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International Journal of Current Research
Vol. 6, Issue, 11, pp.9653-9656, November, 2014

INTERNATIONAL JOURNAL OF CURRENT RESEARCH

ISSN: 0975-833X

RESEARCH ARTICLE

NUMERICAL ALGORITHMS FOR NEWTON-COTES OPEN QUADRATURE FORMULAE

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ARTICLE INFO

Article History:
Received 22nd August, 2014
Received in revised form
04th September, 2014
Accepted 08th October, 2014
Published online 18th November, 2014

Key words:
Algorithm, Improper integrals,
Newton-Gregory forward difference interpolation formula.

ABSTRACT

In this paper, we have designed three new algorithms for the solutions of improper integrals which can't be solved by means of Newton-Cotes closed integration formulae (Trapezoidal rule, Simpson's 1/3 rule, Simpson's 3/8 rule, Boole's rule and Weddle's rule). The results presented here are presumably new.

INTRODUCTION

In this age of computer, the knowledge of numerical integration is necessary because computers do not go through the analytic process of integration. With the rapid advancement in the field of computer based solution of engineering problems, the importance of numerical integration need not be over emphasized. So far as the techniques of the numerical integration are concerned, the following five Newton-Cotes open integration formulae (1.1) to (1.5) in simplest forms, are fairly well known in the literature (Booth, 1958; Conte, 1965; Froberg, 1965; Hildebrand, 1956; Mathews, 1994; McCormic and Salvadori, 1961; Phillips and Taylor, 1973; Scheid, 1968; Stanton, 1967) of numerical analysis.

$$\int_{x_0}^{x_2} f(x)\,\mathrm{d}x \approx 2h f_1 \qquad \ldots\ldots\ldots\ldots (1.1)$$

$$\int_{x_0}^{x_3} f(x)\,\mathrm{d}x \approx \frac{3h}{2}(f_1 + f_2) \qquad \ldots\ldots\ldots\ldots (1.2)$$

$$\int_{x_0}^{x_4} f(x)\,\mathrm{d}x \approx \frac{4h}{3}(2f_1 - f_2 + 2f_3) \qquad \ldots\ldots\ldots\ldots (1.3)$$

$$\int_{x_0}^{x_5} f(x)\,\mathrm{d}x \approx \frac{5h}{24}(11f_1 + f_2 + f_3 + 11f_4) \qquad \ldots\ldots\ldots\ldots (1.4)$$

$$\int_{x_0}^{x_6} f(x)\,\mathrm{d}x \approx \frac{3h}{10}(11f_1 - 14f_2 + 26f_3 - 14f_4 + 11f_5) \qquad \ldots\ldots\ldots\ldots (1.5)$$

In (1.1) to (1.5), $h = \dfrac{x_n - x_0}{n},\ n \in \{2,3,4,5,6\}$ respectively.

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Recently in 2012, Azad *et al.* (2012) developed a formula for $n = 10$ as follows:

$$\int_{x_0}^{x_{10}} f(x)\,\mathrm{d}x \approx \frac{5h}{4536}(4045f_1 - 11690f_2 + 33340f_3 - 55070f_4 +$$
$$+67822f_5 - 55070f_6 + 33340f_7 - 11690f_8 + 4045f_9) \qquad \text{...........(1.6)}$$

2. NEW OPEN QUADRATURE FORMULAE

Motivated from the above work (1.1)-(1.6), we developed the three new Newton-Cotes open integration formulae for $n \in \{7, 8, 9\}$.

$$\int_{x_0}^{x_7} f(x)\,\mathrm{d}x \approx \frac{7h}{1440}(611f_1 - 453f_2 + 562f_3 + 562f_4 - 453f_5 + 611f_6) \qquad \text{.........(2.1)}$$

$$\int_{x_0}^{x_8} f(x)\,\mathrm{d}x \approx \frac{8h}{945}(460f_1 - 952f_2 + 2196f_3 - 2459f_4 + 2196f_5 - 952f_6 + 460f_7) \qquad \text{..........(2.2)}$$

$$\int_{x_0}^{x_9} f(x)\,\mathrm{d}x \approx \frac{9h}{4480}(1787f_1 - 2803f_2 + 4967f_3 - 1711f_4 - 1711f_5 + 4967f_6 - 2803f_7 + 1787f_8) \qquad \text{......(2.3)}$$

