



RESEARCH ARTICLE

RESCALED RANGE ANALYSIS AND DETRENDED FLUCTUATION ANALYSIS IN THE STUDY OF
NORTHEAST MONSOON RAINFALL OF TAMIL NADU

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ABSTRACT

Rescaled range analysis (R/S analysis), detrended fluctuation analysis (DFA) are widely-used methods for detection of long-range correlations in time series. This paper employs R/s analysis and detrended fluctuation analysis method to investigate the trend of Northeast monsoon rainfall of Tamil Nadu time series. The data used for this analysis is from the year 1901 to 2004 through about 104 years. The different scaling exponents estimated by R/S method and DFA method indicates the same result of anti-persistence in the Northeast monsoon rainfall of Tamil Nadu.

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INTRODUCTION

Tamil Nadu, located in southeast Peninsular India, receives the major part of its annual rainfall during the northeast monsoon (NEM) season (the three-month period from October to December). While coastal Tamil Nadu receives about 60% of its annual rainfall, interior Tamil Nadu receives about 40–50% of annual rainfall during NEM season. In comparison with Indian Summer Monsoon (ISM), the NEM is characterized by limited aerial extent and average lesser rainfall amount. During NEM season, Tamil Nadu generally receives rainfall due to the formation of trough of low, cyclonic circulation, easterly waves, low pressure area, depression and cyclonic storm over Bay of Bengal Balachandran *et al.*, (2006). It has, however, been noted that the rainfall during northeast monsoon is highly variable. Therefore, if its behavior could be predicted in advance, it would go a long way toward helping the agricultural and industrial activities of the region Dhar and Rakhecha (1983). Statistical analysis of rainfall records for long periods is essential to provide information about rainfall variability and to better manage the rainfed agricultural activities such that the impact of climate change as well as changes in land use can be realistically assessed. Understanding rainfall variability is essential to optimally manage the scarce water resources that are under continuous stress due to the increasing water demands, increase in population and the economic development Herath and Ratnayake (2004). The Hurst exponent as well as detrended fluctuations analysis (DFA) are used as technical tools. First we compute the Hurst exponent H using the empirical method based on the R/S parameter.

Next, for the same sets of data, we calculate the parameter α using the detrended fluctuations analysis (DFA) method. It will serve as a verification of the Hurst method. If both R/S analysis and DFA will show some long-term correlations in the investigated time series, we can be more convinced that there is something real behind our results Beben and Orłowski (2001). The R/S analysis has a strong theoretical back up, its statistical limitations for scaling (Hurst) exponent estimation are well established and it finds wide acceptability for application in diverse econometric fields Lo (1991). On the other hand, due to the simplicity in implementation, the DFA is now becoming a widely used method in physics and engineering.

Data Used

We have used the Northeast monsoon rainfall data of Tamil Nadu from the period 1901–2004. The data are obtained from the Regional Meteorological centre, Chennai.

METHODOLOGY

R/S analysis - Hurst exponent

Studying the Nile river and the problems related to water storage, Hurst created the (R/S) method which gives a reliable measure of some statistical aspects of time series records. Hurst (1951). Based on Feder (1988), (R/S) analysis can be introduced as follows. Given a time series $\{x(1), x(2), \dots, x(t)\}$ of a natural phenomena recorded at discrete time over a time span, we follow this procedure

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1. Calculate the mean;

$$m = \frac{1}{n} \sum_{i=1}^n X_i.$$

2. Create a mean-adjusted series;

$$Y_t = X_t - m \quad \text{for } t = 1, 2, \dots, n.$$

3. Calculate the cumulative deviate series Z ;

$$Z_t = \sum_{i=1}^t Y_i \quad \text{for } t = 1, 2, \dots, n.$$

4. Compute the range R ;

$$R(n) = \max(Z_1, Z_2, \dots, Z_n) - \min(Z_1, Z_2, \dots, Z_n).$$

5. Compute the standard deviation S ;

$$S(n) = \sqrt{\frac{1}{n} \sum_{i=1}^n (X_i - m)^2}.$$

6. Calculate the rescaled range $R(n) / S(n)$ and average over all the partial time series of length n . The Hurst exponent (H) is

$$E \left[\frac{R(n)}{S(n)} \right] = C n^H$$

estimated by fitting the power law to the data.

Here c is a constant and H is called the Hurst exponent. To estimate the Hurst exponent, we plot (R/S) versus n in log-log axes. The slope of the regression line approximates the Hurst exponent. The values of the Hurst exponent range between 0 and 1. Based on the Hurst exponent value H , the following classifications of time series can be realized:

$H = 0.5$ indicates a random series;

$0 < H < 0.5$ indicates an anti - persistent series, which means an up value is more likely followed by a down value, and vice versa; $0.5 < H < 1$ indicates a persistent series, which means the direction of the next value is more likely the same as current value Alina Barbulescu *et al.*, (2007).

Detrended fluctuation analysis

In recent years, the detrended fluctuation analysis (DFA) proposed by Peng *et al.* (1994) has been established as an important tool for the detection of long-range correlations in time series with non-stationarities.

