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RESEARCH ARTICLE

A MODEL FOR THE TRANSIENT FLOW OF NATURAL GAS THROUGH A PIPELINE IN TWO-DIMENSIONAL CYLINDRICAL COORDINATES - PART I

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ABSTRACT

This paper presents a two-dimensional gas pipeline model in cylindrical coordinates. The flow is assumed compressible and adiabatic. A modification in the energy equation of the compressible Navier-Stokes is made. First, a substantial derivative of the equation of state for an ideal gas (where pressure is the dependent variable) is obtained. A substitution for the substantial derivative of pressure in the energy equation is then made. This eliminates any derivative of pressure in the energy equation. This paper is a precursor to a second paper in which the computer simulation of the model will be presented and analysed.

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INTRODUCTION

A gas distribution network is composed of pipe segments connected by simple junctions or other components (compressors stations, pressure regulators, valves, etc.) (Luongo, 1986). The mass of gas flowing through one of these components at any given time is negligible compared with the gas packed in the pipe segments. This assumption allows only a quasi-steady state treatment of these components and leaves the pipe sections as the only distributed system requiring the solution of partial differential equations for its characterization. Unsteady gas flow in pipelines occurs due to rapid and slow disturbances (Nouri-Borujerdi, 2011). In general, pressure and mass-flow fluctuations cause slow disturbances whereas rapid disturbances are associated with compression wave effects caused by sharp closure of a shut-off valve, the system startup or expansion wave related to the pipeline rupture. The unsteady flow of gas in a long pipeline after an accidental rupture is of considerable interest to the natural gas industry due to the enormous amount of flammable gas release and its potential hazards. The accurate prediction of outflow and its variation with time following pipeline rupture or (any other disturbance in the system) are therefore extremely important since this information dictates all the major consequences associated with such failure including fire, explosion and environmental pollution (Mahgerefteh *et al*., 2006). Flow of gas (or any other fluid) is usually modeled by the Navier – Stokes system of equations. The Navier-Stokes system of equations may be simplified by disregarding one or more equations and/or some of the terms of each equation. For example, the momentum equations alone are often called the Navier-Stokes equations.

Nouri-Borujerdi (2011) simulated transient compressible adiabatic gas flows in a long pipeline following a catastrophic failure using an implicit high order finite difference scheme as a discretisation technique for convective terms. On the other hand, in a study by Kessal (2000), a set of equations governing an isothermal compressible transient fluid flow in one dimension was resolved numerically for two practical cases. Abbaspour *et al*. (2010) simulated the non-isothermal, one-dimensional, transient homogenous two-phase flow gas pipeline system using two-fluid conservation equations by the application of the fully implicit finite difference method.

Nouri-Borujerdi *et al*. (2007) investigated the numerical modeling of the dynamic behavior of compressible flow of gas in pipelines. The numerical simulation was performed by solving the coupled conservation form of the governing equations for two dimensional, laminar, viscous, supersonic flows in developing region under different thermal boundary

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conditions. Ziaei-Rad and Nouri-Borujerdi (2008) investigated numerically the compressible gas flow through a pipe subjected to wall heat flux in unsteady condition in the entrance region. The coupled conservation equations governing turbulent compressible viscous flow in the developing region of a pipe were solved numerically under different thermal boundary conditions. Hassan *et al*. (1991) applied a numerical scheme for the solution of two-dimensional problems of steady compressible inviscid and laminar viscous high-speed flows. Lobão (2010) presented a work describing the numerical simulations of transient flows in two dimensional geometries with shock waves involving multiple shocks and reflections of shockwaves, and their interactions. The numerical formulations were used to solve the fully compressible Navier Stokes equations set using explicit Mac Cormack method.

