Available online at http://www.journalcra.com

INTERNATIONAL JOURNAL OF CURRENT RESEARCH

International Journal of Current Research
Vol. 8, Issue, 01, pp.25269-25276, January, 2016

ISSN: 0975-833X

RESEARCH ARTICLE

AN IMPROVED RATIO ESTIMATOR OF POPULATION MEAN AT CURRENT OCCASION

*Sandeep

Centre for Women's Development Studies, New Delhi-01, India

ARTICLE INFO

Article History:
Received 15th October, 2015
Received in revised form
25th November, 2015
Accepted 09th December, 2015
Published online 31st January, 2016

Key words:

Successive sampling,
Ratio method of estimation.

ABSTRACT

The present paper deals with the estimation of finite population meanusing ratio method of estimation when two phase sampling scheme is used. In the present work we have proposed a pooled estimator for population mean at the current occasions by using ratio type estimator involving a suitably chosen scalarθ. We have calculated its variances upto the first order of approximations and obtained the optimum replacement policy. We have also investigated the efficiency of the proposed estimator compared with other conventional estimators with and without cost considerations.

Citation: **Sandeep, 2016.** "An improved ratio estimator of population mean at current occasion", *International Journal of Current Research,* 8, (01), 25269-25276.

INTRODUCTION

In many situations the study character of a finite population changes over time. A survey over a single occasion does not provide any information on the nature or rate of change of the characteristic or the average value of the characteristic at the current occasion. To meet this requirement, a portion of the sample taken on the previous occasion is retained and a few new units of the population are selected from the population to complete the sample. This sampling is known as successive sampling and it provides with a strong tool to obtain reliable estimates of population parameter at the current occasion. Theory of successive sampling started with the work of Jessen (1942) where the he collected information on the previous occasions and later this theory was extended by the Patterson (1950), Rao and Graham(1964), Gupta(1979), Das(1982), among others. Sen (1971, 1972,1973), Singh *et al.,* (1991), Singh and Singh(2001), Singh(2003), Feng and Zou(1997), Biradar and Singh (2001), Singh(2005), Singh and Priyanka (2006, 2008) used the auxiliary information on current or on both the occasions to estimate the population mean at the current occasions in two occasions successive sampling. Generally the regression estimatoris used to formulate the first estimator from the information available from the sample taken on the previous occasion and formed the matched (common) portion of the sample selected on the second occasion.

However, in many practical situations it is more favourable to use the ratio estimator not only on the grounds of efficiency but also on account of ease and simplicity in its calculations. Another advantage of using ratio estimator over regression estimator is that the optimum matched portion, the portion which minimizes the variance of the pooled estimator, is larger in case of ratio estimator than of optimum matched portion in case of the regression estimator. In this present work we have proposed a pooled estimator for population mean at the current occasions by using ratio type estimator involving a suitably chosen scalarθ. We have calculated its variances upto the first order of approximations and obtained the optimum replacement policy. We also have investigated the efficiency of the proposed estimator compared with other conventional estimators with and without cost considerations.

Formulation of the estimator

Let $U = U_1, U_2, \cdots, U_N$ be a finite population of size N which is available for sampling over two occasions. The characteristic is denoted by $x(y)$ on the first (second) occasion respectively. It is assumed that the population is considerably large. A simple random sample of size n is selected without replacement from the population on the first occasion. A random sub-sample of$m$ units is retained (matched) out of n units selected at the first occasion for use on the second occasion while a fresh simple random

Corresponding author: Sandeep
Centre for Women's Development Studies, New Delhi-01, India

sample of $u = n - m$ units is selected without replacement from the population of $N - n$ units on the second occasion so that the sample size on the second occasion is also$n$. On the basis of the information available on two occasions, some statistics are defined as follows:

$\bar{x}(n) = (1/n)\sum_{i=1}^{n} x_i$; $\bar{y}(u) = (1/u)\sum_{i=1}^{u} y_i$ are the sample means on the first occasion and unmatched portion of the sample drawn on the second occasion.Similarly,$\bar{x}(m) = (1/m)\sum_{i=1}^{m} x_i$, and $\bar{y}(m) = (1/m)\sum_{i=1}^{m} y_i$ are the sample means based on the matched portion of the sample on the first and second occasions respectively. The corresponding sample variances, covariance and the regression coefficient for the matched portion of the sample are given by:

