



RESEARCH ARTICLE

PHASE SHIFT CALCULATION FOR THE NUCLEI ($^{16}\text{O}, ^{17}\text{F}$) & ($^{40}\text{Ca}, ^{41}\text{Sc}$)
NEAR & FAR FROM STABILITY LINE

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ABSTRACT

The phase shift calculations for the mirror nuclei by applying Gaussian 15 point quadrature method are done. The form factor, binding energy, normalization constants are the inputs in this case. The singularities are overcome by suitable techniques.

Key words:

Special points and weights,
Gaussian quadrature method,
Stripping reactions, Drip line nuclei.

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INTRODUCTION

The calculation of phase shifts by the scattering theory was one of the main method for a long time. Based on the Darboux transforms, inverse scattering methods are successfully applied to determine the nucleon nucleon potential for uncoupled partial waves from the corresponding phase shifts whose analyses are quite uncertain (Leeb and Leidinger, 1992). Kim and Zubarev used the effective linear two-body method for many body problem in atomic and nuclear physics to convert to two-body one by applying variational ones to find out the phase shift values (Kim and Zubarev, 2000). Nuclei having very different N/Z ratio compared to stable nuclei with the same A, large radii cut-off, high momentum distribution are generally known as drip-line nuclei. The magic nuclei having N=Z appear near the β -stability line for which $v(r) \sim r$ curve is symmetric. For proton drip-line nuclei, $v(r) \sim r$ is deeper for n than p-halo in the medium heavy nuclei (Wang *et al.*, 2013). By choosing Wood-Saxon potential it is observed that nuclei with $l=2$ higher orbital angular momentum lying deep are more stable compared to the $l=0$ state for mass

range $A \sim 20 - 40$. Considering the Schrodinger's equation in radial form for a finite well potential in the mass range $A \sim 20 - 220$ nuclei having shell structure level $2s_{1/2}, 1d_{3/2}, 1d_{5/2}$ are seen to be strongly bound and nuclei having shell structure $1d_{3/2}, 2s_{1/2}, 1d_{5/2}$ are stable sd-shell nuclei in the mass range $A \sim 20 - 40$ (Nayak *et al.*, 2009). The nuclei such as $^{40}_{12}\text{Mg}_{28}, ^{42}_{13}\text{Al}_{29}, ^{44}_{14}\text{Si}_{30}$ and their isotopes are the recently found neutron drip-line nuclei. Since $N > Z$ they occur near the β^- -stability zone. More the coulombic force between the $1n, 2n \dots$ and the core these unstable nuclei can come closer to the β^- -stability line and become more stable. The proton drip-line nuclei which are against the proton emission are approximately known up to Pb region. Proton drip-line nuclei lie much closer to the β -stability line than the neutron drip-line due to the higher coulombic barrier (Rodberg and Thaler)

Phase-shift calculation

Schrodinger's equation for S-wave is written as

$$\frac{d^2\Psi}{dx^2} + \frac{2\mu}{\hbar^2}(E - V)\Psi = 0 \quad (1)$$

In radial form

$$\frac{d^2U}{dr^2} + \frac{2\mu}{\hbar^2}(E - V)U = 0 \quad (2)$$

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$$U = rR(r)$$

$R(r)$ being the radial spatial wave function

The partial wave equation of Schrodinger equation (Rodberg and Thaler)

$$\frac{d^2 U_l(r)}{dr^2} + \left[k^2 - U(r) - \frac{l(l+1)}{r^2} \right] U_l(r) = 0 \quad 03$$

$$\Psi_{lm}(r) \equiv \frac{U_l(r)}{r} Y_{lm}(\theta, \phi) \quad 04$$

$$k^2 = \left(\frac{2mE}{\hbar^2} \right)$$

$$U(r) = \left(\frac{2m}{\hbar^2} \right) V(r) \quad 05$$

For $r > R$

$$F_l(kr) = kr j_e(kr) = \left(\frac{1}{2} \pi kr \right)^{\frac{1}{2}} J_{l+\frac{1}{2}}(kr) \quad 06$$

and

$$G_l(kr) = \left(\frac{1}{2} \pi kr \right)^{\frac{1}{2}} (1) J_{-l+\frac{1}{2}}(kr) \quad 07$$

$$U_l(r) = AF_l(kr) + BG_l(kr) \quad 08$$

A, B are complex but this ratio A/B is real near origin

$$F_l(kr) \xrightarrow{kr \ll 1} \frac{(kr)^{l+1}}{1.3.5 \dots (2l+1)} \quad 09$$

$$\text{And } G_l(kr) \xrightarrow{kr \ll 1} \frac{1.3.5 \dots (2l-1)}{(kr)^l} \quad 10$$

