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REVIEW ARTICLE

A NEW APPROACH TO SOLVE TRAVELLING SALESMAN PROBLEM UNDER FUZZY ENVIRONMENT

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ABSTRACT

In this paper a new approach is proposed to solve travelling salesman problem under fuzzy environment. The travelling distance d_{ij} from the city i to city j is considered to be not certain and is taken as triangular fuzzy numbers. In the proposed method a yager's ranking technique is used to transform fuzzy travelling salesman problem into crisp travelling salesman problem and it is solved by using classical Hungarian method. Numerical examples are given to validate the proposed method.

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1. INTRODUCTION

Travelling salesman problem (TSP) is a special type of linear programming problem in which the objective is to minimize the total travelling cost (time). Travelling salesman problem plays an important role in industry and other applications. Travelling salesman problem is similar to assignment problem except that there is an additional restriction. The aim of travelling salesman problem is to find the cheapest way of visiting all elements in a given set of cities where the cost (time) of travel between each pair of them is given, including the return to starting point. There are different approaches for solving travelling salesman problems. Linear programming method, heuristic methods like cutting plan algorithms and branch and bound method, markov chain, simulated annealing and tabu search methods. Few more algorithms like particle swarm optimization, neural networks, evolutionary computations, ant system, artificial be colony, etc., are also there. The basic concepts of fuzzy sets and its applications were discussed by Dubois and Prade, (1980). In recent years, Fuzzy TSP has got great attention and the problems in Fuzzy TSP have been approached using several technique.

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The Fuzzy TSP has been solved for LR-fuzzy parameters by Amit kumar and Anil gupta (Amit kumar and Anil gupta, 2012), Mukerjee and Basu (Mukherjee and Basu, 2010), developed a new method to solve fuzzy TSP. In the work of Chaudhuri *et al*. (2011), the fuzzy linear programming was used to solve the problem. Buckly, (1988) applied the concept of triangular fuzzy numbers in linear programming problems. Ranking methods for fuzzy numbers have been developed by many authors in recent years (Chen, 1985; Chen and Klein, 1997; Choobinesh and Li, 1993; Liou and Wang, 1992). Kikuchi, (2000) developed the method of defuzzification of fuzzy numbers. H.W.Kuhn (10) introduced an algorithm called as Hungarian method for solving assignment problems in crisp environment. Chen (1985) solved the assignment problem with fuzzy cost. In this paper, the discussion may be done on more realistic problem, namely travelling salesman problem with fuzzy cost (time) $\tilde{d}_{ij}$. Since the objective is to minimize the total travelling cost (time) subject to some crisp constraints, the objective function is considered under fuzzy environment. In the proposed method, Yager's ranking technique (Yager, 1986) has been adopted to transform the fuzzy travelling salesman problem to a crisp one so that the conventional solution method called "Hungarian method" may be applied to solve travelling salesman problem.

2. Preliminaries

Definition 2.1: The membership grade corresponds to the degree to which an element is compatible with the concept represented by fuzzy set.

Definition 2.2: Let X denote a universal set. Then the characteristic function which assigns certain values or a membership grade to the elements of this universal set within a specified range (0,1) is known as membership function & the set thus defined is called a fuzzy set.

Definition 2.3: Let X denote a universal set. Then the membership function μ_A by using a fuzzy set A is usually denoted as $\mu_A : X \to I$, where I = (0,1)

Definition 2.4: An α -cut of a fuzzy set A is a crispest A^α that contains all the elements of the universal set X that have a membership grade in A greater or equal to specified value of α . Thus

$$A^\alpha = \{x \in X, \mu_A(x) \geq \alpha\}, 0 \leq \alpha \leq 1$$

Definition 2.5: A triangular membership function is specified by three parameters (a,b,c) as follows

$$\mu\,(\text{x:a,b,c})=\begin{cases}(x-a)/(b-a), a \leq x \leq b \\ 1 & , x = b \\ (c-x)/(c-b), b \leq x \leq c \\ 0 & , otherwise\end{cases}$$

This function is determined by the choice of the parameter a, b, c where $x_{ij} \in [0,1]$