Chorin (1968) presented a finite-difference method for solving the time-dependent Navier-Stokes equations for an incompressible fluid. The method is applicable to two- and three-space dimensions. Kim and Moin (1985) presented a numerical method for computing three-dimensional, time-dependent incompressible flows. Luchini and Quadrio (2006) published a numerical method for the direct numerical simulation of incompressible wall turbulence in rectangular and cylindrical geometries. Bao *et al*. (2010) developed two versions of a second-order characteristic-based split scheme (a finite element method) in the framework of incremental projection method for the solution of incompressible flow problem. Barron and Zogheib (2010) presented a new numerical method for solving the two-dimensional, steady, incompressible, viscous flow equations on a curvilinear staggered grid.AL-Shannag (2002) studied the shear-driven incompressible flow in a toroidal cavity of square cross-section (DxD) and radius of curvature Rc both experimentally and numerically. Numerical solutions were obtained by integrating the incompressible time-dependent Navier-Stokes equations using a fourth-order accurate code. Shah *et al*. (2010) presented a time-accurate numerical method using high-order accurate compact finite difference scheme for the incompressible Navier–Stokes equations. The method relies on the artificial compressibility formulation, which gives the governing equations a hyperbolic–parabolic nature. Gupta and Kalita (2005) proposed a new method for solving Navier–Stokes equations. The proposed methodology is based on a streamfunction–velocity formulation of the two-dimensional steady-state Navier–Stokes equations representing incompressible fluid flows in two-dimensional domains Pulliam and Steger (1980) presented an implicit finite-difference procedure for unsteady three-dimensional flow capable of handling arbitrary geometry through the use of general coordinate transformations. Viscous effects were optionally incorporated with a "thin-layer" approximation of the Navier-Stokes equations. Jameson and Yoon (1985) applied a numerical method to the solution of the Euler equations for transonic flow over an airfoil. Greyvenstein (2002) presented an implicit finite difference method based on the simultaneous pressure correction approach which is valid for both liquid and gas flows, for both isothermal and non-isothermal flows and for both fast and slow transients. The problematic convective acceleration term in the momentum equation, often neglected in other methods, was retained but eliminated by casting the momentum equation in an alternative form. Zhu (2005) presented a dissertation on a numerical study in primitive variables of three-dimensional Navier-Stokes equations and energy equation in an annular geometry. A fast direct method was developed to solve the Poisson equation for pressure with Neumann boundary conditions in radial and axial directions, and periodic boundary conditions in azimuthal direction. Srivastava *et al*. (2013) proposed an implicit exponential finite-difference scheme (Expo FDM) for solving two dimensional nonlinear coupled viscous Burgers' equations (VBEs) with appropriate initial and boundary conditions. Accurate results were also obtained. Li (1993), made a comparative study of three finite volume formulations to investigate the efficacy of flux limiters and damping coefficients on three-dimensional Euler and Navier-Stokes solutions.

This work presents a model derived from the Compressible Navier-Stokes system of equations in 2-dimensional cylindrical (axisymmetric) coordinates. The derivation was effected in the energy equation. It is a follow-up on the work published by Nouri-Borujerdi (2011). His was carried out in one-dimensional Cartesian coordinate. The model is to be used to simulate transient compressible adiabatic gas flow in a long pipeline. Viscous flows are to be considered. An explicit finite element method is to be employed for the simulation. Trials will also be made using implicit forms. The second part of this work, to be presented later, will border on the solution and validation of the model.

Nomenclature

V_r	velocity in the radial (r) direction, m/s
V_z	velocity in the axial (z)direction, m/s
V_θ	velocity in the azimuthal direction
r	radial direction, m
z	axial direction, m
T	temperature, K
t	time, s
E	total internal energy, J
k	thermal conductivity, W/m.K
P	pressure, N/m^2
C_p	constant pressure specific heat capacity, J/KgK
q	heat flux, W/m^2
g_r, g_z	acceleration due to gravity in the r and z directions respectively
$\bar{Z}$	average value for compressibility factor
R	gas constant

Greek Characters

μ	dynamic viscosity, Ns/m^2
ρ	density, kg/m^3
τ	shear stress tensor, N/m^2
θ	azimuthal direction

THE MODEL EQUATIONS

Continuity Equation in cylindrical coordinates, r and z

$$\frac{\partial \rho}{\partial t} + \frac{1}{r}\frac{\partial}{\partial r}(\rho r V_r) + \frac{\partial}{\partial z}(\rho V_z) = 0 \quad \ldots\ldots (1)$$

Expanding the equation above, we have;

$$\frac{\partial \rho}{\partial t} + \rho \frac{\partial V_r}{\partial r} + \rho \frac{V_r}{r} + V_r \frac{\partial \rho}{\partial r} + \rho \frac{\partial V_z}{\partial z} + V_z \frac{\partial \rho}{\partial z} = 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots..(2)$$

Equation of motion in terms of $\boldsymbol{\tau}$ gives in the r-direction:

$$\rho\left(\frac{\partial V_r}{\partial t} + V_r \frac{\partial V_r}{\partial r} + V_z \frac{\partial V_r}{\partial z}\right) = -\frac{\partial P}{\partial r} - \left[\frac{1}{r}\frac{\partial}{\partial r}(r\tau_{rr}) + \frac{\partial}{\partial z}\tau_{zr} - \frac{\tau_{\theta\theta}}{r}\right] + \rho g_r \ldots\ldots (3)$$

$$\tau_{rr} = -\mu\left(2\frac{\partial V_r}{\partial r}\right) + \frac{2}{3}\mu(\nabla . v) \ldots\ldots\ldots\ldots. (4)$$