3. DERIVATIONS OF (2.1)-(2.3)

The integrand $f(x)$ is undefined at $x = x_0$ and $x = x_7$ i.e., $f(x)$ is well defined in the open interval (x_0, x_7)

then $I = \int_{x_0}^{x_7} f(x)\,\mathrm{d}x$ ....(3.1)

where $x_7 = x_0 + p\,h.$

Substituting $x = x_1 + p\,h$ in (3.1), then

$$I = h\int_{-1}^{6} f(x_1 + p\,h)\,\mathrm{d}p \qquad \text{...(3.2)}$$

Now using the well known Newton-Gregory forward difference interpolation formula for six equally spaced points, we get

$$I \approx h\int_{-1}^{6}\{f_1 + p\Delta f_1 + \frac{p(p-1)}{2!}\Delta^2 f_1 + \frac{p(p-1)(p-2)}{3!}\Delta^3 f_1 +$$
$$+\frac{p(p-1)(p-2)(p-3)}{4!}\Delta^4 f_1 + \frac{p(p-1)(p-2)(p-3)(p-4)}{5!}\Delta^5 f_1\}\mathrm{d}p$$

$$I \approx h\int_{-1}^{6}\{f_1 + p\Delta f_1 + \frac{(p^2 - p)}{2!}\Delta^2 f_1 + \frac{(p^3 - 3p^2 + 2p)}{3!}\Delta^3 f_1 +$$
$$+\frac{(p^4 - 6p^3 + 11p^2 - 6p)}{4!}\Delta^4 f_1 + \frac{(p^5 - 10p^4 + 35p^3 - 50p^2 + 24p)}{5!}\Delta^5 f_1\}\mathrm{d}p$$

$$\approx h\left\{7f_1 + \frac{35}{2}\Delta f_1 + \frac{329}{12}\Delta^2 f_1 + \frac{189}{8}\Delta^3 f_1 + + \frac{9107}{720}\Delta^4 f_1 + \frac{4277}{1440}\Delta^5 f_1\right\}$$

$$\approx h\{7f_1 + \frac{35}{2}(f_2 - f_1) + \frac{329}{12}(f_3 - 2f_2 + f_1) + \frac{189}{8}(f_4 - 3f_3 + 3f_2 - f_1) +$$

$$+\frac{9107}{720}(f_5 - 4f_4 + 6f_3 - 4f_2 + f_1) + \frac{4277}{1440}(f_6 - 5f_5 + 10f_4 - 10f_3 + 5f_2 - f_1)$$

After simplification, we get (2.1).

Similarly, on the same parallel lines of derivation of (2.1), we get (2.2) and (2.3).

4. ALGORITHMS

Algorithm of (2.1):

Step 1: Define the given function $f(x)$.

Step 2: Enter the values of upper limit $= x_7$ and lower limit $= x_0$.

Step 3: Compute $h = \frac{x_7 - x_0}{7}$.

Step 4: Initialize $\text{sum} = 0$.

Step 5: Calculate $\text{sum} = \text{sum} + 611(f(x_0 + h) + f(x_0 + 6h))$

$\text{sum} = \text{sum} - 453(f(x_0 + 2h) + f(x_0 + 5h))$

$\text{sum} = \text{sum} + 562(f(x_0 + 3h) + f(x_0 + 4h))$

$\text{sum} = \frac{7 * h * \text{sum}}{1440}$

Step 6: Write the value of given integral = "sum".

Step 7: Exit.

Algorithm of (2.2)

Step 1: Define the given function $f(x)$.

Step 2: Enter the values of upper limit $= x_8$ and lower limit $= x_0$.

Step 3: Compute $h = \frac{x_8 - x_0}{8}$.

Step 4: Initialize $\text{sum} = 0$.

Step 5: Calculate $\text{sum} = \text{sum} + 460(f(x_0 + h) + f(x_0 + 7h))$

$\text{sum} = \text{sum} - 952(f(x_0 + 2h) + f(x_0 + 6h))$

$\text{sum} = \text{sum} + 2196(f(x_0 + 3h) + f(x_0 + 5h))$

$\text{sum} = \text{sum} - 2459 * f(x_0 + 4h)$

$\text{sum} = \frac{8 * h * \text{sum}}{945}$

Step 6: Write the value of given integral = "sum".

Step 7: Exit.

Algorithm of (2.3)

Step 1: Define the given function $f(x)$.

Step 2: Enter the values of upper limit x_9 and lower limit $= x_0$.

Step 3: Compute $h = \frac{x_9 - x_0}{9}$.

Step 4: Initialize $\text{sum} = 0$.

Step 5: Calculate $\text{sum} = \text{sum} + 1787(f(x_0 + h) + f(x_0 + 8h))$

$$\text{sum} = \text{sum} - 2803(f(x_0 + 2h) + f(x_0 + 7h))$$

$$\text{sum} = \text{sum} + 4967(f(x_0 + 3h) + f(x_0 + 6h))$$

$$\text{sum} = \text{sum} - 1711(f(x_0 + 4h) + f(x_0 + 5h))$$

$$\text{sum} = \frac{9 * h * \text{sum}}{4480}$$

Step 6: Write the value of given integral = "sum".

Step 7: Exit.

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