Definition 2.6

A trapezoidal fuzzy number $\widetilde{A} = (a,b,c,d)$ is a fuzzy number with membership function of the form

$$\mu(x:a,b,c,d) = \begin{cases}(x-a)/(b-a) & , a \leq x \leq b \\ 1 & , \; b \leq x \leq c \\ (d-x)/(d-c) & , \; c \leq x \leq d \\ 0 & , \; otherwise\end{cases}$$

3. Yager's ranking method

The Yager's ranking (13) is defined as

$$r(\widetilde{d}) = \int_0^1 0.5\left(d_\alpha{}^L + d_\alpha{}^U\right) d\alpha$$

where d_α^L = Lower α- level cut, d_α^U = Upper α- level cut.

Yager's ranking technique satisfies compensation, linearity and additive property which provides results that are consistent with human intuition. If r (s) ≤ r(i) then s ≤ i .

4. Formulation of Fuzzy Travelling Salesman Problem

The travelling salesman problem deals with finding shortest path in a n-city where each city is visited exactly once. The travelling salesman problem is similar to assignment problem that excludes sub paths. Specifically in an n-city situation define

$$x_{ij} = \begin{cases}1 \;, if\ city\ j\ is\ reached\ from\ city\ i \\ 0 \;, otherwise\end{cases}$$

The distance matrix d_{ij} for TSP is formulated as

	City1	City2	City3	…	Cityn
City1	∞	d_{12}	d_{13}	…	d_{1n}
City2	d_{21}	∞	d_{23}	…	d_{2n}
City3	d_{31}	d_{32}	∞	…	d_{3n}
⋮	⋮	⋮	⋮	⋮	⋮
Cityn	d_{n1}	d_{n2}	d_{n3}	…	∞

where d_{ij} is the distance from city i to city j. Mathematically TSP can be stated as

Minimize $z = \sum_{i=1}^{n}\sum_{j=1}^{n} d_{ij}x_{ij}, \quad d_{ij} = \infty \; for\ all\ i = j$

Subject to $\sum_{j=1}^{n} x_{ij} = 1 \quad, i = 1,2...n$

$$\sum_{i=1}^{n} x_{ij} = 1 \quad, j = 1,2...n \quad , x_{ij} \in [0,1]$$

The fuzzy travelling salesman problem can be formulated with the help of membership function. Here the distance $\widetilde{d}_{ij}$ is taken as triangular fuzzy number. When the distance $\widetilde{d}_{ij}$ are fuzzy numbers, then the total distance becomes a fuzzy number

$$\widetilde{Z}^* = Min\,\widetilde{Z} = \sum_{i=1}^{n}\sum_{j=1}^{n} \widetilde{d}_{ij}x_{ij}$$

The above fuzzy travelling salesman problem cannot be solved directly. Defuzzification of fuzzy cost coefficient into crisp one is done by the ranking method. By applying ranking method given in section 3, the objective function of the travelling salesman problem is defuzzified as

$$\mathrm{r}\left(\widetilde{\mathrm{Z}}^*\right) = \mathrm{Min}\ \widetilde{\mathrm{Z}} = \sum_{\mathrm{i=1}}^{\mathrm{n}}\sum_{\mathrm{j=1}}^{\mathrm{n}} \mathrm{r}\left(\widetilde{\mathrm{d}}_{\mathrm{ij}}\right)\mathrm{x}_{\mathrm{ij}}$$

Thus the fuzzy travelling salesman problem is transformed into crisp travelling salesman problem using yager's Ranking technique.

Then the crisp travelling salesman problem is solved by classical Hungarian method and the results are interpreted.

5. Numerical Example

Let us consider a fuzzy travelling salesman problem with five cities A,B,C,D,E. The distance matrix $\left[\widetilde{d}_{ij}\right]$is given whose elements are triangular fuzzy numbers. A salesman must travel from city to city to maintain his accounts. The problem is to find the optimal assignment, so that the assignment minimize the total distance of visiting all cities and return to starting city.