$$S_x^2(m) = \frac{1}{m-1}\sum_{i=1}^{m} (x_i - \bar{x}(m))^2$$

$$S_y^2(m) = \frac{1}{m-1}\sum_{i=1}^{m} (y_i - \bar{y}(m))^2$$

$$S_{xy}(m) = \frac{1}{m-1}\sum_{i=1}^{m} (x_i - \bar{x}(m))\,(y_i - \bar{y}(m))$$

$$b(m) = \frac{S_{xy}(m)}{S_x^2\,(m)}$$

To estimate the population mean $T = \bar{y}$ on the second occasion, two estimates are suggested. One is the ordinary sample mean i.e.$T(u)$=$\bar{y}(u)$ on the basis of sample of size$n$ drawn afresh on the second occasion and the second

$$T(m) = \frac{\bar{x}(n)\bar{y}(m)}{\bar{x}(n) + \theta[\bar{x}(m) - \bar{x}(n)]} \qquad (1)$$

On combining the estimators $T(u)$ and $T(m)$ we have the final estimator of $T = \bar{y}$

$$\hat{T}_{R\theta} = (1-\phi)T(u) + \phi T(m)$$

$$\hat{T}_{R\theta} = (1-\phi)\bar{y}(u) + \phi\frac{\bar{x}(n)\bar{y}(m)}{\bar{x}(n) + \theta[\bar{x}(m) - \bar{x}(n)]} \qquad (2)$$

Where$0 \le \phi \le 1$ is an unknown constant to be suitably obtained. Note that at $\theta = 1$ the estimator $\hat{T}_{R\theta}$becomes$\hat{T}_R = (1-\phi)\bar{y}(u) + \phi(\bar{y}(m)/\bar{x}(m))\bar{x}(n)$, which is a ratio estimator. It is assumed that sample size is large so that the biases of $T(u)$and $T(m)$can be ignored. Consequently, the bias of $\hat{T}_{R\theta}$ will be negligible. The variances of $\hat{T}_{R\theta}$ is given in the following theorem:

Theorem 2.1. $V(\hat{T}_{R\theta})$ is obtained as

$$V(\hat{T}_{R\theta}) = (1-\phi)^2 V[T(u)] + \phi^2 V\lfloor T(m)\rfloor \qquad (3)$$

Where $V\lfloor T(u)\rfloor = \frac{S_y^2}{u}$

And

$$V\lfloor T(m)\rfloor = \left\lfloor \frac{1-t_\theta}{m} - \frac{t_\theta}{n} \right\rfloor S_y^2 \qquad (5)$$

Where $t_\theta = 2\theta\rho\frac{cv(x)}{cv(y)} - \theta^2\left\lfloor\frac{cv(x)}{cv(y)}\right\rfloor^2$ (6)

Proof: The expression for $V\lfloor T(u)\rfloor$ is regular expression for survey sampling ignoring fpc. The expression for $V\lfloor T(m)\rfloor$ is obtained upto first order of approximation and for large population size N approaching infinity (ignoring fpc). By assuming large sample approximation:

$$\frac{\bar{y}(m) - \bar{Y}}{\bar{Y}} = e_0$$

$\frac{\bar{x}(m)-\bar{X}}{\bar{X}} = e_1$

$$\frac{\bar{x}(m)-\bar{X}}{\bar{X}} = e_1{}'$$

Under the above transformations$T(m)$ takes the following form

$$T(m) = \bar{Y}(1+e_0)\frac{\bar{X}(1+e_1{}')}{\bar{X}(1+e_1{}') + \theta\bar{X}[(1+e_1)-(1-e_1{}')]}$$

after some algebraic manipulation, we get

$$T(m) - \bar{Y} = \bar{Y}[e_0 - \theta(e_1 - e_1{}')]$$

Squaring and taking expectation on both sides, we get

$$V[T(m)] = E[T(m)-\bar{Y}]^2$$
$$= \bar{Y}^2 E\left[e_0^2 + \theta^2\left(e_1^2 + e_1^{2'} - 2e_1e_1{}'\right) - 2\theta(e_0e_1 - e_1e_1{}'\right]$$