At large distances for $kr \gg 1$

$$F_l(kr) \rightarrow \sin \left(kr - \frac{1}{2} \pi l \right) \quad 11$$

$$G_l(kr) \rightarrow \cos \left(kr - \frac{1}{2} \pi l \right) \quad 12$$

$$r_{tot} = \int_0^\infty \frac{4\pi}{k^2} \sin^2 \delta_l d\Omega \quad 13$$

$$\left\{ \frac{d^2}{dr^2} - \frac{l(l+1)}{r^2} + \frac{2m}{\hbar^2} (S_{nlj} - V(r) - V_{ls}(r)) \right\} R_{nlj}(r) = 0 \quad 14$$

Centrifugal potential:

$$\frac{l(l+1)}{2mr^2}$$

Wood-Saxon Potential:

$$\frac{V_0}{1 + \exp \left(\frac{r-R}{a} \right)}$$

The barrier height of the potentials (centrifugal + Wood-Saxon) $\propto \frac{l(l+1)}{r^2}$

$$r \sim r_0 A^{1/3}$$

The integration in equation (13) can be divided into three intervals (Anjana Acharya, 2007)

(i) From 0 to \sqrt{E}

(ii) From \sqrt{E} to $\sqrt{E + 2\varepsilon_B}$

(iii) From $\sqrt{E + 2\varepsilon_B}$ to ∞

Within these intervals six, three and six mesh points (altogether 15 points) are chosen. These are known as Gauss Legendre points mapped onto the respective intervals. In the 2nd and 3rd intervals the Gauss-Legendre points t , which are three and six in number in the two cases are mapped onto the respective intervals by the transformation (Jansen *et al.*, 2011)

$$U_k = [(t+1)\varepsilon_B + E]^{\frac{1}{2}} \quad \text{and} \quad U_k = \frac{2}{(1-t)} (E + 2\varepsilon_B)^{1/2} \quad 16$$

Thus requiring the on-shell point $U_k = Q_k$ to be one of the 15 mesh points in the U_k integration. The mesh points in the 2nd and 3rd intervals are different for different partitions, $k=1, 2, 3$. But in all cases the 8th mesh point is the corresponding on-shell momentum.

The expression for the phase shift is given as (Acharya *et al.*, 2015)

$$\delta = -\frac{\pi}{2} k \lambda_x \frac{v^2(k)}{(1+Re I)}$$

$$Re I = \lambda x \int P \frac{dQ Q^2 v^2(k)}{Q^2 - k^2}$$

'x' stands for last neutron or proton with specific l and j values.

Table 1. Values of the parameters to calculate phase shift

Reaction Type	State(l_j)	l_j in MeV	α^2 in fm^{-2}	β in fm^{-1}	in fm^{-7}
$^{16}\text{O} + n = ^{17}\text{O}$	5^+	-4.146	0.1878	1.7	917
	$\frac{3}{2}^+$				
	1^+	-3.27	0.1481	1.7	4.1378
$^{16}\text{O} + p = ^{17}\text{F}$	5^+	-0.596	0.027	1.7	1109.34
	$\frac{3}{2}^+$				
	1^+	-0.10	0.0045	1.7	9.85
$^{40}\text{Ca} + n = ^{41}\text{Ca}$	3^+	-6.41	0.3010	0.548	93
	$\frac{3}{2}^+$				
$^{40}\text{Ca} + p = ^{41}\text{Sc}$	3^+	-0.13	0.00611	0.548	0.183
	$\frac{3}{2}^+$				

