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RESEARCH ARTICLE

CONVOLUTION THEOREM FOR GENERALIZED TWO DIMENSIONAL FRACTIONAL COSINE TRANSFORM

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ABSTRACT

The Fractional Fourier Transform (FrFT) is a generalization of the ordinary Fourier transform. The ordinary Fourier transform and related techniques are of importance in various different areas like communications, signal processing and control systems. In fact, the FrFT has already found many applications in the areas of signal processing and communications. The success of FrFT in its application has promoted the development of other kinds of fractional transforms like fractional Hartley transform, fractional Hadamard transform, fractional cosine transform and fractional sine transform (FrST). Fractional cosine transform is the extension of cosine transform and it has been widely used in domain of digital signal and image processing. In this paper convolution theorem for generalized two dimensional fractional cosine transform is proved.

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INTRODUCTION

Nowadays there is no doubt the use and applications of the continuous and discrete convolution operations in many branches of science. Moreover, the number of applications is so large that trying to name and count them would take a long time. Some of these applications are in signal and image processing, electric circuits, telecommunications, probability, statistics, etc. All FrCTs and FrSTs possess convolution — multiplication property which is a powerful tool for performing digital filtering in the transform domain. The convolution operation in the transform domain realized by taking an inverse transform of the product of forward transforms of two data sequences is equivalent to symmetric convolution of those symmetrically extended sequences in the spatial domain. Convolution plays a very important role in the theory of integral transform. Almeida (1997) had defined convolution for fractional Fourier transform. Zayed (1998) had revised the definition in order to follow the standard Convolution theorem. In our previous work we already defined following terms.

1.1. Generalized two dimensional fractional Cosine transform

Two dimensional fractional Cosine transform with parameter α f(x, y) denoted by $F_C^{\alpha}(x, y)$ perform a linear operation given by the integral transform.

$$F_C^{\alpha}\{f(x,y)\}(u,v) = \int_0^{\infty}\int_0^{\infty} f(x,y)K_{\alpha}(x,y,u,v)\,dx\,dy \tag{1.1}$$

Where the kernel,

$$K_c^{\alpha}(x,y,u,v) = \sqrt{\frac{1-icot\alpha}{2\pi}}\,e^{\frac{i(x^2+y^2+u^2+v^2)cot\alpha}{2}}\cos(cosec\alpha.ux).\cos(cosec\alpha.vy) \tag{1.2}$$

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The test function space E

An infinitely differentiable complex valued function on R^n belongs to $E(R^n)$ if for each copactset$I \subset S_{a,b}$, where,

$S_{a,b} = \{x, y: x, y \in R^n, |x| \leq a, |y| \leq b, a > 0, b > 0\}, I \in R^n$

$\gamma_{E_{p,q}}(\ ) = \sup_{x,y} |D_{x,y}^{p,q}\ (x, y)| < \infty$Where, p, q =1, 2, 3….

Thus $E(R^n)$ will denote the space of all $\in E(R^n)$ with support contained in $S_{a,b}$

Note that the space E is complete and therefore a Frechet space. Moreover, we say that f is a fractional Cosine transformable, if it is a member of$E$, the dual space of E.

This paper emphasizes to deriving convolution theorem for two dimensional fractional cosine transform and defined distributional two-dimensional fractional Cosine transform

Distributional two-dimensional fractional Cosine transform

The two dimensional distributional fractional Cosine transform of $f(x, y) \in E\ (R^n)$ defined by

$$F_c^\alpha\{f(x, y)\} = F^\alpha(u, v) = f(x, y), K_\alpha(x, y, u, v) \tag{2.1}$$

$$K_c^\alpha(x, y, u, v) = \sqrt{\frac{1 - icot\alpha}{2\pi}}\, e^{\frac{i(x^2+y^2+u^2+v^2)cot\alpha}{2}} \cos(cosec\alpha . ux) . \cos(cosec\alpha . vy) \tag{2.2}$$

