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RESEARCH ARTICLE

ON WEAK BI-IDEALS IN NEAR SUBTRACTION SEMIGROUP

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ABSTRACT

In this paper we introduce the notation of weak bi-ideals in near-subtraction semigroup. (ie) A subalgebra B of (X,–) is said to be a weak bi-ideal in near-subtraction semigroup if $B^3 \subseteq B$ and obtain equivalent conditions for various regularities in terms of weak bi-ideals and bi-ideals.

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1. INTRODUCTION

Schein (Steinfeld, 1956) considered systems of the form (X; o;/), where X isa set of functions closed under the composition "o" of functions (and hence (X; o) is a function semigroup) and the set theoretic subtraction "/" (and hence (X;/) is a subtraction algebra in the sense of (Abbott, 1969)). He proved that every subtraction semigroup is isomorphic to a difference semigroup of invertible functions. Zelinka (1995) discussed a problem proposed by Schein concerning the structure of multiplication in a subtraction semigroup. He solved the problem for subtraction algebras of a special type, called the atomic subtraction algebras. Jun *et al*. (2003) introduced the notion of ideals in subtraction algebras and discussed characterization of ideals. In (Jun and Kim, 2007), Jun and Kim established the ideal generated by a set, and discussed related results. For basic definition one may refer to Pilz (1992). In ring theory the notation of quasi-ideal introduced by Steinfield in (Tamizh Chelvam and Ganesan, 1987). Motivated by the study of strongly regular in "On Strongly Regular Near Subtraction Semigroups" by Dheena and Satheesh Kumar and bi-ideals in "Bi-ideals of Near-Subtraction Semigroup" by Maharasi, Mahalakshmi and Jayalakshmi, the new concept of 'Weak bi-ideal' is introduced and also motivated by the study of strongly regular in "Strongly regular and bi-ideals of near subtraction semigroup" by Maharasi, Mahalakshmi and Jayalakshmi and In this paper, with a new idea, we define weak bi-ideal and investigate some of its properties. We characterize weak bi-ideal by sub algebra of near subtraction semi group. A characterization of weak bi-ideals of near subtraction semigroup is given. In the case of zero symmetric and left self-distributive s-near subtraction semigroup we establish weak bi-ideal B is strongly regular under necessary and sufficient condition. In this paper we shall introduce weak bi-ideal in near subtraction semigroups and obtain equivalent conditions for weak bi-ideals in near subtraction semigroups using quasi-ideals. This concept motivates the study of different kinds of new ideas in algebraic theory especially ideas in subtraction bialgebra and Fuzzy Prime ideal in semigroup and maximal fuzzy field.

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2. Preliminaries on Near Subtraction Algebra

Definition 2.1. A nonempty set X together with binary operations " " and is said to be a subtraction algebra if it satisfies the following:

(i) x (y x) = x.
(ii) x (x y) = y (y x).
(iii)(x y) z = (x z) – y, for every x, y, z∈X.

Definition 2.2. A nonempty set X together with two binary operations " " and "•" is said to be a subtraction semigroup if it satisfies the following:
(i) (X,) is a Subtractionalgebra.
(ii) (X,•) is a Semigroup.
(iii) x(y z) = xy xz and (x y)z = xz yz , for every x, y, z∈X.

Definition 2.3. A nonempty set X together with two binary operations " " and "•" is said to be a near subtraction semigroup (right) if it satisfies the following:

(i) (X,) is a Subtraction algebra.
(ii) (X,•) is a Semigroup.
(iii) (x y)z = xz yz , for every x, y, z∈X.

Definition 2.4. A nonempty subset S of a near subtraction semigroup X is said to be a subalgebra of X, if x – x'∈S whenever x, x'∈S.

Note 2.5. Let X be a near subtraction semigroup. Given two subsets A and B of X, AB = {ab/a∈A, b∈B}. Also we define another operation "∗ " A∗B = {ab a(a′ b)/a, a′∈A, b∈B}.