After some simplifications, we get

$$V[T(m)] = \frac{S_y^2}{n} + \left(\frac{1}{m} - \frac{1}{n}\right)\left(S_y^2 + \theta^2R^2S_x^2 - 2\theta RS_x\right)$$

where $R = \bar{Y}/\bar{X}$ and $S_{xy} = \rho S_x S_y$. On further simplifications, we get

$$V[T(m)] = \left[\frac{1-t_\theta}{m} - \frac{t_\theta}{n}\right]S_y^2$$

where $t_\theta = 2\theta\rho[cv(x)/cv(y)] - \theta^2[cv(x)/cv(y)]^2$□

Since the $V(\hat{T}_{R\theta})$ given in equation (3) is a function of unknown constant ϕ, therefore, it can be minimized to yield the optimum value of ϕ as:

$$\phi_{\text{opt}} = \frac{V[T(u)]}{V[T(m)] + V[T(u)]} \quad (7)$$

and on substituting the value of ϕ_{opt} in (3)we get the optimum variance of $\hat{T}_{R\theta}$ as

$$V(\hat{T}_{R\theta})_{\text{opt}} = \frac{V[T(u)].V[T(m)]}{V[T(m)] + V[T(u)]}$$

Further substituting the values of (4) and (5) in (7) and (8), we getϕ_{opt} and $V(\hat{T}_{R\theta})_{\text{opt}}$ as shown in the theorem 2.2.

Theorem 2.2

$$\phi_{\text{opt}} = \frac{1-\mu}{1-\mu^2t_\theta} \quad (9)$$

$$T_{\text{opt}} = \frac{1}{1-\mu^2t_\theta}\{\mu(1-\mu t_\theta)T(u) + (1-\mu)T(m)\} \quad (10)$$

$$V(\hat{T}_{R\theta})_{\text{opt}} = \frac{1-\mu t_\theta}{1-\mu^2t_\theta}\frac{S_y^2}{n} \quad (11)$$

Where $\mu = u/n$ is the fraction of fresh sample taken at second (current) occasion.
The optimum value of μ (the fraction of the sample taken a fresh at the second occasion) so that the mean $\bar{y}$ may be estimated with high precision. We minimize $V(\hat{T}_{R\theta})_{\text{opt}}$ in equation (11) with respect to μ, we get

$$\mu_{\text{opt}} = \frac{1}{1+(1-t_\theta)^{1/2}} \quad (12)$$

This gives the optimal estimator under the optimum replacement policy as

$$\left(\hat{T}_{R\theta}\right)_{\text{opt}} = \frac{1}{2}\{T(u) + T(m)\} \quad (13)$$

and the corresponding optimum variance is given by

$$V\left(\hat{T}_{R\theta}\right)_{opt} = \frac{1+(1-t_\theta)^{1/2}}{2n} S_y^2 \quad (14)$$

Comparison of the Proposed Estimator with Conventional Estimator

The proposed estimator defined in section 2 has been compared with the following two conventional estimators, namely regression estimator and product ratio estimator

$$\hat{T}_{lr} = (1-\phi)\bar{y}_u + \phi\{\bar{y}(m) + b(m)(\bar{x}(n) - \bar{x}(m)\} \quad (15)$$

And

$$\hat{T}_{PR\theta} = (1-\phi)\bar{y}_u + \phi\left\{\frac{\bar{y}_m \bar{x}_m}{\bar{x}_n} + (1-\theta)\bar{y}_m\right\}$$
$$= (1-\phi)T(u) + \phi T(m)_{PR\theta} \quad (16)$$

The results relating to the above estimators are summarized in the following theorem:

Theorem 3.1.

$$V(T(m))_{lr} = \left[\frac{1-\rho^2}{m} + \frac{\rho^2}{n}\right] S_y^2 \quad (17)$$

$$V(\hat{T}_{lr}) = \left[\frac{1-\mu\rho^2}{n(1-\mu^2\rho^2)}\right] S_y^2 \quad (18)$$

$$(\mu_{\text{opt}})_{lr} = \frac{1}{1+(1-\rho^2)^{1/2}} \quad (19)$$

$$(\hat{T}_{lr})_{\text{opt}} = \left[\frac{1+(1-\rho^2)^{1/2}}{2n}\right] S_y^2 \quad (20)$$

$$(T(m))_{PR\theta} = \left[\frac{1-t_\theta}{m} - \frac{t_\theta}{n}\right] S_y^2 \quad (21)$$

Where

$$t_\theta = -2\theta\rho\frac{cv(x)}{cv(y)} - \theta^2\left[\frac{cv(x)}{cv(y)}\right]^2 \quad (22)$$

$$(\mu_{\text{opt}})_{PR\theta} = \frac{1}{1+(1-t_\theta)^{1/2}} \quad (23)$$

$$V\left(\hat{T}_{R\theta}\right)_{opt} = \frac{1+(1-t_\theta)^{1/2}}{2n} S_y^2 \quad (25)$$

Theorem 3.2.$(\mu_{R\theta})_{\text{opt}} \leq (\mu_{lr})_{\text{opt}}$

The proof of theorem is easy.
Usually the cost per unit of the information for matched portion is less expensive, therefore, the sample information for the matched portion on the second occasion will be cheaper for the proposed estimator as compared to the conventional estimator.

