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RESEARCH ARTICLE

AN ASYMPTOTIC SOLUTION TO BIO-POROUSCONVECTION IN A SUSPENSION OF MICROSCOPIC SWIMMING PHOTOTACTIC ALGAE

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ABSTRACT

This paper deals with a detailed study of asymptotic solution to bio-porous convection in a suspension of microscopic swimming phototactic algae. Experimental observations indicate that bio convection patterns are modified by illumination. This phenomenon will be observed in a large number of different algal species where the cells are denser than water and tend to swim upward on average. The continuum model for phototaxis and suspension shading was formulated in a porous medium. Here, the length scale of the bulk motions and the concentration distribution are large when compared to cell diameters and cell spacing. Pure phototaxis is a valid limiting case to consider in order to understand the complexities of the effects of photo taxis on BPC (bio-porous-convection) before moving in to a higher dimension. Further the diffusion tensor is a constant orthotropic tensor. The linear stability problem is discussed in detail and solvability conditions are derived up to the third order. Analyses for the two cases namely a) upper rigid and b) upper stress free boundary are discussed. Extensive graphs are drawn for the various computed results and the permeability number has strong influence on the inhibition and enhancement of bio convection.

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INTRODUCTION

World's major part constitutes biomass. Complex bioconvection patterns are observed when suspensions of some microscopic swimming algae are placed in a shallow dish. The pattern formation depends on many factors; in particular on the tactic nature of the microorganism. In this chapter we discuss a new generic model for phototaxis in a suspension of microscopic swimming algae and study the influence of permeability parameter on bioconvection (BPC) in a suspension of phototactic algae. Phototaxis is a directed swimming response dependent upon the light conditions sensed by the microorganisms. The model that is going to be discussed for phototaxis also incorporates the effect of shading whereby microorganisms nearer the light source absorb and scatter the light before it reaches those farther away. Some of the recent and important works in bioconvection include Platt, 1961, Rudraiah and Srimani, 1980, Harashima,et al. 1988, Srimani and Sudhakar, 1992, vincent and Hill, 1996,Srimani and Padmasini 2001, Shu and Ramapriyan, 2005, Ghorai and Hill , 2005, Srimani and Anuradha, 2007, Basak *et al.,* 2008, Srimani and Roopa, 2011 and Srimani and Sujatha, 2011.

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A detailed review of the literature is available in Padmasini (2003), Anuradha (2006) and Hill and Pedley (2005). The present model constitutes five dimensionless parameters together with a parameter that specifies the vertical position of the sublayer in the fluid. Our model considers two cases (i) Rigid upper surface (ii) Stress free upper surface. The asymptotic analysis is carried upto the fourth order approximation. The cumulative effect of the other governing parameters on the stability conditions as well as on the different profiles is remarkable. The position of the sublayer actually depends on the intensity of the light source and the solutions are obtained through the solvability conditions. The computed results are presented through graphs and are in excellent agreement with the available results in the limiting cases.

MATERIALS AND METHODS

In this section the continuum model in boundary conditions and asymptotic analysis are discussed.

Nomenclature

d-Depth parameter, R-Rayleigh number,Sc-Schimid number, C-Sub layer positionparameter which specifies the vertical position of the sublayer in the fluid, $\hat{p}$-Orientation vector, f(p)-The probability density function, $\bar{p}$ - The ensemble average, V-Cell swimming velocity, I_c-Critical intensity, Φ-Microorganisms volume fraction, ϑ - Cell volume,ρ-Density of the fluid, t*-Time, g-Acceleration gravity, n*-Cell contraction, α*-Extinction coefficient, μ-Dynamic viscosity ,k-Permeability, P_e-Excess pressure, D-Diffusion tensor, H-Depth of the layer, v-Kinematic viscosity, V_A-Average cell swimming speed, D_v-Vertical diffusion parameter, N_o-Uniform cell concentration, K-Dimensionless horizontal wavenumber, σ-Growth rate, u*-Average velocity of the material in δv, Pl-Darcian/porous parameter.

The Continuum Model

In this study it is assumed that the length scale of the bulk motions and the concentration distribution are large compared to typical cell diameters and cell spacing. The algal cells themselves are modeled as internally homogeneous, pigmented particles of volume v and density $\rho + \Delta\rho(\Delta\rho << \rho,$ where ρ is the density of the fluid) and possess the same light transmittance in all orientations. The number of cells in a small volume δ v defined relative to Cartesian axis O x* y* z* is n*(x*,t*) δv, where z* is the axis in the vertical direction and t* is time. Neglecting all inertia in the cells motion and supposing the suspension is dilute ($n^* \vartheta \ll 1$) and incompressible then, if u*(x*, t*) is the average velocity of all the material in δ V,

$$\nabla^* \cdot u^* = 0 \qquad \ldots(1)$$

For simplicity, we shall assume that the effect of the cells, on the suspension is dominated by the stokelets due to their negative buoyancy and that all other contributions to the bulk stress are sufficiently small to be neglected. Neglecting all the forces on the fluid except the cells negative buoyancy, n*vg $\Delta\rho\delta v$ where g is the acceleration due to gravity, the momentum equation under the Boussinesq approximation is .

$$\rho\left[\frac{Du^*}{Dt^*}\right] = -\nabla^* P_e^* - n^* \vartheta g \Delta\rho \hat{k} + \mu \nabla^{*2} u^* - \frac{\mu}{k_p} u^* \qquad \ldots(2)$$

