Available online at http://www.journalcra.com

International Journal of Current Research
Vol. 7, Issue, 11, pp.22448-22450, November, 2015

INTERNATIONAL JOURNAL OF CURRENT RESEARCH

ISSN: 0975-833X

RESEARCH ARTICLE

COMMON FIXED POINT THEOREM IN BANACH SPACE

*,[1]Aradhana Sharma and [2]Gauri Shanker Sao

[1]Department of Mathematics, Govt. Bilasa Girls P.G.College, Bilaspur (C.G.)
[2]Department of Mathematics, Govt.ERR P G Science College, Bilaspur (C.G.)

ARTICLE INFO

Article History:
Received 23rd August, 2015
Received in revised form
05th September, 2015
Accepted 19th October, 2015
Published online 30th November, 2015

Key words:
Fixed point, Contractive map, Contraction map, Banach space

AMS Subject classification:
Primary 47H10, Secondary 54H25.

ABSTRACT

In this paper, we establish the generalization of T-contractive type of mappings on Banach space. In (Huang and Zhang, 2007) Huang and Zhang generalized the concept of metric spaces, replacing the set of real numbers by an ordered Banach space.

Citation: Aradhana Sharma and Gauri Shanker Sao, 2015. "Common fixed point theorem in banach space", *International Journal of Current Research*, 7, (11), 22448-22450.

INTRODUCTION

Recently Beiranvand *et al*. (2009) introduced a new class of contractive mappings. T- contraction and T-contractive extended by the Banach's contraction principle and the Edelstein fixed point theorem.

2. PRELIMINIES

Definition 2.1:A norm on X is a real-valued function $\|.\|: X \to R$ defined on X such that for any x, y ϵX and for $\lambda \epsilon K$

(i) $\|x\|=0$ only if $x=0$
(ii) $\|x+y\| \leq \|x\|+\|y\|$
(iii) $\|\lambda x\|=|\lambda|\|x\|$

Definition 2.2:Normed linear space is a pair $(X,\|.\|)$ consisting of a linear space X and a norm $\|.\|$

Definition 2.3: A sequence $\{x_n\}$ in a nls X is a Cauchy sequence if for any given $\varepsilon>0$ there exits an $n_0 \epsilon N$ such that $\|x_m-x_n\| < \varepsilon$ for $m,n \geq n_0$

Definition 2.4: A norm linear space X is said to be complete if every Cauchy sequence in X converges to an elements of X.

Definition 2.5: A Banach space $(X,\|.\|)$ is a complete nls.

Definition 2.6 : The Banach fixed point theorem stated that each self mapping T of a complete metric space (X, d) such that $d(Tx,Ty)< kd(x,y)$, $(x \neq y, 0<k<1)$ has a unique fixed point. the assumption $k < 1$

****Corresponding author: Aradhana Sharma,***
Department of Mathematics, Govt. Bilasa Girls P.G.College, Bilaspur (C.G.)

is nonsuperfluous with k=1 the mapping of this sort need not have a fixed point, however X is compact then T has a unique fixed point.

Definition 2.7 : Let X be normed linear space and d(x,y) = ‖x-y ‖ ,x , y ϵ X. If X is complete with respect to the metric d(x,y) , X is said to be Banach space.

3. MATERIALS AND METHODS

Theorem: Let CB(X) be closed Banach space where X itself is Banach space, if T : X → X , R, S : X → X satisfied

$\|TR^{2n+1}x_0 - TR^{2n+2}x_0\|$ =0 and $\|TS^{2n+1}x_0 - TS^{2n+2}x_0\|$ =0

such that there exist subsequences { $R^{2n+1}x_0$ } , { $S^{2n+2}x_0$ } then Rx = x= Sx.

So they have common fixed point.