Where , RHS of equation (2.1) has a meaning as the application of $f \in E$ to$K_\alpha(x, y, u, v) \in E$

Convolution Theorem

If $(u, v) = (f . g)(u, v)$ and $FC_\alpha, GC_\alpha, HC_\alpha$ denote two dimensional the fractional cosine transform

$f, g,$ Respectively, then $HC_\alpha = FC_\alpha\{f(x, y)\}(u, v)\quad GC_\alpha\{g(t, s)\}(u, v) = \sqrt{\frac{1 - icot\alpha}{2\pi}}\frac{e^{\frac{i}{2}(u^2+v^2)cot\alpha}}{4}$

$$\begin{bmatrix} FC_\alpha\left\{e^{\frac{-i}{2}(\tau^2+\rho^2)cot\alpha}(\bar{f} . \overline{g_1})\right\} + FC_\alpha\left\{e^{\frac{-i}{2}(\tau^2+\xi^2)cot\alpha}(\bar{f} . \overline{g_2})\right\} \\ + FC_\alpha\left\{e^{\frac{-i}{2}(\eta^2+\rho^2)cot\alpha}(\bar{f} . \overline{g_3})\right\} + FC_\alpha\left\{e^{\frac{-i}{2}(\eta^2+\xi^2)cot\alpha}(\bar{f} . \overline{g_4})\right\} \end{bmatrix}$$

Proof: From the definition of two dimensional the fractional cosine transform, we have

$$FC_\alpha\{f(x, y)\}(u, v)\quad GC_\alpha\{g(t, s)\}(u, v) = \begin{matrix} \sqrt{\frac{1\quad icot\alpha}{2\pi}} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{\frac{i}{2}(x^2+y^2+u^2+v^2)cot\alpha} \\ cos(cosec\alpha . ux)cos(cosec\alpha . vy)f(x, y)dxdy \\ \sqrt{\frac{1\quad icot\alpha}{2\pi}} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{\frac{i}{2}(t^2+s^2+u^2+v^2)cot\alpha} \\ cos(cosec\alpha . ut)cos(cosec\alpha . vs)g(t, s)dtds \end{matrix}$$

$$FC_\alpha\{f(x, y)\}(u, v)\quad GC_\alpha\{g(t, s)\}(u, v)$$

$$= \left(\sqrt{\frac{1\quad icot\alpha}{2\pi}}\right)^2 e^{i(u^2+v^2)cot\alpha} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{\frac{i}{2}(x^2+y^2+t^2+s^2)cot\alpha}$$

$$cos(cosec\alpha . ux)cos(cosec\alpha . vy)f(x, y)cos(cosec\alpha . ut)cos(cosec\alpha . vs)g(t, s)dxdydtds$$

$$FC_\alpha\{f(x,y)\}(u,v)\quad GC_\alpha\{g(t,s)\}(u,v) = \left(\sqrt{\frac{1\quad icot\alpha}{2\pi}}\right)^2 e^{i(u^2+v^2)cot\alpha}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{\frac{i}{2}(x^2+y^2+t^2+s^2)cot\alpha} f(x,y)g(t,s)\frac{1}{2}[\cos(csc\alpha.u(x+t))+\cos(csc\alpha.u(x\quad t))]\frac{1}{2}[\cos(csc\alpha.v(y+s))+\cos(csc\alpha.v(y\quad s))]dxdydtds$$

Let $A=\left(\sqrt{\frac{1-icot\alpha}{2\pi}}\right)^2 \quad B=\frac{e^{i(u^2+v^2)cot\alpha}}{4}$

$$FC_\alpha\{f(x,y)\}(u,v)\quad GC_\alpha\{g(t,s)\}(u,v) = AB\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{\frac{i}{2}(x^2+y^2+t^2+s^2)cot\alpha}f(x,y)g(t,s)\begin{bmatrix}\cos(csc\alpha.u(x+t))\cos(csc\alpha.v(y+s))+\cos(csc\alpha.u(x+t))\cos(csc\alpha.v(y\quad s))\\ \cos(csc\alpha.u(x\quad t))\cos(csc\alpha.v(y+s))+\cos(csc\alpha.u(x\quad t))\cos(csc\alpha.v(y\quad s))\end{bmatrix} dxdydtds$$