The DFA procedure consists of the following steps. In the first step, the profile

$$X_t = \sum_{i=1}^t (x_i - \langle x_i \rangle)$$

X_t is called cumulative sum or profile..

Next, X_t is divided into time windows Y_j of length L samples, and a local least squares straight-line fit (the local trend) is calculated by minimising the squared error E^2 with respect to the slope and intercept parameters a, b :

$$E^2 = \sum_{j=1}^L (Y_j - a_j - b)^2.$$

Trends of higher order, can be removed by higher order DFA, where the linear function $a_i + b$ is replaced by a polynomial of order n . Next, the root-mean-square deviation from the trend,

the fluctuation, is calculated over every window at every time scale:

$$F(L) = \left[\frac{1}{L} \sum_{j=1}^L (Y_j - a_j - b)^2 \right]^{\frac{1}{2}}.$$

This detrending followed by fluctuation measurement process is repeated over the whole series at a range of different window sizes L , and a log-log graph of L against $F(L)$ is constructed.

A straight line on this log-log graph indicates statistical self-affinity expressed as $F(L) \propto L^\alpha$. The scaling exponent α is calculated as the slope of a straight line fit to the log-log graph of L against $F(L)$ using least-squares. This exponent is a generalization of the Hurst exponent. Similarly to the Hurst analysis, interpretation of the results depends on the value of α . If $0 < \alpha < 0.5$, the time series is long-range anti-correlated; if $\alpha > 0.5$, the time series is long-range correlated. $\alpha = 0.5$ corresponds to data series with no correlations or short-range correlation.

RESULTS AND DISCUSSION

The behavior of Northeast monsoon rainfall of Tamil Nadu is shown in Fig. 3. As shown in fig. 1 a graph is plotted for $\log n$ vs $\log R/S$ and the slope is calculated for the given time series. The slope for the dataset is found to be 0.2105, which is the Hurst exponent. As shown in Fig. 2 a graph is plotted for $\log L$ vs $\log F$ and the slope is calculated as 0.2295 using DFA method. The value correlates very much with Hurst exponent method. From the results obtained we can say that, the Northeast monsoon rainfall of Tamil Nadu follows anti-persistence pattern and is verified by these methods.

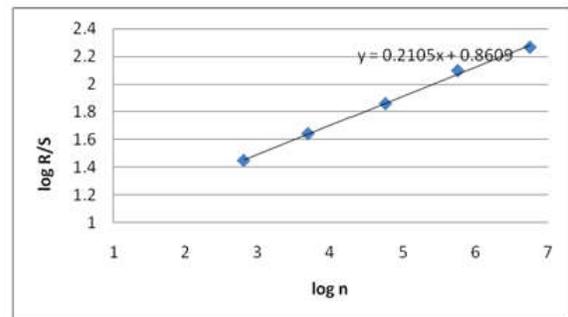


Fig 1. Plot for the calculation of slope in the Hurst exponent

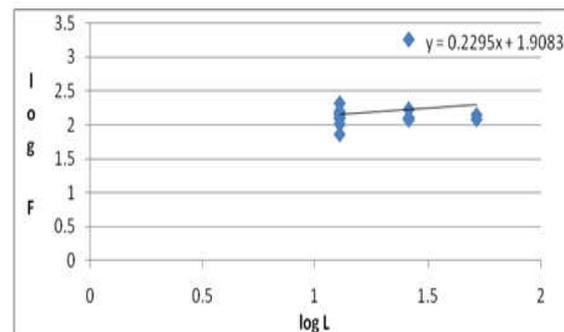


Fig 2. Plot for the calculation of slope in the DFA

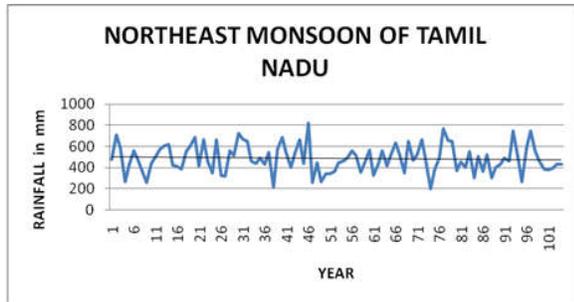


Fig. 3. Northeast monsoon rainfall of Tamil Nadu from 1901-2004

CONCLUSION

We use Rescaled Range Analysis (R/S) and Detrended Fluctuation Analysis (DFA) methods to determine long-range correlations. Both methods characterize fractional behavior, but R/S analysis can yield more accurate results for small and stationary data sets and DFA analysis yields more accurate result for non stationary data sets. The exponents calculated could serve as verification and comparison of the results; therefore, both methods are used. It shows that the Northeast monsoon rainfall of Tamil Nadu exhibits anti-persistence in both the methods. Since the Hurst exponent and Detrended fluctuation analysis provides a measure for predictability, we can use this value to guide data selection before forecasting. This can save time and effort and lead to better forecasting.

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