

*Ziegler-Nichols (ZN) PID controller tuning method.
*Modified Ziegler-Nichols PID controller tuning method.
*Tyreus-Luyben (TL) PID controller tuning method.

The quadruple conical tank in the industrial production has non-linear, time-varying and delay characteristics. Hence, we cannot create an absolute mathematical model. It is always a painful and challenging task to select proper values for K_P, K_I, and K_D gains. To reduce the above problems and to improve transient response specifications, the outer loop PID is tuned by using tuning algorithms. Tuning of PID controller involves the best selection of values for proportional (K_P), integral (K_I) and derivative (K_D) gains.

The following are the steps to calculate critical gain (K_C) and critical time period (T_C).

1. Reduce integral and derivative actions to their minimum effect i.e. design the system with proportional controller only and with unity feedback.
2. Gradually begin to increase the proportional gain value until the system exhibits the sustained oscillations.
3. This gain at which the system exhibits steady cycling or sustained oscillations about the set point iscalled critical gain (Kc). The time period corresponding to these oscillations is called as critical time period (Tc).
4. Note the values of these Kc and Tc.
5. From these values, calculate Kp& Ki gain values based on the method considered as shown below

Conclusion

In this paper, we designed and compared PI controller against modified PI controller and applied to the 4 conical tank system with minimum and no minimum phase system. The output response of the dyanamic system is observed using both control actions. we conclude that the Modified PI controller have minimum peak overshoot and faster settling time with same K_p and K_i value for both controller (1&2) over PI controller (1&2).

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