The system is a sparsely packed fluid saturated porous layer $\frac{D}{Dt^*} = \frac{\partial}{\partial t^*} + u^* \cdot \nabla^*$ is the material time

derivative, $\hat{k}$ is a unit vector in the z* direction, P_e is the excess pressure above hydrostatic and μ is the dynamic viscosity of the suspension which, since the suspension is dilute, is considered to be that of the fluid, which is effectively water. The equation for cell conservation is

$$\frac{\partial n^*}{\partial t^*} = -\nabla^* \cdot J^* \qquad \ldots(3)$$

where J* (x*, t*) δs the net flux across a surface element, δs, at x*, j* can be written as

$$J^* = n^* u^* + n^* V_C \bar{p} - D^* \cdot \nabla^* n^* \qquad \ldots(4)$$

where $\bar{p}$ is given by

$$V_A \bar{p} = v_A \int p f(p) dp \ldots(5)$$

The first term, (n*u*) is the flux due to advection of cells by the bulk flow. The other terms represent fluxes arising from the stochastic nature of the cell swimming: n*$V_c \bar{p}$ is a mean flux due to cell swimming and –D*. $\nabla^* n^*$ is a diffusive flux of cells down the cell concentration gradients. We shall consider a fluid saturated porous layer with horizontal boundaries at z* = -H, O and shall assume that the vertical boundaries are far away so that the layer has an effectively infinite width. The suspension is illuminated by parallel light from a uniform source of intensity I_s ,vertically above the layer. If the light absorption by the pure fluid is negligible then the light intensity I (x*)received by the photo receptors of an algal cell at the position x*=(x*,y*,z*) is

$$I(x^*) = I_S \exp(-\alpha^* \int_{Z*}^{0} n^*(x', t^*) dz') \qquad \ldots(6)$$

Here n*(x*) is the concentration of cells at any position x* and α^* is the extinction co-efficient, which quantifies the strength of absorption and scattering of light in the suspension. If the critical intensity I_c occurs at position *z* =-CH (0 ≤ C ≤ 1)* for a vertically uniform concentration profile N_0 then, for cells at x^* correct to O($\alpha^* n^* H$):

$$\bar{P} = \Lambda I_C \alpha^* (\int_{Z^*}^{O} n^*(x', t^*) dz' - CHN_0)\hat{k} \qquad \ldots(7)$$

The continuum model is now complete.

Basic State Solution

This study presents the results for both free and rigid upper horizontal boundaries.

The boundary conditions for the problem are.

$$u^* \cdot \hat{k} = 0 \quad on \quad z^* = -H, 0 \qquad \ldots (8)$$

$$J^* \cdot \hat{k} = 0 \quad on \quad z^* = -H, 0 \qquad \ldots(9)$$

For a rigid boundary

$$u^* \times \hat{k} = 0 \quad on \quad z^* = -H, 0 \qquad \ldots(10)$$

whilst for a free boundary,

$$\frac{\partial^2}{\partial z^{*2}}(u^{*}.\hat{k}) = 0 \quad on \quad z^{*} = -H.O \qquad \ldots(11)$$

When there is no fluid motion the steady equation for cell flux becomes on substituting, (4) and (7) into (3)

$$\frac{d}{dz^{*}}\{n^{*} V_A \Lambda I_c \alpha^{*} (\int_{Z^*}^{0} n(x')dz' - CHN_0) - D_v \frac{dn^{*}}{dz^{*}}\} = 0 \qquad \ldots(12)$$

Solving (12) we get,

$$n^{*}(z^{*}) = \frac{k^{*2} D_v}{2V_A \Lambda I_c \alpha^{*}} Sech^2\left(\frac{1}{2}k^{*}(z^{*} + CH)\right) \qquad \ldots (13)$$

where k* is a constant of integration which can be related to N_0 the number of cells per unit volume for the whole layer, by the relation

$$\int_{-H}^{0} n^{*}(x')dz' = N_0 H \qquad \ldots (14)$$

It is apparent from (13) that the region above z* = -CH is locally gravitationally stable and convective motions occurring in the unstable region below z*= -CH will penetrate the upper layer.

Non-Dimensionalization

Non-dimensionalizing the governing equations (1) to (3) by using the following scale length scales(H: H is depth of the layer.)Bulk fluid velocity: D ν /H, Time : H^2 / D_v, Cell Concentration : N_0 (Which is the uniform cell concentratin),Diffusion:D_v, Pressure : $\nu D_v \rho / H^2$

Further,

$$U^{*} = \frac{D_v u}{H} \quad n^{*} = N_0 n \quad t^{*} = \frac{H^2}{D_v} t$$

$$\nabla^{*2} = \frac{I}{H^2}\nabla^2 \quad p_c^{*} = \frac{VD_v \rho}{H^2} p_e$$

$$p_c^{*} = p_0 + \in p_e \quad \nabla^{*} = \frac{\nabla}{H} \quad \frac{D}{Dt^{*}} = \frac{\partial}{\partial t} + u^{*}\nabla^{*} \quad Pl = \frac{H^2}{k_p} \quad :$$

porous parameter

where ν is the kinematic viscosity and D_ν is the diffusion parameter. In terms of the new symbols the basic equilibrium state can be written as.

$$u=0, \quad n(z) = \frac{k^2}{2d}\sec h^2(\frac{1}{2}k(z+C)) \qquad \ldots(15)$$

where the horizontal boundaries are at z=-1,0 and

$$d = \frac{HV_A P}{DV}, \quad P = \Lambda I_c \alpha^{*} N_0 H \qquad \ldots(16)$$

Now, $K = k^{*}H$ which is obtained by the equation (14) in turn gives a transcendental equation.

$$K[\tanh(\frac{1}{2}KC) - \tanh(\frac{1}{2}K(C-1))] = d \qquad \ldots(17)$$

Here P is directly proportional to $\alpha^{*}, I_c \; and \; \Lambda.$ It represents the strength of the photo taxis in the suspension. Suppose K << 1, then equation (17) can be solved approximately to give $K \approx (2d)^{1/2} (d \ll 1)$ so that (17) Becomes

$$n(z) = \sec h^2\left(\left(\frac{d}{2}\right)^{1/2} (z+C)\right) \qquad \ldots(18)$$