$$\left[\widetilde{d}_{ij}\right]=\begin{bmatrix} \infty & (2,3,4) & (5,6,7) & (2,3,7) & (1,3,4) \\ (3,4,6) & \infty & (3,5,6) & (2,3,8) & (2,4,5) \\ (6,7,8) & (5,6,7) & \infty & (4,6,7) & (4,5,7) \\ (2,3,12) & (2,3,4) & (6,6,7) & \infty & (5,6,7) \\ (1,3,4) & (3,3,4) & (2,4,5) & (6,7,8) & \infty \end{bmatrix}$$

The fuzzy travelling salesman problem can be formulated in the following mathematical programming form

$$Min\left\{\begin{array}{l} r(2,3,4)x_{12}+r(5,6,7)x_{13}+r(2,3,7)x_{14}+r(1,3,4)x_{15} \\ +r(3,4,6)x_{21}+r(3,5,6)x_{23}+r(2,3,8)x_{24}+r(2,4,5)x_{25} \\ +r(6,7,8)x_{31}+r(5,6,7)x_{32}+r(4,6,7)x_{34}+r(4,5,7)x_{35} \\ +r(2,3,12)x_{41}+r(2,3,4)x_{42}+r(6,6,7)x_{43}+r(5,6,7)x_{45} \\ +r(1,3,4)x_{51}+r(3,3,4)x_{52}+r(2,4,5)x_{53}+r(6,7,8)x_{54} \end{array}\right\}$$

subject to

$$x_{12}+x_{13}+x_{14}+x_{15}=1, x_{21}+x_{31}+x_{41}+x_{51}=1$$
$$x_{21}+x_{23}+x_{24}+x_{25}=1, x_{12}+x_{32}+x_{42}+x_{52}=1$$
$$x_{31}+x_{32}+x_{34}+x_{35}=1, x_{13}+x_{23}+x_{43}+x_{53}=1$$
$$x_{41}+x_{42}+x_{43}+x_{45}=1, x_{14}+x_{24}+x_{34}+x_{54}=1$$
$$x_{51}+x_{52}+x_{53}+x_{54}=1, x_{15}+x_{25}+x_{35}+x_{45}=1$$

Now, defuzzifying the distance d_{12}=(2,3,4) using Yager's ranking method. The membership function of triangular fuzzy number (2,3,4) is

$$\mu(x)=\begin{cases} (x-2)/1 & ,2\le x\le 3 \\ 1 & ,x=3 \\ (4-x)/1 & ,3\le x\le 4 \\ 0 & ,otherwise \end{cases}$$

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The α-cut of the fuzzy number (2,3,4) is

$$\left[d_1^{(\alpha)},d_2^{(\alpha)}\right]=\left[\alpha+2,4-\alpha\right]$$

Now the rank of d_{12} is

$$r(d_{12}) = r(2,3,4) = \int_0^1 0.5\left(d^1_{(\alpha)}+d^2_{(\alpha)}\right)d\alpha$$

$$=\int_0^1 0.5(6)d\alpha$$

$$r(d_{12}) = 3$$

In similar manner, the remaining distances are defuzzified using Yager's ranking method and are given below

$$r(d_{13})=6, r(d_{14})=3.75, r(d_{15})=2.75$$
$$r(d_{21})=4.25, r(d_{23})=4.75, r(d_{24})=4$$
$$r(d_{25})=3.75, r(d_{31})=7, r(d_{32})=6$$
$$r(d_{34})=5.25, r(d_{35})=5.75, r(d_{41})=5$$
$$r(d_{42})=3, r(d_{43})=6.5, r(d_{45})=6$$
$$r(d_{51})=2.75, r(d_{52})=4.25, r(d_{53})=3.75,$$
$$r(d_{54})=7$$

the crisp value of the distances, form the distance matrix is solved by Hungarian method. The optimal solution is obtained as

$$B\to C\to D\to B, A\to E\to A$$

This is not a complete solution, so the next best solution can be obtained by choosing next minimum value in the matrix, which does not affect the extra restriction. Now the resulting optimal solution is obtained as

$$A\to E\to C\to D\to B\to A$$

Optimal distance is 19 and corresponding fuzzy optimal distance is (12, 20, 26)

6. Conclusion

In this paper, a new method is proposed to solve fuzzy travelling salesman problem, which occurs in real life situations. The ranking method is simple and suitable for all type of optimization problem like project scheduling and transportation problem. The different types of membership function can also be used instead of triangular membership function.

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