$$\tau_{zr} = \tau_{rz} = -\mu\left(\frac{\partial V_r}{\partial z} + \frac{\partial V_z}{\partial r}\right) \ldots\ldots\ldots\ldots\ldots\ldots (5)$$

$$\nabla . v = \frac{1}{r}\left(\frac{\partial}{\partial r}\right)(rV_r) + \frac{\partial V_z}{\partial z} \ldots\ldots\ldots\ldots\ldots\ldots. (6)$$

$$\tau_{\theta\theta} = -\mu\left[2\left(\frac{1}{r}\frac{\partial V_\theta}{\partial \theta} + \frac{V_r}{r}\right)\right] + \frac{2}{3}\mu[\nabla . v] \ldots\ldots\ldots\ldots\ldots\ldots\ldots..(7)$$

Similarly, equation of motion in terms of $\boldsymbol{\tau}$ in the z-direction:

$$\rho\left(\frac{\partial V_z}{\partial t} + V_r \frac{\partial V_z}{\partial r} + V_z \frac{\partial V_z}{\partial z}\right) = -\frac{\partial P}{\partial z} - \left[\frac{1}{r}\frac{\partial}{\partial r}(r\tau_{rz}) + \frac{\partial}{\partial z}\tau_{zz}\right] + \rho g_z \ldots\ldots\ldots. (8)$$

$$where \qquad \tau_{zz} = -\mu\left(2\frac{\partial V_z}{\partial z}\right) + \frac{2}{3}\mu(\nabla . v) \ldots\ldots\ldots\ldots. (9)$$

$$\tau_{zr} = \tau_{rz} = -\mu\left(\frac{\partial V_r}{\partial z} + \frac{\partial V_z}{\partial r}\right) \ldots\ldots\ldots\ldots..(10)$$

Energy Equation in r and z direction

$$\rho C_p\left(\frac{\partial T}{\partial t} + V_r \frac{\partial T}{\partial r} + V_z \frac{\partial T}{\partial z}\right) = -\left[\frac{1}{r}\frac{\partial}{\partial r}(rq_r) + \frac{\partial q_z}{\partial z}\right] - \left(\frac{\partial ln\rho}{\partial lnT}\right)_p \frac{DP}{Dt} - (\tau : \nabla_v) \ldots (11)$$

$$where \qquad q_z = -k\frac{\partial T}{\partial z} \; and \; q_r = -k\frac{\partial T}{\partial r} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (12)$$

$$\tau : \nabla_v = \tau_{rr}\left(\frac{\partial V_r}{\partial r}\right) + \tau_{rz}\left(\frac{\partial V_r}{\partial z}\right) + \tau_{zr}\left(\frac{\partial V_z}{\partial r}\right) + +\tau_{zz}\left(\frac{\partial V_z}{\partial z}\right) \ldots\ldots. (13)$$

The total derivative

$$\frac{DP}{Dt} = \frac{\partial P}{\partial t} + V_r \frac{\partial P}{\partial r} + V_z \frac{\partial P}{\partial z} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (14)$$

Substituting for (12) and (13) in (11) gives

$$\rho C_p\left(\frac{\partial T}{\partial t} + V_r \frac{\partial T}{\partial r} + V_z \frac{\partial T}{\partial z}\right) = -\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\left(-k\frac{\partial T}{\partial r}\right)\right) + \frac{\partial}{\partial z}\left(-k\frac{\partial T}{\partial z}\right)\right] - \left(\frac{\partial ln\rho}{\partial lnT}\right)_p \left(\frac{\partial P}{\partial t} + V_r \frac{\partial P}{\partial r} + V_z \frac{\partial P}{\partial z}\right) - (\tau : \nabla_v) \ldots\ldots\ldots (15)$$

Expansion of the first and second terms on the right hand side of equation (15) gives

$$\rho C_p\left(\frac{\partial T}{\partial t} + V_r \frac{\partial T}{\partial r} + V_z \frac{\partial T}{\partial z}\right) = k\frac{\partial^2 T}{\partial r^2} + \frac{k}{r}\frac{\partial T}{\partial r} + k\frac{\partial^2 T}{\partial z^2} - \left(\frac{\partial ln\rho}{\partial lnT}\right)_p \left(\frac{\partial P}{\partial t} + V_r \frac{\partial P}{\partial r} + V_z \frac{\partial P}{\partial z}\right) - (\tau : \nabla_v) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots..(16)$$