The governing equations after non demensionalizing are:-

$$\nabla \cdot u = O \qquad \ldots(19)$$

$$Sc^{-1}\frac{\partial u}{\partial t} = -\nabla P_C - \beta n\hat{k} + \nabla^2 u - Plu \qquad \ldots(20)$$

$$\frac{\partial n}{\partial t} = -(\nabla n \cdot u) - d\frac{\partial n}{\partial z}\left[\int n(x')dz - C\right]$$
$$-dn\frac{\partial}{\partial z}\left[\int n(x1)dz - C\right] + k\nabla_h^2 n + \frac{\partial^2 n}{\partial z^2} \qquad \ldots(21)$$

Linear Stability analysis

In this section, the linear stability problem is discussed by considering a small perturbation to the equilibrium state (15) of amplitude $\in$ where $0 < \in \leq 1$ in this case.

$$\left.\begin{aligned} u &= \in u_1 \\ n &= n_0 + \in n_1(x,t) \\ p_c &= p_o + \in p_e \end{aligned}\right\} \qquad \ldots(22)$$

$$\text{where } n_0 = \frac{K^2}{2d} Sech^2\left(\frac{1}{2}K(z+C)\right) \qquad \ldots(23)$$

If $u_1 = (u_1 v_1 w_1)$ and $D_H = \kappa D_V$ where κ is a positive real number then, substituting the above perturbation quantities into (8), (9) and (01) and linearizing about the (basic state by collecting the 0(ε) terms), we get the following governing equations :

$$\nabla \cdot u_1 = 0 \qquad \ldots (24)$$

$$Sc^{-1} \frac{\partial u_1}{\partial t} = -\nabla P_e - \beta n_1(x,t)\hat{k} + \nabla^2 u_1 - Plu_1 \ \ldots \ (25)$$

$$\frac{\partial n_1}{\partial t} + d\left[\frac{dn_0}{dz}\int_z^0 n_1(x,t)dz + \frac{\partial n_1}{\partial z}\left(\int_z^0 n_0(s)ds - C\right) - 2n_0 n_1\right] \quad \text{Where}$$

$$-k\nabla_h^2 n_1 - \frac{\partial^2 n_1}{\partial z^2} = -w_1\frac{dn_0}{dz} \qquad (26)$$

$$\beta = \frac{N_0 \vartheta \Delta \rho g H^3}{\rho D_V v} \quad \text{and}$$

$S_c = \dfrac{v}{D_v}$ is the Schimid Number ... (27)

$$\text{and } \nabla_h^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2},$$

The pressure is eliminated from the above equations by taking the curl of (25) twice and retaining the z-component of the result. This reduces the system into the following equation:

$$Sc^{-1}\frac{\partial}{\partial t}\left[+\nabla_h^2 w_1 + \frac{\partial^2 w_1}{\partial z^2}\right] = \nabla_h^4 w_1 + 2\nabla_h^2 \frac{\partial^2 w_1}{\partial z^2} + \frac{\partial^4 w_1}{\partial z^4}$$
$$-\beta\nabla_h^2 n_1 Pl\nabla_h^2 w_1 - Pl\frac{\partial^2 w_1}{\partial z^2} \qquad \ldots(28)$$

Now w_1 *and* n_1 can then be resolved into normal modes using following substitutions in (28) and (26) i.e.

$$n_1 = \phi(z) f(x,y)\exp(\sigma t); \ w_1 = w(z) f(x,y)\exp(\sigma t) \qquad \ldots (29)$$

Also taking $\nabla_h^2 f = -k^2 f$ *and* $\nabla_h^4 f = k^4 f$ we get the governing equations as

$$\left(Sc^{-1}\sigma + k^2 \frac{-d^2}{dz^2} + Pl\right)\left(k^2 - \frac{d^2}{dz^2}\right)w_1 = -\beta k^2 \phi \qquad \ldots(30)$$

$$d\frac{dn_0}{dz}\int_z^0 \phi(s)ds + \left(\sigma - 2dn_0 + \kappa k^2\right)\phi +$$
$$d\left(\int_z^0 n_0(s)ds - C\right)\frac{d\phi}{dz} = -\frac{dn_0}{dz}w \qquad \ldots(31)$$

subject to the boundary conditions

$$\frac{d\phi}{dz} - d\left\{\left(\int_z^0 n_0(s)ds - C\right)\phi + dn_0\int_z^0 \phi(s)ds = 0 \text{ at } z = -1,0 \qquad \ldots(32)\right.$$

Rigid Boundary: $w = \dfrac{dw}{dz} = 0$ at z = -1,0 …(33)

Stress free surface: $w = \dfrac{d^2 w}{dz^2} = 0$ at z=0 …(34)

Here k is a dimensionless horizontal wave number and σ is the growth rate, and the effects of phototactic motions, are incorporated in the terms that are found in the L.H.S of cell flux equation.

Equation (31) is a linear integro-differential equation with non-constant co-efficients, the solution of which represents a problem of considerable difficulty. Certainly it may be reduced to ODE by substituting

$$\phi^*(z) = \int_z^0 \phi(s)ds \qquad \ldots(35)$$

Using the above substitutions in (30) and (31) we get

$$\left(\sigma Sc^{-1} + k^2 - \frac{d^2}{dz^2} + Pl\right)\left(k^2 - \frac{d^2}{dz^2}\right)w = \beta k^2 \frac{d\phi^*}{dz} \qquad \ldots(36)$$

$$d\frac{dn_0}{dz}\phi^*(z) - \left(\sigma - 2dn_0 + \kappa k^2\right)\frac{d\phi^*}{dz} - d\left(\int_z^0 n_0(s)ds - C\right)$$
$$\frac{d^2\phi^*}{dz^2} + \frac{d^3\phi^*}{dz^3} = -\frac{dn_0}{dz}w \qquad \ldots(37)$$

The system of equations (36) and (37) is of seventh order, as opposed to the original system which was sixth order, and therefore the additional boundary conditions are required. This is supplied by observing equation (35) i.e.,

i.e., $\phi^*(z) = 0$ at z=0 …(38)

Asymptotic solutions

The asymptotic solution to the normal mode problem for rigid and stress free upper surfaces when 0<d<<1 is presented in this section.