Definition 2.6. A subalgebra B of (X,) is said to be bi–ideal of X if BXB∩BX ∗B⊆B. In the case of a zero-symmetric near subtraction semigroup, a subalgebra B of X is a quasi-ideal if BXB ⊆B.

Definition 2.7. A subalgebra Q of (X,–) is said to be a quasi-ideal of X if QX∩XQ∩X∗Q⊆Q. In the case of a zero-symmetric near subtraction semigroup, a subalgebra Q of X is a quasi-ideal if QX∩XQ⊆Q.

Definition 2.8. We say that X is an s(s′) near-subtraction semigroup if a∈Xa(aX), for all a∈X.

Definition 2.9. A s-near subtraction semigroup X is said to be a $\bar{S}$-near subtraction semigroup if x∈xX, for all x∈X.

Definition 2.10. A near subtraction semigroup X is said to be subcommutative if aX =Xa, for every a∈X.

Definition 2.11. A near subtraction semigroup X is said to be left-bipotent if Xa=Xa^2, for every a∈X.

Definition 2.12. An element a∈X is said to be regular if for each a∈X, a=aba, for some b∈X.

Definition 2.13. A near subtraction semigroup X is said to have property (α) if xX is a subalgebra of (X,), for every x∈X.

Definition 2.14. A subalgebra S of (X,) is called an left(right)x-subalgebra of X if X S(SX)⊆S.

3. WEAK BI-IDEALS

In this section, with a new idea, we define weak bi-ideal and investigate some of its properties.

Definition 3.1. A near subtraction semigroup X is said to be left self-distributive s-near subtraction semigroup If abc = abac, for all a,b,c∈X.

Definition 3.2. A near subtraction semigroup is said to be strict weakly regular if A^2= A, for every left X-subalgebra A of X.

Definition 3.3. A subalgebra B of (X,–) is said to be a weak bi-ideal if B^3 ⊆ B.

Example 3.3.1. Let X = {0,a,b,1} in which " " and "•" are defined by,

–	0	a	b	1
0	0	0	0	0
a	a	0	a	0
b	b	b	0	0
1	1	b	a	0

•	0	a	b	1
0	0	0	0	0
a	0	a	0	a
b	0	0	b	b
1	0	a	b	1

Then X is weak bi-ideal.

Example 3.3.2. Let X = {0,a,b,c} in which " " and "•" are defined by,

–	0	a	b	c
0	0	0	0	0
a	a	0	c	b
b	b	0	0	b
c	c	0	c	0

•	0	a	b	c
0	0	0	0	0
a	0	0	a	a
b	0	0	c	b
c	0	0	b	c

Clearly {0,b} is not weak bi-idal, Since {0,b}^{3} = {0,c}⊄{0,b}

Note 3.4. Every Bi-ideal is a weak bi-ideal, but the converse is not true.

Example 3.4.1. Let X = {0,1,2} in which " " and "•" are defined by,

–	0	1	2
0	0	0	0
1	1	0	2
2	2	0	0

•	0	1	2
0	0	0	0
1	0	1	2
2	0	2	1

Clearly {0,1} is weak bi-ideal. But not bi-ideal, Since {0,1}X{0,1}= X⊄{0,1}.

Proposition 3.5. The set of all weak bi-ideals of a near subtraction semigroup X form a Moore system on X.

Proof. Let $\{Bi\}_{i\in I}$ be a set of all weak bi-ideals of X. Let B = ∩Bi. Then B^3=BBB⊆ BiBiBi = Bi^3⊆Bi⊆B for every i∈I. Therefore B is a weak bi-ideal of X.

Proposition 3.6. If B is a weak bi-ideal of a near subtraction semigroup X and S is a semigroup of X, then B∩S is a weak bi-ideal of X.

Proof. Let C = B∩S. Now C^3 = (B∩S)((B∩S)(B∩S)) ⊆ (B∩S)(BB∩SS) ⊆ (B∩S)BB ∩(B∩S)SS ⊆ BBB∩SSS = B^3∩SS ⊆ B∩S = C. (i.e.) C^3⊆C. Therefore B∩S is a weak bi-ideal of X.