$FC_\alpha\{f(x,y)\}(u,v)\quad GC_\alpha\{g(t,s)\}(u,v)$
$= AB\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{\frac{i}{2}(x^2+y^2+t^2+s^2)cot\alpha}f(x,y)g(t,s)$
$\cos(csc\alpha.u(x+t))\cos(csc\alpha.v(y+s))\,dxdydtds + AB\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{\frac{i}{2}(x^2+y^2+t^2+s^2)cot\alpha}f(x,y)g(t,s)$
$\cos(csc\alpha.u(x+t))\cos(csc\alpha.v(y\quad s))\,dxdydtds + AB\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \begin{matrix} e^{\frac{i}{2}(x^2+y^2+t^2+s^2)cot\alpha}f(x,y)g(t,s) \\ \cos(csc\alpha.u(x\quad t))\cos(csc\alpha.v(y+s))\end{matrix}\, dxdydtds$

$+ AB\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{\frac{i}{2}(x^2+y^2+t^2+s^2)cot\alpha}f(x,y)g(t,s)$
$\cos(csc\alpha u(x\quad t))\cos(csc\alpha v(y\quad s))\,dxdydtds$

$$FC_\alpha\{f(x,y)\}(u,v)\quad GC_\alpha\{g(t,s)\}(u,v) = I_1+I_2+I_3+I_4 \qquad (3.1)$$

Let $I_1 = AB\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{\frac{i}{2}(x^2+y^2+t^2+s^2)cot\alpha}f(x,y)g(t,s)$

$$\cos(csc\alpha.u(x+t))\cos(csc\alpha.v(y+s))\,dxdydtds$$

Let $x+t=\tau, t=\tau\quad x,\ y+s=\rho, s=\rho\quad y$ $if\ t=\quad\infty,\tau=\quad\infty\ if\ t=\infty,\tau=\infty$ $if\ s=\quad\infty,\rho=\quad\infty\ if\ s=\infty,\rho=\infty$

$$I_1 = AB\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{\frac{i}{2}(x^2+y^2)cot\alpha}f(x,y)\left\{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{\frac{i}{2}((\tau-x)^2+(\rho-y)^2)cot\alpha}g((\tau\quad x),(\rho\quad y))d\tau d\rho\right\}\cos(csc\alpha.u\tau)\cos(csc\alpha.v\rho)\,dxdy$$

Let $\overline{g_1(}\tau\quad x,\rho\quad y) = e^{\frac{i}{2}((\tau-x)^2+(\rho-y)^2)cot\alpha}g((\tau\quad x),(\rho\quad y))$
$\bar{f}(x,y) = e^{\frac{i}{2}(x^2+y^2)cot\alpha}f(x,y)$
$I_1 = AB\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\bar{f}(x,y)\{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\overline{g_1(}\tau\quad x,\rho\quad y)d\tau d\rho\}$
$\cos(csc\alpha.u\tau)\cos(csc\alpha.v\rho)\,dxdy$
$I_1 = AB\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\bar{f}(x,y)\overline{g_1(}\tau\quad x,\rho\quad y)dxdy\}$
$\cos(csc\alpha.v\rho)\cos(csc\alpha.u\tau)\,d\tau d\rho$