Physically this is for shallow layer approximation when the scale for the equilibrium cell distribution is large enough when compared to the depth of the layer that is H. This analysis provides an insight into the fluid mechanics of bio-convection. When d<<1, (17) can be solved approximately. This gives $k \approx (2d)^{1/2}$ *and* $n(z)$ is given by (15).

Now. Put $R = \beta d$

$$\tilde{k} = \frac{k}{d} \Rightarrow k = \tilde{k}d \ \therefore \beta k^2 = dR\tilde{k}^2$$

(36) and (37) yields

$$\left(\sigma Sc^{-1} + d^2\tilde{k}^2 - \frac{d^2}{dz^2} + Pl\right)\left(d^2\tilde{k}^2 - \frac{d^2}{dz^2}\right)w = dR\tilde{k}^2\frac{d\phi^*}{dz} \quad \ldots(39)$$

$$d\frac{dn_0}{dz} - \left[s - 2dn_0 + k\tilde{k}^2 d^2\right]\frac{d\phi^*}{dz} - d\left(\int_z^o n_0(s)ds - C\right)$$

$$\frac{d^2\phi^*}{dz^2} + \frac{d^3\phi^*}{dz^3} = -\frac{dn_0}{dz}w \quad \ldots(40)$$

$$S_1(z) = -\frac{1}{2}(z^2 + 2zC + C - \frac{1}{3}) \quad (41)$$

$$s_2(z) = \frac{1}{6}(z^4 + 4z^3C + 3z^2C^2 - 2zC^3 + 3z^2C + 6zC^2 - C^3 - z^2) \quad \ldots(42)$$

This leading order balances obtained are solvable by elementary methods, but they either reduced to the balance considered below or need numerical solutions Hill et al.(1989) noted that to get non-trivial solution, the highest derivatives are retained and leading order balance gives:

$$\left(\frac{d^2}{dz^2} - \sigma Sc^{-1}\right)\frac{d^2w}{dz^2} = d\tilde{k}^2 R\frac{d\phi^*}{dz} \quad \ldots\ldots(43)$$

Case I: Rigid upper surface

Expanding the quantities ϕ^*, w, R and in power series of d,

$$\phi^* = \sum_{n=0}^{\infty} d^n\phi_n^*,\ w = \sum_{n=1}^{\infty} d^n w_n,\ R = \sum_{n=0}^{\infty} d^n R_n,\ \sigma = \sum_{n=0}^{\infty} d^n\sigma_n$$

we get the following systems.

Leading order system:

$$\frac{d^4w_1}{dz^4} - (\sigma_0 Sc^{-1} + Pl)\frac{d^2w_1}{dz^2} = K^2R_0\frac{d\phi_0^*}{dz} \ldots(44)$$

$$\frac{d^3\phi_0^*}{dz^3} - \sigma_0\frac{d\phi_0^*}{dz} = 0 \quad \ldots(45)$$

Boundary conditions

Rigid upper surface

$$\frac{d^2\phi_0^*}{dz^2} = 0 \text{ at } z=-1,0 \quad \ldots(46)$$

$$w_1 = \frac{dw_1}{dz} = 0 \text{ at } z=-1,0 \quad \ldots(47)$$

Further

$$\frac{d^2\phi_0^*}{dz^2} = 0 \text{ at } z = 0,\ \phi_0^*(z) = 0 \quad \text{at } z=0$$

Equation (45) reduces to

$$\Rightarrow \phi_0^*(z) = -z \quad \text{when } \sigma_0 = 0 \quad \ldots(48)$$

And equation (44) yields

$$w_1 = -\tilde{k}^2R_0\left[Pl\frac{z^6}{6!} + \frac{z^4}{4!} + k_1\frac{z^3}{3!} + k_2\frac{z^2}{2!}\right] \quad \ldots(49)$$

$$k_1 = \frac{4Pl}{5!} + \frac{1}{2},\quad k_2 = \frac{Pl}{5!} + \frac{1}{12} \quad (50)$$

Second order system

$$\frac{d^4w_2}{dz^4} - \left(\sigma_0 Sc^{-1} + Pl\right)\frac{d^2w_2}{dz^2} - \sigma_1 Sc^{-1}\frac{d^2w_1}{dz^2} = \tilde{k}^2R_0\frac{d\phi_0^*}{dz} + \tilde{k}^2R_1\frac{d\phi_0^*}{dz} \quad \ldots(51)$$

$$\frac{d^3\phi_1^*}{dz^3} + C\frac{d^2\phi_0^*}{dz^2} - \int_z^o ds$$

$$\frac{d^2\phi_0^*}{dz^2} - \sigma_0\frac{d\phi_1^*}{dz} - \sigma_1\frac{d\phi_0^*}{dz} + 2\frac{d\phi_0^*}{dz} = 0 \quad \ldots(52)$$

With the boundary conditions

$$\frac{d^2\phi_1^*}{dz^2} - 2z - C = 0 \quad \text{at } z=-1,0$$

$$w_2 = \frac{dw_2}{dz} = 0 \quad \text{at } z=-1,0$$

$$\phi_1^*(z) = 0 \quad \text{at } z=0$$

Solution

Integrating and applying the boundary conditions we get

$$\phi_1^* = \frac{1}{3}z^3 + \frac{1}{2}Cz^2 \quad \ldots(53)$$

$$w_2 = \left(2\tilde{k}^2R_0Pl\right)\frac{z^8}{8!} + \left(\tilde{k}R_0PlC\right)\frac{z^7}{7!} + \tilde{k}\left(2R_0 - PlR_1\right)\frac{z^6}{6!} + \left(C\tilde{k}R_0\frac{z^5}{5!} - \tilde{k}^2R_1\frac{z^4}{4!}\right) + k_3\frac{z^3}{3!} + k_4\frac{z^2}{2!} \quad \ldots(54)$$

$$k_3 = \frac{\tilde{k}^2 R_0}{5!}\left[2(4-9C)+\frac{Pl}{14}(3-10C)\right]-\frac{\tilde{k}R_1}{2}(1+\frac{Pl}{15}) \quad \ldots(55)$$

$$k_4 = \frac{\tilde{k}^2 R_0}{5!}\left[2(1-2C)+\frac{Pl}{42}(\frac{5}{2}-8C)\right]-\frac{\tilde{k}R_1}{12}(1-\frac{Pl}{10}) \quad \ldots(56)$$