Preposition 3.7. **Let B be a weak bi-ideal of X. Then Bb and b′B are the weak bi-ideals of X where b, b′∈B and b′ is a distributive element.**

Proof. **Clearly Bb is a subalgebra of (X,–). Then $(Bb)^3$ = BbBbBb ⊆ BBBb ⊆ B^3b ⊆ Bb. Since b′ is distributive, b′B is a subalgebra of (X,–) and $(b'B)^3$ = b′Bb′Bb′B ⊆ b′BBB = b′B^3 ⊆ b′B. Hence Bb and b′B are weak bi-ideals of X.**

Corollary 3.8. Let B be a weak bi-ideal of X. For b, c∈B, if b is distributive, then bBc is a weak bi-ideal of X.

Proposition 3.9. Let X be a left self-distributive s-near subtraction semigroup. Then B^3 = B for every weak bi-ideal B of X if and only if X is strongly regular.

Proof. Let B be a weak bi-ideal of X. If X is strongly regular, then X is regular. Let b∈B. Since X is regular, b = bab, for some a∈X. By our assumption that X is left self-distributive, we have bab = babb. Thus b = bab = babb = bab^2 = bb^2 = b^3∈B^3. (i.e.) B⊆B^3. Hence B = B^3, for every weak bi-ideal B of X.

Conversely let a∈X. Since Xa is a weak bi-ideal of X and X is a s-near subtraction semigroup, we get a∈Xa = $(Xa)^3$ = XaXaXa ⊆ XaXa. (i.e) a = x_1ax_2a. Since X is left self-distributive, a = $x_1ax_2a^2$. (i.e) X is strongly regular.

Remark 3.10. Let X be a left self-distributive s-near-subtraction semigroup. If B=B^3, for every weak bi-ideal B of X, then L = {0}.

Proposition 3.11. Let X be a left self-distributive s-near subtraction semigroup. Then B=XB^2, for every strong bi-ideal B of X if and only if X is strongly regular.

Proof. Assume that B=XB^2, for every strong bi-ideal B of X. Since Xa is a strong bi-ideal of X and X is a s-near subtraction semigroup, we have a∈Xa=X$(Xa)^2$ =XXaXa ⊆ XaXa. (i.e.) a = x_1ax_2a. Since X is a left self-distributive near-subtraction semigroup, a = x_1ax_2a = $x_1ax_2a^2$∈Xa^2. (i.e.) X is strongly regular.

Conversely, let B be a strong bi-ideal of X. Since X is strongly regular, for all b∈B there exist x∈X such that b = xb^2∈XB^2. (i.e.) B⊆XB^2. Hence XB^2 = B, for every strong bi-ideal B of X.

Theorem 3.12. Let X be a left self-distributive s-near subtraction semigroup. Then B^3=B, for every weak bi-ideal B of X if and only if XB^2=B for every strong bi-ideal B of X.

Proof: Follows from the Propositions 3.9 and 3.11.

Proposition 3.13. Let X left self-distributive s-near-subtraction semigroup. Then the following conditions are equivalent:

(i) X is strongly regular.
(ii) X is regular.
(iii) X is left-bipotent.

Proof. (i) ⇒ (ii) is true.
(ii) ⇒ (iii)

Assume that X is regular. Let a∈X and a = aba. If x∈Xa, then x = na = naba. Since X is a left self-distributive s-near-subtraction semigroup, x = naba = nabaa = $naba^2$∈Xa^2. (i.e) Xa⊆Xa^2. But trivially Xa^2⊆Xa and so Xa=Xa^2.(i.e) X is left-bipotent.

(iii) ⇒ (i)

Assume that X is left-bipotent. Then Xa=Xa^2. Since s-near-subtraction semigroup,

a∈Xa=Xa^2. (i.e) X is strongly regular.

Theorem 3.14. Let X be a left self-distributive s-near-subtraction semigroup. Then B^3 = B, for every weak bi-ideal B of X if and only if X is left-bipotent.

Proof. Follows from the Propositions 3.9 and 3.13.