$I_1 = \left(\sqrt{\frac{1-icot\alpha}{2\pi}}\right)^2\frac{e^{i(u^2+v^2)cot\alpha}}{4}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\bar{f}(x,y)\overline{g_1(}\tau\quad x,\rho\quad y)dxdy\}$
$\cos(csc\alpha.v\rho)\cos(csc\alpha.u\tau)\,d\tau d\rho$
Let $\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\bar{f}(x,y)\overline{g_1(}\tau\quad x,\rho\quad y)dxdy = (\bar{f}.\overline{g_1})$

$$I_1 = \sqrt{\frac{1-icot\alpha}{2\pi}}\frac{e^{\frac{i}{2}(u^2+v^2)cot\alpha}}{4}e^{\frac{-i}{2}(\tau^2+\rho^2)cot\alpha}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}e^{\frac{-i}{2}(\tau^2+\rho^2)cot\alpha}e^{\frac{i}{2}(u^2+v^2+\tau^2+\rho^2)cot\alpha}\sqrt{\frac{1-icot\alpha}{2\pi}}(\bar{f}.\overline{g_1})$$
$$\cos(csc\alpha.v\rho)\cos(csc\alpha.u\tau\ )\,d\tau d\rho$$

$$I_1 = \sqrt{\frac{1\quad icot\alpha}{2\pi}}\frac{e^{\frac{i}{2}(u^2+v^2)cot\alpha}}{4}FC_\alpha\left\{e^{\frac{-i}{2}(\tau^2+\rho^2)cot\alpha}(\bar{f}.\overline{g_1})\right\}$$

$$I_2 = AB\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}e^{\frac{i}{2}(x^2+y^2+t^2+s^2)cot\alpha}f(x,y)g(t,s)$$
$$\cos\big(csc\alpha.u(x+t)\big)\cos\big(csc\alpha.v(y\quad s)\big)\,dxdydtds$$

$x + t = \tau \ \ y \quad s = \xi$

Let $\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\bar{f}(x,y)\overline{g_2}(\tau\quad x\,,\xi\quad y)dxdy = (\bar{f}.\overline{g_2})$

For I_2 we do similar calculations in I_1 we get

$$I_2 = \sqrt{\frac{1\quad icot\alpha}{2\pi}}\frac{e^{\frac{i}{2}(u^2+v^2)cot\alpha}}{4}FC_\alpha\left\{e^{\frac{-i}{2}(\tau^2+\xi^2)cot\alpha}(\bar{f}.\overline{g_2})\right\}$$

Similarly

$$I_3 = AB\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}e^{\frac{i}{2}(x^2+y^2+t^2+s^2)cot\alpha}f(x,y)g(t,s)$$
$$\cos\big(csc\alpha.u(x\quad t)\big)\cos\big(csc\alpha.v(y+s)\big)\,dxdydtds$$

Here

$x \quad t = \eta \ \ y + s = \rho$ Let $\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\bar{f}(x,y)\overline{g_3}(\eta\quad x\,,\rho\quad y)dxdy = (\bar{f}.\overline{g_3})$

$$I_3 = \sqrt{\frac{1\quad icot\alpha}{2\pi}}\frac{e^{\frac{i}{2}(u^2+v^2)cot\alpha}}{4}FC_\alpha\left\{e^{\frac{-i}{2}(\eta^2+\rho^2)cot\alpha}(\bar{f}.\overline{g_3})\right\}$$

Similarly

$$I_4 = AB\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}e^{\frac{i}{2}(x^2+y^2+t^2+s^2)cot\alpha}f(x,y)g(t,s)$$
$$\cos\big(csc\alpha.u(x\quad t)\big)\cos\big(csc\alpha.v(y\quad s)\big)\,dxdydtds$$

$x \quad t = \eta \ \ y \quad s = \xi$ Let $\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\bar{f}(x,y)\overline{g_3}(\eta\quad x\,,\xi\quad y)dxdy = (\bar{f}.\overline{g_4})$

$$I_4 = \sqrt{\frac{1\quad icot\alpha}{2\pi}}\frac{e^{\frac{i}{2}(u^2+v^2)cot\alpha}}{4}FC_\alpha\left\{e^{\frac{-i}{2}(\eta^2+\xi^2)cot\alpha}(\bar{f}.\overline{g_4})\right\}$$