Third order system

$$\frac{d^4 w_3}{dz^4}-Pl\frac{d^2 w_3}{dz^2}-\sigma_0 Sc^1\frac{d^2 w_3}{dz^2}-\sigma_1 Sc^{-1}\frac{d^2 w_2}{dz^2}$$
$$-\sigma_2 Sc^{-1}\frac{d^2 w_1}{dz^2}-2\tilde{k}^2\frac{d^2 w_1}{dz^2}+\sigma_0 Sc^1\tilde{k}^2 w_1+Pl\tilde{k}^2 w_1$$

$$=\tilde{k}^2 R_0\frac{d\phi_2^*}{dz}+\tilde{k}^2 R_1\frac{d\phi_1^*}{dz}+\tilde{k}^2 R_1\frac{d\phi_0^*}{dz} \quad \ldots(57)$$

$$\frac{ds_1}{dz}f_0^* - s_2\frac{df_0^*}{dz}+2\frac{df_1^*}{dz}+2S_1\frac{df_0^*}{dz}-k\tilde{k}^2\frac{df_0^*}{dz}$$
$$-\int_z^0 dz\frac{d^2\phi_1^*}{dz^2}+C\frac{d^2\phi_1^*}{dz^2}+\frac{d^3\phi_2^*}{dz^3}=-w_1\frac{ds_1}{dz} \quad \ldots(58)$$

Boundary Conditions

$$\frac{d^2\phi_2^*}{dz^2}=0 \qquad \text{at } z=0$$

$$\frac{d^2\phi_2^*}{dz^2}=C^2-3C+\frac{5}{3} \qquad \text{at } z=-1 \quad w_3=\frac{dw_3}{dz}=0$$
at z=-1,0

$$\phi_2^*(z)=0 \qquad \text{at } z=0$$

Solution

$$s_2 = k_5 - k\tilde{k}^2 \quad (59)$$
$$k_5=\tilde{k}^2 R_0\left[\frac{Pl}{8!}(7-8C)+\frac{1}{6!}(5-6C)-\frac{k_1}{5!}(4-5C)+\frac{k_2}{4!}(3-4C)\right]$$

…(60)

Taking $\sigma_2 = 0$ on the neutral curve expansion (58) gives

$$R_0=\frac{k}{\left[\frac{Pl}{8!}(7-8C)+\frac{1}{6!}(5-6C)-\frac{k_1}{5!}(4-5C)+\frac{k_2}{4!}(3-4C)\right]} \quad \ldots(61)$$

Now Substituting the value of σ_2 and integrating (58) three times and applying the boundary conditions we get the expression for ϕ_2^* as follows.

$$\phi_2^*=-\tilde{k}^2 R_0\left\{(\frac{7Pl}{10!})z^{10}+\frac{CPl}{9!}z^9+\frac{5}{8!}z^8+\frac{1}{7!}(4k_1+C)z^7+\frac{1}{6!}(3k_2+k_1C)z^6\right\}$$
$$-\frac{1}{5!}\left(\tilde{k}^2 R_0 k_2 C+12\right)z^5-\frac{C}{3}z^4-\frac{1}{6}\left((C^2+C-\frac{1}{3})+k_5\right)z^3+z \quad \ldots(62)$$

where k_1 and k_2 are given by (50)

Considering the fourth order system,

$$\frac{ds_1}{dz}f_1^*+\frac{ds_2}{dz}f_0^* - s_0\frac{df_3^*}{dz}-s_1\frac{df_2^*}{dz}-s_2\frac{df_1^*}{dz}$$
$$-s_3\frac{df_0^*}{dz}+2\frac{df_2^*}{dz}+2s_1\frac{df_1^*}{dz}+2S_2\frac{df_0^*}{dz}-k\tilde{k}^2\frac{df_1^*}{dz}$$
$$-\int_z^0 ds\frac{d^2f_2^*}{dz^2}-\int_z^0 S_1 ds\frac{d^2f_1^*}{dz^2}-\int_z^0 S_2 ds\frac{d^2f_0^*}{dz}$$
$$+C\frac{d^2f_2^*}{dz}+\frac{d^3f_3^*}{dz^3}=-\frac{ds_1}{dz}w_2-\frac{ds_2}{dz}w_1 \quad \ldots(63)$$

Boundary Conditions

$$\frac{d^2\phi_3^*}{dz^2}=-C \qquad \text{at } z=0$$

$$\frac{d^3\phi_3^*}{dz^2}=\frac{C^3}{3}-\frac{19}{12}C^2+\frac{5C}{3}-\frac{23}{45}+(2-C)+\frac{\tilde{k}R_0}{1440}\left(\frac{5C}{42}-\frac{11}{84}\right) \text{ at } z=-1$$