This establishes the economic efficiency of the proposed estimator in many of the practical situation.Moreover larger matched portion also ensures negligibility of bias in the estimator.

Efficiency and the Cost Efficiency

The sampling efficiency of the proposed estimator as compared to $\hat{T}_{lr}$ is given by

$$e_s(\hat{T}_{R\theta},\hat{T}_{lr})=\frac{V(\hat{T}_{lr})}{V(\hat{T}_{R\theta})}=\frac{1-\mu\rho^2}{1-\mu t_\theta}\frac{1-\mu^2 t_\theta}{(1-\mu^2\rho^2)}$$

For optimum sampling fraction at fresh occasion, it reduces to

$$e_s(\hat{T}_{R\theta},\hat{T}_{lr})_{\text{opt}}=\frac{1+(1-\rho^2)^{1/2}}{1+(1-t_\theta)^{1/2}}$$

Cost Analysis

Consider the total cost on the second occasion, apart from the overhead cost

$$C_2=mC_m+uC_\mu \tag{26}$$

where C_m and C_u are per unit cost matched and unmatched portion of the sample. This can equivalently be written as

$$\frac{C_2}{C_u}=m\delta+u=n(\delta+(1-\delta)\mu) \tag{27}$$

where $\delta=C_m/C_u$.

Following Cochran, 1977 for this cost structure the optimum proportion μ for the proposed estimator $\hat{T}_{R\theta}$ can be obtained by minimizing $V(\hat{T}_{R\theta})C_2$, or equivalently minimizing

$$\frac{V(\hat{T}_{R\theta})C_2}{C_u S_y^2}=\frac{(1-\mu t_\theta)[\delta+(1-\delta)\mu]}{1-\mu^2 t_\theta} \tag{28}$$

with respect to μ, we get

$$(\mu_{R\theta})_{opt}=\frac{-(2\delta-1)\pm[(2\delta-1)^2-\left\{\frac{1-\delta-\delta t_\theta}{t_\theta}\right\}^{1/2}]}{1-\delta-\delta t_\theta} \tag{29}$$

Similarly the optimal unmatched portion for $\hat{T}_{lr}$ with the same cost structure can be obtained by minimizing

$$\frac{V(\hat{T}_{lr})C_2}{C_u S_y^2}=\frac{(1-\rho^2\mu)[\delta+(1-\delta)\mu]}{1-\rho^2\mu^2} \tag{30}$$

with respect to μ to get

$$(\mu_{lr})_{opt}=\frac{-(2\delta-1)\pm[(2\delta-1)^2-\left\{\frac{1-\delta-\delta\rho^2}{\rho^2}\right\}]}{1-\delta-\delta\rho^2} \tag{31}$$

It is important to note that in (29) and (31) only real values of μ are to be considered.
Thus the cost efficiency of the proposed estimator with matching over the estimator with no matching is given by

$$\frac{V(\bar{y}(n))nC_u}{V(\hat{T}_{R\theta})C_2}=\frac{\frac{S_y^2}{n}.n.C_u}{\frac{(1-\mu t_\theta)[\delta+(1-\delta)\mu]}{1-\mu^2 t_\theta}S_y^2 C_u} \tag{32}$$

$$= \frac{1-\mu^2 t_\theta}{(1-\mu t_\theta)[\delta+(1-\delta)\mu]}$$

Further, under the above cost structure, the percent proportional gain of the proposed estimator due to matching over no matching is given by

$$= \frac{\frac{S_y^2}{n} n\, C_u - V(\hat{T}_{R\theta})C_2}{V(\hat{T}_{R\theta})C_2} 100\%$$

$$= \left[\frac{1-\mu^2 t_\theta}{(1-\mu t_\theta)[\delta+(1-\delta)\mu]} - 1\right] 100\% \qquad (33)$$

The gain for $\hat{T}_{lr}$ is given by

$$= \left[\frac{1-\mu^2 \rho^2}{(1-\mu \rho^2)[\delta+(1-\delta)\mu]} - 1\right] 100\% \qquad (34)$$