Then (3.1) implies

$$I = I_1 + I_2 + I_3 + I_4$$

$$FC_\alpha\{f(x,y)\}(u,v)\quad GC_\alpha\{g(t,s)\}(u,v) = \sqrt{\frac{1-icot\alpha}{2\pi}}\frac{e^{\frac{i}{2}(u^2+v^2)cot\alpha}}{4}FC_\alpha\left\{e^{\frac{-i}{2}(\tau^2+\rho^2)cot\alpha}(\bar{f}.\overline{g_1})\right\}+$$

$$\sqrt{\frac{1\quad icot\alpha}{2\pi}}\frac{e^{\frac{i}{2}(u^2+v^2)cot\alpha}}{4}FC_\alpha\left\{e^{\frac{-i}{2}(\tau^2+\xi^2)cot\alpha}(\bar{f}.\overline{g_2})\right\}+\sqrt{\frac{1\quad icot\alpha}{2\pi}}\frac{e^{\frac{i}{2}(u^2+v^2)cot\alpha}}{4}FC_\alpha\left\{e^{\frac{-i}{2}(\eta^2+\rho^2)cot\alpha}(\bar{f}.\overline{g_3})\right\}$$

$$+\sqrt{\frac{1\quad icot\alpha}{2\pi}}\frac{e^{\frac{i}{2}(u^2+v^2)cot\alpha}}{4}FC_\alpha\left\{e^{\frac{-i}{2}(\eta^2+\xi^2)cot\alpha}(\bar{f}.\overline{g_4})\right\}$$

$$FC_\alpha\{f(x,y)\}(u,v)\quad GC_\alpha\{g(t,s)\}(u,v)=\sqrt{\frac{1-icot\alpha}{2\pi}}\frac{e^{\frac{i}{2}(u^2+v^2)cot\alpha}}{4}$$

$$\begin{bmatrix} FC_\alpha\left\{e^{\frac{-i}{2}(\tau^2+\rho^2)cot\alpha}(\bar{f}.\overline{g_1})\right\}+FC_\alpha\left\{e^{\frac{-i}{2}(\tau^2+\xi^2)cot\alpha}(\bar{f}.\overline{g_2})\right\} \\ +FC_\alpha\left\{e^{\frac{-i}{2}(\eta^2+\rho^2)cot\alpha}(\bar{f}.\overline{g_3})\right\}+FC_\alpha\left\{e^{\frac{-i}{2}(\eta^2+\xi^2)cot\alpha}(\bar{f}.\overline{g_4})\right\} \end{bmatrix}$$

Conclusion

In the present work Convolution theorem for generalized two dimensional fractional cosine transform is proved

REFERENCES

Almeida, L.B. 1997. Product and convolution theorems for the fractional Fourier transform. IEEE Signal Processing Letters,4 (1): Page No.15—17

Doetsch, G. 1943. Theorie und Anwendung der Laplace-Transformation. New York: Dover.

Gardner, M. F. and Barnes, J. L. 1963. Transients in Linear Systems: Lumped-Constant Systems, 16th ed., vol. I. New York: Wiley.

Pie S. C. and Ding J. J. 2002. Fractional cosine, sine and Hartley transform; trans. On signal processing, Vo. 50, No. 7, P. 1661-1680.

Sharma, V. D. and Khapre, S. A. 1927. Analyticity of the generalized two dimensional fractional Cosines transforms, *J. Math. Computer Sci*., ISSN. -5307.

Sharma, V. D. and Khapre, S.A. 2012. Generalized two dimensional fractional Sine transforms, In Proc. IJCA. *Intconf, Recent Trends in information Technology and Compsuter Science*.

Zayed A. I.1998. A convolution and product theorem for the fractional Fourier transform, IEEE Sig. Proc. Letters, Vol. 5, No. 4, 101-103.

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