$\phi_3^*(z)=0$ at z = 0, $w_4=\dfrac{dw_4}{dz}=0$ at z=-1,0

Applying boundary conditions and substituting for

$$\phi_0^*,\phi_1^*,S_1,S_2,w_1,w_2, \quad \sigma_0=\sigma_1=0$$

Integration of Equation (63) between -1 to 0 yields

$$\sigma_3=\frac{\tilde{k}^2 R_0}{6!}\left[\begin{array}{l}(\frac{2}{3}C^3-\frac{3}{2}C^2+\frac{95}{84}C-\frac{17}{84})+\\ \frac{Pl}{10}(\frac{20}{21}C^3-\frac{27}{28}C^2+\frac{17}{72}C+\frac{1}{42})\end{array}\right]$$
$$+\frac{\tilde{k}R_1}{6!}\left[(5-6C)-\frac{Pl}{56}(8C-7)\right]$$
$$+\frac{k_3}{5!}\left[(4-5C)-\frac{k_4}{4!}(3-4C)\right] \quad \ldots(64)$$

where k_3, k_4 are given by (55) and (56)

Taking $\sigma_3 = 0$, we get the first order Rayleigh number as

$$R_1 = R_0 \frac{\left[\frac{1}{420}\left(280C^3 - 420C^2 + 223C - 43\right) + \frac{Pl}{7!}\left(480C^3 - 546C^2 + 149C + 3\right)\right]}{\left[(C - \frac{1}{2}) + Pl(\frac{C}{7} - \frac{3}{40})\right]} \qquad \ldots(65)$$

Which is exactly same as that of Vincent and Hill(1996) when Pl=0.

Case II: Stress free upper surface.

An analysis for stress free upper surface was carried out in a similar way. The solution to the leading order system with boundary conditions is as follows.

$$w_1 = -\tilde{k}^2 R_0 \left(\frac{Pl}{6!} z^6 + \frac{1}{4!} z^4 + \frac{A_1}{3!} z^3 + A_2 Z \right) \qquad \ldots(66)$$

$$A_1 = \frac{3}{8} + \frac{Pl}{48} \qquad A_2 = \frac{-1}{48} + \frac{7Pl}{2.6!} \qquad \ldots(67)$$

$$\sigma_2 = \frac{\tilde{k}^2 R_0}{6!} \left[(1 - 9C) + \frac{25}{8} Pl(\frac{5C}{7} - 1) \right] \qquad \ldots(68)$$

Solving the second order system we get

$$w_2 = \left(\frac{2\tilde{k}^2 R_0 Pl}{8!} \right) z^8 + \left(\frac{\tilde{k}^2 R_0 Pl C}{7!} \right) z^7 + \frac{\tilde{k}^2}{6!} \left(2R_0 - R_1 Pl \right) Z^6$$

$$+ \left(\frac{\tilde{k}^2 R_0 C}{5!} \right) z^5 - \left(\frac{\tilde{k}^2 R_1}{4!} \right) z^4 + \frac{\tilde{k}^2 A_3}{3!} z^3 + \left(\tilde{k}^2 A_4 \right) z \qquad \ldots(69)$$

$$A_3 = R_0 \left[\frac{1}{5!} \left(\frac{5}{3} - 6C \right) + \frac{Pl}{7!} \left(\frac{21}{8} - 9C \right) \right] - \frac{R_1}{8} \left[\frac{9}{5} + \frac{Pl}{12} \right] \qquad \ldots(70)$$

$$A_4 = \frac{R_0}{8!} \left[\frac{56}{3} - Pl(5 - 4C) \right] - \frac{R_1}{6!} \left[15 - \frac{Pl}{4} \right] \qquad \ldots(71)$$

The results are computed and presented through graphs.

RESULTS AND DISCUSSION

The results of the present investigation are presented in figures 1 to22. The results are computed for different sets of governing parameters (κ, d, C, Pl) for rigid upper and stress free upper surfaces. The results for small as well as large i.e. in the range $0 \leq Pl \leq 10^3$ are presented through graphs. The following important observations are made from the figures:

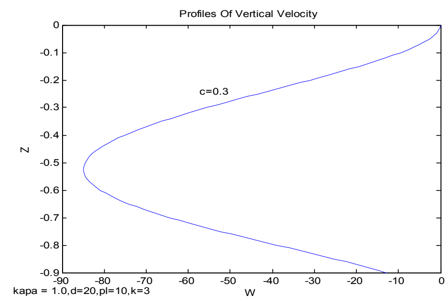


Figure 1: Z vs. W;

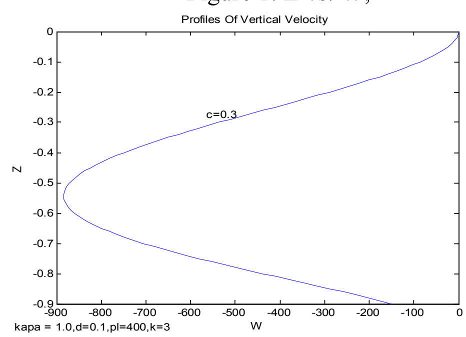


Figure 2: Z vs. W;

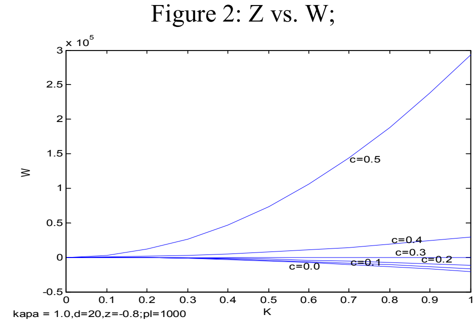


Figure 3: W vs. K;

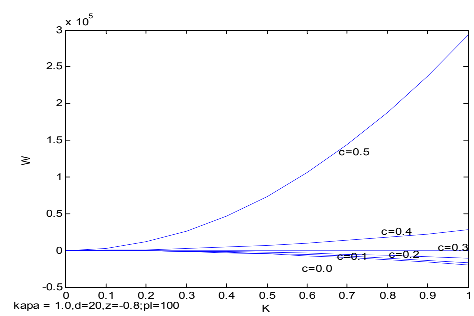


Figure 4: W vs. K;