The percentage proportional gain in efficiency for the proposed estimator due to matching over no matching, when the cost of sampling is ignored, is given by

$$\frac{\frac{S_y^2}{n} - V(\hat{T}_{R\theta})}{V(\hat{T}_{R\theta})} 100\% = \left[\frac{(1-\mu^2 t_\theta)}{(1-\mu t_\theta)} - 1\right] 100\% \qquad (35)$$

This gain for $\hat{T}_{lr}$ is given by

$$= \left[\frac{(1-\mu^2 \rho^2)}{(1-\mu \rho^2)} - 1\right] 100\% \qquad (36)$$

Emprical study of the proposed estimator

From the Table 1 and 6 show the optimum matched portion of the sample, percentage gain in precision with optimum matching compared with no matching and gain in precision with $m/n = 1-\mu = 1/2, 1/3$ and $1/4$ with no matching for the regression and ratio estimators namely $\hat{T}_{lr}$ and $\hat{T}_R$ respectively under the assumption that $CV(X) = CV(Y)$. It can easily be observed that the optimum matched percentage for the estimator $\hat{T}_R$ is larger than for $\hat{T}_{lr}$, it decreases with increase in ρ however it never exceeds 50%. Further, the percentage gain in precision for both the estimators with matching compared with no matching increases with increase in ρ and it reaches at 100% for $\rho = 1$. The optimum matched percentage and percent gain in precision for optimum matching and matching with $m/n = 1-\mu = 1/2, 1/3$ and $1/4$ compared with no matching differs very little if $\rho > 0.8$. This suggests that if the correlation is high, which is generally a case in most of the situations concerning sampling on two occasions, the estimator $\hat{T}_R$ could be used with a very little loss in precision.

Table 1. Optimum Percentage Matched and Percentage Gain in Precision when Regression Estimator is Used

ρ	Optimum matched %	% Gain in precision	$m/n = 1/2$	$m/n = 1/3$	$m/n = 1/4$	% gain
0.6	44	11	10.97	11	9	
0.7	42	17	16.20	17	15	
0.8	38	25	23.52	25	23	
0.9	30	39	34.05	39	39	
0.95	24	52	47.50	50	52	
1.0	0	100	50.00	67	75	

Table 2. Optimum Percentage Matched and Percentage Gain in Precision when Ratio Estimator is Used

ρ	Optimum matched %	% Gain in precision	$m/n = 1/2$	$m/n = 1/3$	$m/n = 1/4$	% gain
0.6	47.21	5.57	5.55	5.13	4.41	
0.7	43.65	12.70	12.50	12.12	10.71	
0.8	38.74	22.51	21.40	22.22	20.45	
0.9	30.90	38.19	33.33	38.09	37.50	
0.95	24.02	51.94	40.90	50.00	51.92	
1.0	0	100	50	66.66	75.00	

Table 3. Proportional Gain in percent when Ratio Estimator is Used

ρ	m/n		
	1/2	1/3	1/4
0.6	0.01635	0.42293	1.11199
0.7	0.17926	0.51771	1.79501
0.8	0.89459	0.23943	1.71045
0.9	3.64749	0.07344	0.50666
1.0	33.3333	20.00000	14.28571

Table 4. Proportional Gain in percent when proposed Estimator is Used, when m/n=1/2

ρ θ	0.5	0.6	0.7	0.8	0.9	1.0
0.1	.0012791	.0023987	.0040684	.0064245	.0096176	.0138141
0.2	.0079064	.0163454	.0299521	.0505393	.0803362	.1221001
0.3	.0191981	.0446245	.0895342	.1633077	.2789946	.4550817
0.4	.0299523	.0803362	.1792581	.3580521	.6669561	1.1904762
0.5	.0343654	.1103629	.2789946	.6187994	1.277470	2.5641026
0.6	.0299527	.1221001	.3580521	.8945598	2.081244	4.8824593
0.7	.0191985	-	.3882336	1.109045	2.949253	8.5387105
0.8	-	-	-	1.190476	3.647450	14.035087
0.9	-	-	-	-	3.919041	22.017517
1.0	-	-	-	-	-	33.333333

Table 5. Proportional Gain in percent when proposed Estimator is Used, when m/n=1/3