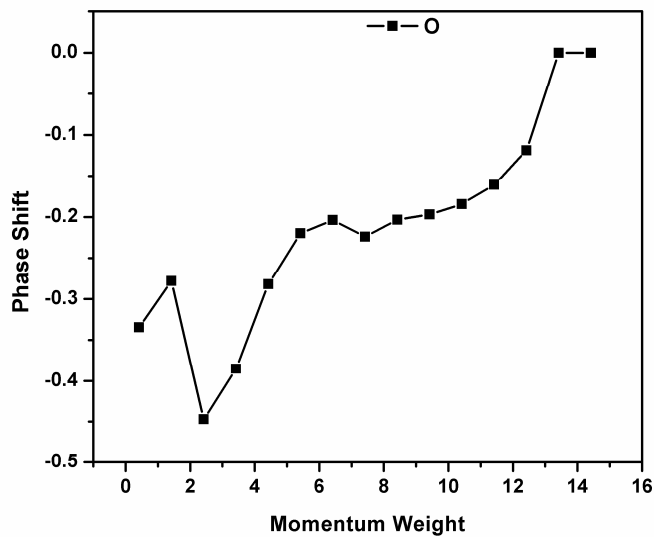


Fig. 1. The phase shift versus momentum weights for $^{16}\text{O}(d,n)$ reaction

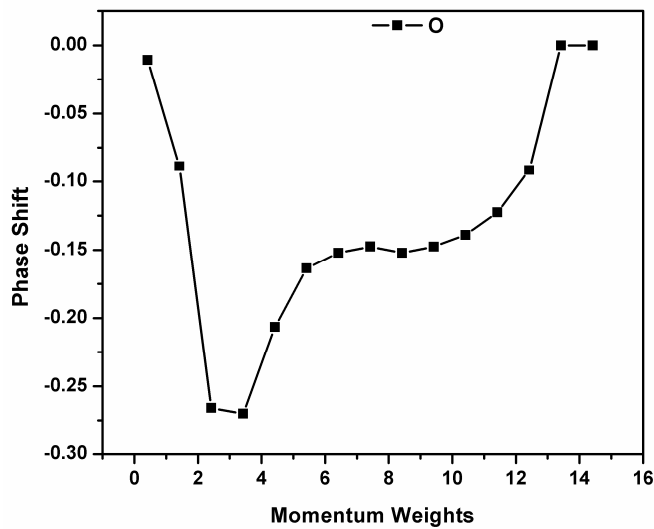


Fig 2. The phase shift versus momentum weights for $^{16}\text{O}(d,p)$ reaction

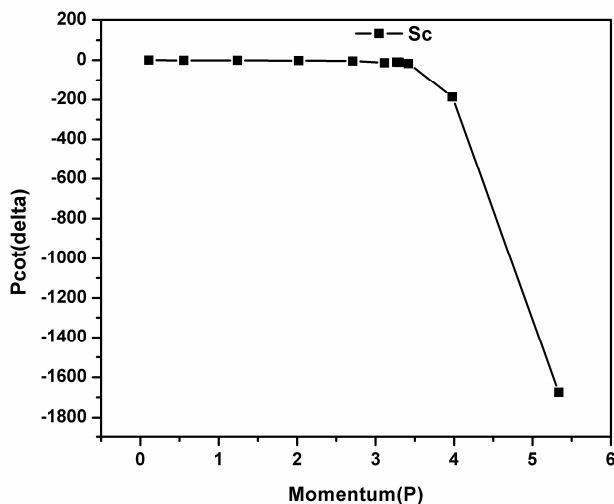


Fig 3. The phase shift versus momentum weights for $^{40}\text{Ca}(d,n)$ reaction

RESULTS AND DISCUSSION

The cross-section values is larger for those reactions where channel spin is conserved where the spin of the i^{th} particle coupled to the i^{th} bound pair in the initial channel is same as the spin of the i^{th} particle coupled to the j^{th} pair in the product channel. So to study the isospin study breaking for heavy halo nuclei is our future plan. These lead to the astrophysical interest. The position of a body (moon or planet) elongation 90° or 270° i.e. body-earth-sun angle is 90° . In signal processing, quadrature amplitude modulation (QAM), a modulation method of using both carrier and modulated wave is necessary. The RMS value for different nuclei gradually increases with respect to the atomic mass no. both for the nuclei near the β^+ and β^- stability line in the mass region 0-220 are calculated, the N~Z graph for the different nuclei near and far from the β -stability line are shown in ref (Nayak *et al.*, 2009) (red line- β^- stability nuclei, blue line- β^+ stability nuclei and black line- β stability nuclei). By designing a specific model to study the internal structure of these halo nuclei near and far from the stability line has wide applications in various fields. In Fig 1 we see that the phase shift increases from -0.35 and attains peak at -0.46 when the momentum weights varies from 0 to 2.5. Then the delta-values tend towards zero with respect to p-weights which depend on the l, j states. The other graphs can be calibrated similarly. Our future plan is to find out the cross-section for different isotopes of oxygen and calcium when coulomb interaction is included.

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