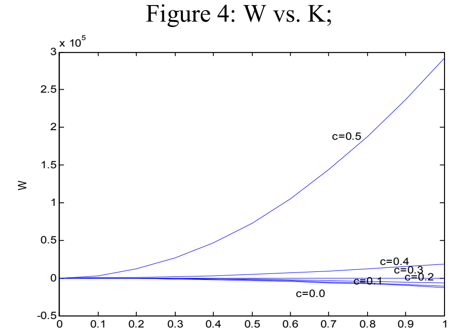


Figure 5: W vs.K;

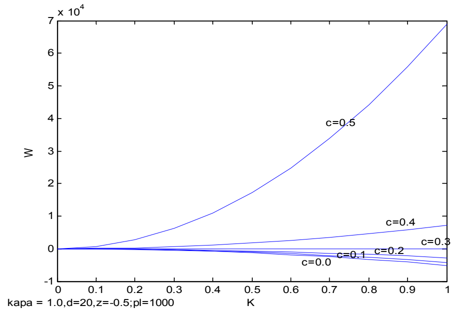


Figure 6: W vs. K;

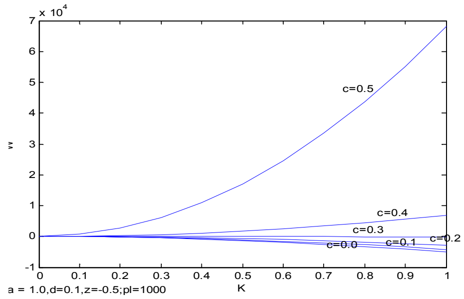


Figure 7: W vs. K;

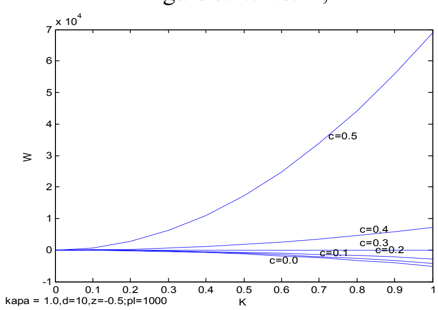


Figure 8: W vs. K;

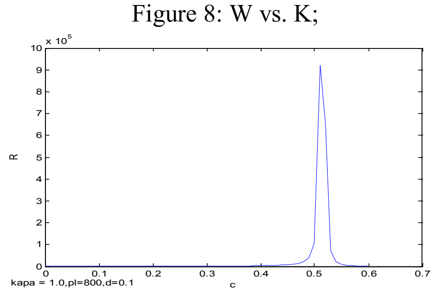


Figure 9: R vs. C;

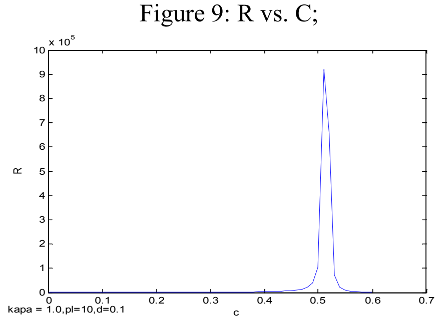


Figure10: R vs. C;

(i) The velocity profiles are drawn for different combinations of the governing parameters. w(z) is negative throughout the

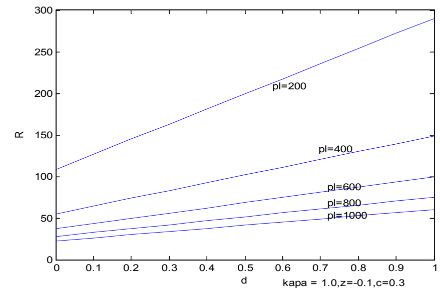


Figure 11: R vs.D;

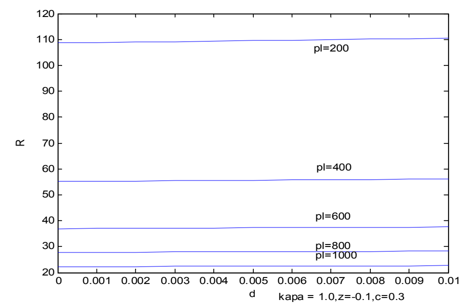


Figure 12: R vs. d;

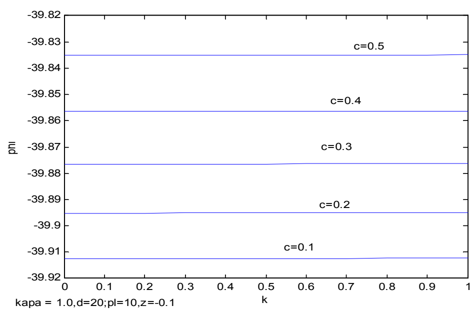


Figure 13: Phi vs.

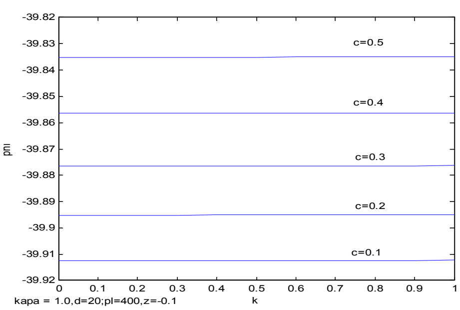
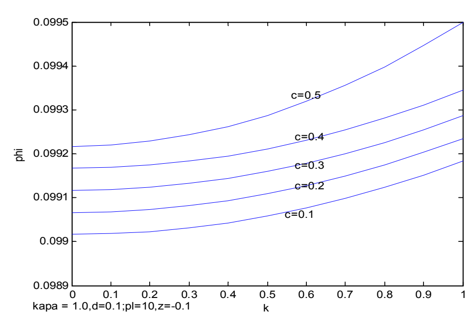