Proposition 3.15. Let X be a left self-distributive s-near subtraction semigroup. Then B=BXB, for every bi-ideal B of X if and only if X is regular.

Proof. Let B be a bi-ideal of X. If X is regular, then B = BXB, for every bi-ideal B of X.

Conversely, let B = BXB, for every bi-ideal B of X. Since Xa is a bi-ideal of X and X is a s-near subtraction semigroup, we get a∈Xa=XaXXa. (i.e.,) a = x_1ax_2a, for some x_1,x_2∈X. Since X is a left self-distributive near subtraction semigroup, a = $x_1ax_2a^2$∈Xa^2. (i.e.) X is strongly regular and so X is regular.

Theorem 3.16. Let X be near subtraction semigroup. Then the following conditions are equivalent:

(i) X is strict weakly regular.
(ii) For every a∈X, a∈$(Xa)^2$.
(iii) For any two left X-subalgebras S_1 and S_2 such that S_1⊆S_2, we have S_2S_1=S_1.

Proof. (i) ⇒ (ii)

Since X is strict weakly regular, for each a∈X, Xa = $(Xa)^2$. Also for an s-near subtraction semigroup, a = xa, for some x∈X. Hence a∈Xa = $(Xa)^2$.

(ii) ⇒ (iii)

Let S_1 and S_2 be any two left X-subalgebras such that $S_1 \subseteq S_2$ and $a \in S_1$. Then by (ii), $a \in (Xa)^2 = XaXa \subseteq S_2S_1$ which means $S_1 \subseteq S_2S_1$. Also $S_2S_1 \subseteq XS_1 \subseteq S_1$.

(iii) ⇒ (i) Take $S_1 = S_2$.

Theorem 3.17. Let X be a left self-distributive s-near subtraction semigroup. Then the following conditions are equivalent:

(i) Q = QXQ for every quasi-ideal Q of X.
(ii) $B^3 = B$, for every weak bi-ideal B of X.
(iii) $XB^2 = B$, for every strong bi-ideal B of X.
(iv) X is regular.
(v) $B_1 \cap B_2 = B_1B_2 \cap B_2B_1$, for every pair of bi-ideals B_1, B_2 of X.
(vi) $Q_1 \cap Q_2 = Q_1Q_2 \cap Q_2Q_1$, for every pair of quasi-ideals Q_1, Q_2 of X.
(vii) $Q^2 = Q$, for every quasi-ideal Q of X.
(viii) $B^2 = B$, for every bi-ideal B of X.
(ix) X is strict weakly regular near subtraction semigroup.
(x) X is strongly regular.
(xi) X is left bi-potent.
(xii) B=BXB, for every bi-ideal B of X.

Proof. (i) ⇒ (ii)

Let $a \in X$. Since Xa is a quasi-ideal of X and X is a s-near subtraction semigroup, we have $a \in Xa = XaXXa \subseteq XaXa$. (i.e.) $a = x_1ax_2a$. Since X is left self-distributive, $a = x_1ax_2a = x_1ax_2a^2 \in Xa^2$. (i.e) X is strongly regular. Therefore, by Proposition 3.9, $B^3 = B$, for every weak bi-ideal B of X.

(ii) ⇒ (iii)

Let B be a strong bi-ideal of X. Every strong bi-ideal is a bi-ideal and so weak bi-ideal. By the assumption $B = B^3 = BBB = BB^2 \subseteq XB^2$. (i.e.) $B \subseteq XB^2$ and so $B = XB^2$, for every strong bi-ideal B of X.

(iii) ⇒ (iv)

By the Proposition 3.11, X is strongly regular and so X is regular.

(iv) ⇒ (v)

Let B_1 and B_2 be a pair of bi-ideals of X. Let $x \in B_1B_2 \cap B_2B_1$. Then $x = b_1b_2$ and $x = b_2'b_1'$. Since X is regular, $b_1 = b_1a_1b_1$ and $b_2 = b_2a_2b_2$, for some $a_1, a_2 \in X$. From this $x = b_1b_2 = b_1a_1b_1b_2 = b_1a_1b_2'b_1' \in B_1XB_1 \subseteq B_1$. (i.e.,) $B_1B_2 \cap B_2B_1 \subseteq B_1$. Similarly $B_1B_2 \cap B_2B_1 \subseteq B_2$. Hence $B_1B_2 \cap B_2B_1 \subseteq B_1 \cap B_2$.