Equation of state to be employed

$$P = Z\rho RT \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (17)$$

A substantial derivative of equation (17) gives (with an average value for Z) is as follows

$$\frac{\partial P}{\partial t} + V_r \frac{\partial P}{\partial r} + V_z \frac{\partial P}{\partial z} = \bar{Z}R\left[\frac{\partial \rho T}{\partial t} + V_r \frac{\partial \rho T}{\partial r} + V_z \frac{\partial \rho T}{\partial z}\right] \ldots\ldots\ldots\ldots\ldots\ldots..(18)$$

$$\frac{\partial \rho T}{\partial t} + V_r \frac{\partial \rho T}{\partial r} + V_z \frac{\partial \rho T}{\partial z} = T\left(\frac{\partial \rho}{\partial t} + V_r \frac{\partial \rho}{\partial r} + V_z \frac{\partial \rho}{\partial z}\right) + \rho\left(\frac{\partial T}{\partial t} + V_r \frac{\partial T}{\partial r} + V_z \frac{\partial T}{\partial z}\right) \ldots\ldots\ldots\ldots..(19)$$

From the continuity equation,

$$\frac{\partial \rho}{\partial t} + V_r \frac{\partial \rho}{\partial r} + V_z \frac{\partial \rho}{\partial z} = -\left[\rho\frac{\partial V_r}{\partial r} + \rho\frac{V_r}{r} + \rho\frac{\partial V_z}{\partial z}\right] \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots..(20)$$

Substituting for the terms in the first bracket of the right hand side of equation (19) above gives

$$\frac{\partial \rho T}{\partial t} + V_r \frac{\partial \rho T}{\partial r} + V_z \frac{\partial \rho T}{\partial z} = T(-\rho)\left(\frac{\partial V_r}{\partial r} + \frac{V_r}{r} + \frac{\partial V_z}{\partial z}\right) + \rho\left(\frac{\partial T}{\partial t} + V_r \frac{\partial T}{\partial r} + V_z \frac{\partial T}{\partial z}\right) \ldots\ldots\ldots\ldots (21)$$

Replacing the terms in the bracket on the right hand side of equation (18) with those on the right hand side of equation (21) above gives

$$\frac{\partial P}{\partial t} + V_r \frac{\partial P}{\partial r} + V_z \frac{\partial P}{\partial z} = \bar{Z}R\left[T(-\rho)\left(\frac{\partial V_r}{\partial r} + \frac{V_r}{r} + \frac{\partial V_z}{\partial z}\right) + \rho\left(\frac{\partial T}{\partial t} + V_r \frac{\partial T}{\partial r} + V_z \frac{\partial T}{\partial z}\right)\right]$$

This on rearrangement gives

$$\frac{\partial P}{\partial t} + V_r \frac{\partial P}{\partial r} + V_z \frac{\partial P}{\partial z} = -\bar{Z}RT\rho\left(\frac{\partial V_r}{\partial r} + \frac{V_r}{r} + \frac{\partial V_z}{\partial z}\right) + \bar{Z}R\rho\left(\frac{\partial T}{\partial t} + V_r \frac{\partial T}{\partial r} + V_z \frac{\partial T}{\partial z}\right) \ldots\ldots\ldots (22)$$

Replacing the total derivative of pressure in equation (16) with its value in equation (22), we have;

$$\rho C_p \left(\frac{\partial T}{\partial t} + V_r \frac{\partial T}{\partial r} + V_z \frac{\partial T}{\partial z}\right) = k\frac{\partial^2 T}{\partial r^2} + \frac{k}{r}\frac{\partial T}{\partial r} + k\frac{\partial^2 T}{\partial z^2} - \left(\frac{\partial ln\rho}{\partial lnT}\right)_p \left[-\bar{Z}RT\rho\left(\frac{\partial V_r}{\partial r} + \frac{V_r}{r} + \frac{\partial V_z}{\partial z}\right) + \bar{Z}R\rho\left(\frac{\partial T}{\partial t} + V_r \frac{\partial T}{\partial r} + V_z \frac{\partial T}{\partial z}\right)\right] - (\tau : \nabla_v) \ldots\ldots\ldots\ldots\ldots (23)$$

Let

$$-\left(\frac{\partial ln\rho}{\partial lnT}\right)_p = \beta$$

On rearrangement, equation (23) gives

$$\left(\rho C_p - \beta\frac{P}{T}\right)\left[\frac{\partial T}{\partial t} + V_r \frac{\partial T}{\partial r} + V_z \frac{\partial T}{\partial z}\right] + \beta P\left[\frac{\partial V_r}{\partial r} + \frac{V_r}{r} + \frac{\partial V_z}{\partial z}\right] = k\frac{\partial^2 T}{\partial r^2} + \frac{k}{r}\frac{\partial T}{\partial r} + k\frac{\partial^2 T}{\partial z^2} - -(\tau : \nabla_v) \ldots\ldots\ldots\ldots\ldots\ldots\ldots (24)$$

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