4. RESULT AND DISCUSION

PROOF: $\|Tx_1 - Tx_2\| \le \|Tx_1 - TRx_1\| + \|TRx_1 - TRx_2\| + \|TRx_2 - Tx_2\|$

$\le \|Tx_1 - TRx_1\| +$ a $\|Tx_1 - Tx_2\| + \|TRx_2 - Tx_2\|$

$\Rightarrow \|Tx_1 - Tx_2\| \le \frac{1}{1-a} [\|Tx_1 - TRx_1\| + \|TRx_2 - Tx_2\|]$

By considering $x_{2n+2} = Rx_{2n+1} = R^{2n+1}x_0$

And $x_{2n+3} = Sx_{2n+2} = S^{2n+2}x_0$

Therefore $\|Tx_{2n+1} - Tx_{2n+2}\| = \|TR^{2n+1}x_0 - TR^{2n+2}x_0\|$

$\le$ a $\|TR^{2n}x_0 - TR^{2n+1}x_0\|$

$\Rightarrow \|TR^{2n+1}x_0 - TR^{2n+2}x_0\| \le a^{2n+1} \|Tx_0 - TRx_0\|$

Similarly $\|TS^{2n+1}x_0 - TS^{2n+2}x_0\| \le b^{2n+1} \|Tx_0 - TSx_0\|$

Hence $\|TR^{2n+1}x_0 - TR^{2n+2}x_0\|$ =0

Also $\|Tx_{2n+1} - Tx_{2m+1}\| = \|TR^{2n+1}x_0 - TR^{2m+1}x_0\|$

$\le \frac{1}{1-a} [\|TR^{2n+1}x_0 - TR^{2n+2}x_0\| + \|TR^{2n+2}x_0 - TR^{2n+3}x_0\| + \ldots\ldots\ldots\ldots]$

$\le \frac{1}{1-a} [a^{2n+1} \|Tx_0 - TRx_0\| + a^{2n+2} \|Tx_0 - TRx_0\| + \ldots\ldots\ldots\ldots\ldots\ldots\ldots]$

$\le \frac{1}{1-a} [a^{2n+1} + a^{2n+2} + \ldots\ldots\ldots\ldots\ldots\ldots] \|Tx_0 - TRx_0\|$

→ $\|TR^{2n+1}x_0 - TR^{2m+1}x_0\|$ =0 (Since $\|Tx_0 - TRx_0\|$ =0)

→ $R^{2n+1}x_0$ = u

→ T $R^{2n+1}x_0$ = Tu = 0

theorem is completed.

5. Conclusion

In this article we have proved the existence of a fixed point in a Banach space and shows that the fixed point is unique.

6. Acknowledgements

The authors are thankful to the reviewers for their valuable suggestions to enhance the quality of our article.

7. REFERENCES

Beiranvand A., S. Moradi, M. Omid, H. Pazandeh: Two fixed point theorem for special mapping, arXiv0903-1504vI (2009) p.1-6

Huang L.G., X. Zhang: Cone metric space and fixed point theorem of T-contractive mappings, *J.Math.Anal.Appl*., 332 No.2 (2007)1468-76.

Naidu, S.V.R. and Prasad, J.R.: Ishikawa iterates for a pair of maps, *Ind. J. Pure Appl. Math*., 17 (1986), 198-200.

Naimpally, S.A. and Singh, K.L.: Extension of some fixed point theorems of Rhoades, *Math. Anal. Appl*., 96(1983), 437-446.

Rhoades, B.E.: A general principle for Mann iterates, *Indian Jou. of Pure and Appl. Math*., 26 (1995),751-762 .

Rhoades, B.E.: Some fixed point iteration procedure, *Int. J. Math. and Math. Sci*., 14(1991),1-16.

Sao, G.S. and Gupta S.N.: Common fixed point theorem in Hilbert space for rational expression, *Impact Jour.of Sci.and Tech*., vol.4, 2010, P.B. No. 1889 Lautoka Fiji Island, p.39-41

Sao, G.S. and Sharma Aradhana: Generalization of common fixed point theorems of Naimpally and singh in Hilbert space, Acta Ciencia indica, 2008, 34(4), p.1733-34.

Sao, G.S.: Common fixed point theorem for compability on Hilbert space, *Applied Sci. Periodical*., vol.9(1), Feb.07, p.27-29

****