On the other hand if $x \in B_1 \cap B_2$, then $x = b_1 = b_2$, for some $b_1 \in B_1$ and $b_2 \in B_2$. Since B_1 is a bi-ideal and X is regular, $B_1 = B_1XB_1$, and so $b_1 = b_1xb_1$, for some $x \in X$. Since X is a left self-distributive near subtraction semigroup, $x = b_1 = b_1xb_1 = b_1xb_1b_1 = b_1xb_1b_2 \in B_1XB_1B_2 \subseteq B_1B_2$. Therefore $B_1 \cap B_2 \subseteq B_1B_2$. Similarly one can prove that $B_1 \cap B_2 \subseteq B_2B_1$. Hence $B_1 \cap B_2 = B_1B_2 \cap B_2B_1$, for every pair of bi-ideals B_1, B_2 of X.

(v) ⇒ (vi)

Since every quasi-ideal is also a bi-ideal of X, we have $Q_1 \cap Q_2 = Q_1Q_2 \cap Q_2Q_1$, for every pair of quasi-ideals Q_1 and Q_2 of X.

(vi) ⇒ (vii)

Take $Q_1 = Q_2 = Q$. By the assumption $Q = Q \cap Q = Q^2 \cap Q^2 = Q^2$. Hence $Q^2 = Q$, for every quasi-ideal Q of X.

(vii) ⇒ (viii)

Let B be a bi-ideal of X and $x \in B$. Since X is a s-near subtraction semigroup and Xx is a quasi-ideal of X, $x \in Xx = (Xx)^2 = XxXx$. (i.e.,) $x = x_1xx_2x$, for some $x_1, x_2 \in X$. Since X is a left self-distributive near-subtraction semigroup, $x = x_1xx_2x = x_1xx_2x^2 \in Xx^2$. From this X is strongly regular and so X is regular. Let $a \in BX \cap XB$. Then $a = xn = n_1x_1$, for some $x, x_1 \in B$ and $n, n_1 \in X$. Since x is regular, $x = xyx$, for some $y \in X$. Hence $a = xn = (xyx)n = (xy)(n_1x_1) \in BXB \subseteq B$. (i.e.,) $BX \cap XB \subseteq B$. Therefore B is a quasi-ideal of X and so $B^2 = B$, for every bi-ideal B of X.

(viii) ⇒ (ix)

For $x \in X$, note that Xx is a bi-ideal of X. Since X is a s-near subtraction semigroup $x \in Xx = (Xx)^2$. Hence by the Theorem 3.16, X is a strict weakly regular near subtraction semigroup.

(ix) ⇒ (x)

Since X is a s-near subtraction semigroup, $a \in Xa = (Xa)^2 = (Xa)(Xa)$. (i.e.) $a = x_1ax_2a$ for some $x_1, x_2 \in X$. Since X is a left self-distributive near-subtraction semigroup, $a = x_1ax_2a = x_1ax_2a^2 \in Xa^2$. (i.e) X is strongly regular.

(x) ⇒ (xi) Follows from the Propositions 3.13.
(xi) ⇒ (xii)

Since X is a s-near-subtractionsemigroup, $a \in Xa = Xa^2$, X is strongly regular and so X is regular. Hence B = BXB, for every bi-ideal B of X.

(xii) ⇒ (i)

Let Q be a quasi-ideal of X. Since every quasi-ideal is a bi-ideal of X, Q = QXQ.

Conclusion

We have introduced the notation of weak bi-ideals and investigate some of their properties. Work is ongoing some important issues future works are

(i) To develop strategies for obtaining more valuable results.
(ii) To apply these definitions and results for studying related notations in other algebraic structures.
(iii) To describe the fuzzy structure of weak bi-ideals in near subtraction semigroup and its applications.

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