Figure 14:Phi vs. K



15 : Phi vs. k

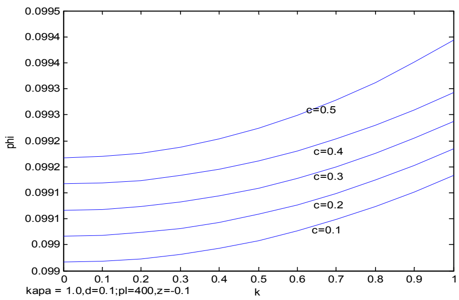


Figure 16: Phi vs.k;

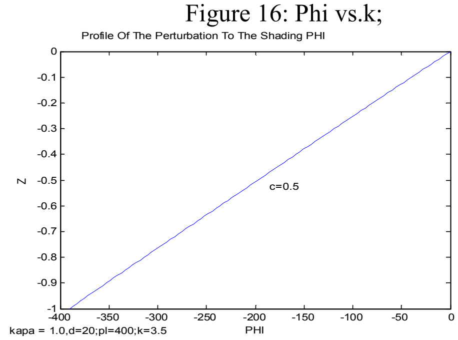


Figure 17: Z vs. Phi;

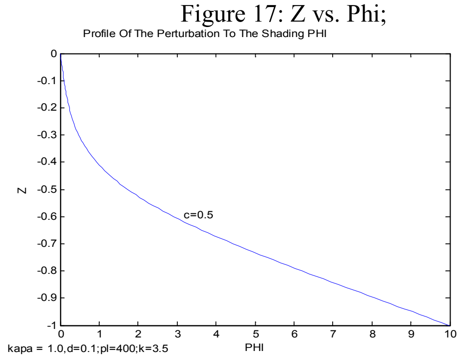


Figure 18 : Z vs. Phi;

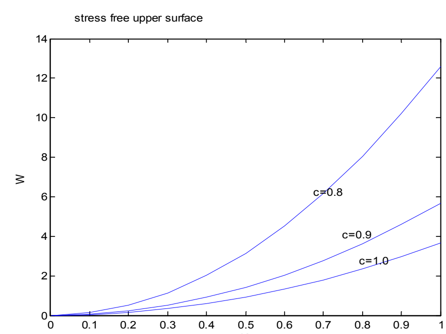


Figure 19:W vs.k

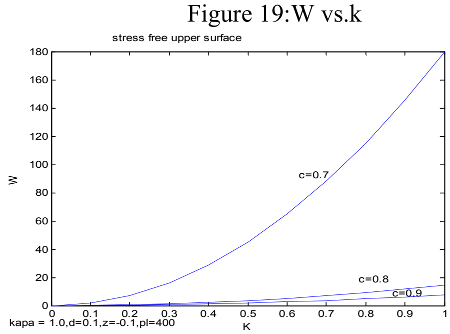


Figure 20 : W vs K;

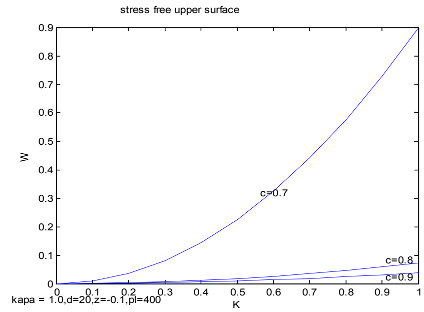


Figure 22: W vs. K;

region only when d = 20 and C ≤ 0.3. when Pl is in the range $10 \leq Pl \leq 10^3$. For d=20, Pl=10, C=0.3 and d=0.1, Pl=400, C=0.3 the profiles are parabolic with a maximum negative value at the middle of the layer (Figures 1 and 2). The parabolic nature strongly depends on d, C and Pl. The effect of increasing C is to decrease w(z) for a particular wave number k. As expected, for small as well as large values of d, w=0 as k→0, irrespective of the values of C (Figures 3 to 8). For values of C≥ 0.3, w increases continuously with k for a particular value of C both in the shallow as well as deep layers as in the case of ordinary bioconvection. Further w increases with Pl for specific values of k and C. (Figures 5 to 8). This clearly predicts that the effect of Darcian parameter is quite remarkable in the case of bioconvection.

(ii) In figures 9 and 10, the variation of total R with respect to C is presented for small as well as large values of Pl. It is observed that the convection cells grows in the range 0.45 < C ≤ 0.55. A single peak is observed in both the cases.

(iii) From figure 11, it is observed that bioconvection is enhanced by Pl contrary to the case of ordinary porous convection (Rudraiah and Srimani 1980, Srimani 1981) for the specified range of the parameters. Further, the effect of increase in d is to increase R. From this it can be concluded that it is possible to control bioconvection through the Darcian parameter.

(iv) In figures 12 to 15, the graphs of the profiles of perturbations to the shading φ (z) vs. k are plotted for different values of C and Pl. It is observed that for small as well as large Pl, φ (z) remains constant for all values of k in the deep layers. Further, φ (z) is negative in the range discussed above. But, in the case of a shallow layer, φ (z) continuously increases with k for values of C in the range of 0.1 ≤ C ≤ 0.5 and is positive for small as well as large values of Pl. In other words, the shading never becomes zero.

(v) In figures 16 and 17, for stress-free upper surface, the graphs of the profiles of perturbations to the shading φ (z) vs. z are plotted for C=0.5 for deep as well as shallow layers. In the case of deep layer φ (z) is negative and is positive for the shallow layer case. The behaviour of φ (z) in a shallow layer is exactly the opposite that of a deep layer.

(vi) In figures 18 to 21, the profiles of w vs. k for a deep layer are presented for values of Pl=10; 400 and C=0.7, 0.8, 0.9, 1.0 for the case of stress– free upper surface. A comparison of the results in figures 18 to 21 with those of 3 and 4 clearly shows that the values of the vertical velocity are very much higher in the case of rigid upper surface and there is a considerable difference in the behaviour of bioconvective porous system with regard to rigid and stress free upper surfaces.

Finally, it is concluded that porous/Darcian parameter has a remarkable influence on the bioconvective porous system irrespective of the nature of the upper boundary. The results could be utilized to suppress or enhance bioconvection.

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