ρ θ	0.5	0.6	0.7	0.8	0.9	1.0
0.1	.234768	.2795737	.3214859	.3604025	.3962197	.428833
0.2	.338705	.4439065	.4956692	.5331736	.555643	.562396
0.3	.458139	.5251777	.5588305	.5568165	.517774	.442194
0.4	.495669	.5556448	.5528973	.4844117	.3540277	.181842
0.5	.506413	.5622149	.5177758	.3732652	.1596305	.001256
0.6	.495669	.5623963	.4844117	.2708017	.027377	.264467
0.7	.458139	-	.4712990	.2042172	.0040492	1.51488
0.8	.378705	-	-	.1818427	.0549602	4.50055
0.9	.234768	-	-	-	.0889265	10.1860
1.0	-	-	-	-	-	19.7604

Table 6. Proportional Gain in percent when proposed Estimator is Used, when m/n=1/4

ρ θ	0.5	0.6	0.7	0.8	0.9	1.0
0.1	.5379438	.649954	.7588202	.864334	.9662773	1.0644175
0.2	.9157661	1.111986	1.291496	1.452116	1.591421	1.7067224
0.3	1.158511	1.413860	1.622608	1.775671	1.862560	1.8714234
0.4	1.291496	1.59142	1.795052	1.877532	1.809971	1.56254
0.5	1.333512	1.680304	1.862560	1.827669	1.517683	.9009009
0.6	-	1.706722	1.877532	1.710418	1.116190	.2031144
0.7	-	-	1.877918	1.604155	.7465138	1.4492754
0.8	-	-	-	1.56253	.5066191	5.6545711
0.9	-	-	-	-	.4269664	14.2857143
1.0	-	-	-	-	-	-

Similar results also hold for the proposed estimator $\hat{T}_{R\theta}$ with an observation that its optimum matched percentage and gain in precision with optimum matching as compared with no matching becomes same as for regression estimator $\hat{T}_{lr}$ when $\theta = \rho$. Note that the percentage gain in precision for the proposed estimator $\hat{T}_{R\theta}$ with optimum matching as compared with no matching reaches maximum over the variations in θ at $\theta = \rho$ for each fixed ρ, and this maximum percentage gain equals the corresponding percentage gain for regression estimator $\hat{T}_{lr}$. In other words the proposed estimator $\hat{T}_{R\theta}$ and the regression estimator $\hat{T}_{lr}$ perform equally when $\theta = \rho$ and $CV(X) = CV(Y)$. In addition to the condition $CV(X) = CV(Y)$, if $\rho = 1$ and $\theta = 1$, i.e. line of regression passes through the origin, then the proposed estimator and the regression estimator are equally efficient. However, for $\rho \geq 0.8$ loss in efficiency is not more than 2.13%. If the coefficients of variations are not equal the performance of the proposed estimator may even be better and it may be as efficient as the estimator based on the regression estimator even if ρ is less than unity.

If we incorporate the cost structure in successive sampling where per unit of cost of information collection for matched portion is low, the proposed estimator $\hat{T}_{R\theta}$ is superior to ratio estimator $\hat{T}_R$ since optimum matched percentage for the former is higher than for later for add values of ρ and θ. However, for small values of ρ and θ the matched portion is higher for both the estimators.

Thus, the economic efficiency of the proposed estimator is above 98% for all values of θ and $\rho \geq 0.8$. Therefore, if $\rho \geq 0.8$ and $\theta = 1$, which is normally true in case of sampling of successive occasion, we can recommend the use of the proposed $\hat{T}_{R\theta}$ for all practical purpose on grounds of simplicity and computational case.

REFERENCES

Biradar, R.S.,Singh, H.P. 2001. Successive sampling using auxiliary information on both occasions, *Cal. Statist. Assoc. Bull.*51 : 243-251.

Chaturvedi, D. K., Tripathi, T.P. 1983. Estimation of population ratio on two occasions using multivariate auxiliary information. *J. Ind. Statist.Assoc.,* 21:113-120.

Gonzalez, M.E., Ogus, J.L., Shapiro, O.G., and Tepping, B.J. 1975. "Standards for discussion and presentation of errors in survey and census data " in *Journal of American Statistical Association*, 70(351), part II, 5-23.

Lepkovski, 1998. "Presentation of Sampling Errors", Methods of Survey Sampling/Applied Sampling-Lecture at Statistics South Africa.

Sukhatme, P. V., Sukhatme, B. V., Sukhatme, S., Ashok, C. 1984. *Sampling Theory of Surveys with Applications*, Indian Society of Agricultural Statistics, New Delhi, India.