Available online at http://www.journalcra.com

INTERNATIONAL JOURNAL
OF CURRENT RESEARCH

International Journal of Current Research
Vol.8, pp.042-047, September, 2010

ISSN: 0975-833X

RESEARCH ARTICLE

INTERDEPENDENT QUEUEING SYSTEM WITH VARYING MODES OF OPERATION

Udaya Chandrika, K[1]., Ebenesar Anna Bagyam, J[2]. and Muthulakshmi, S[1].

[1]Department of Mathematics, Avinashilingam University for Women, Coimbatore – 641 043
[2]Department of Mathematics, SNS College of technology, Coimbatore - 641022

ARTICLE INFO

Article History:
Received 14th July, 2010
Received in revised form
25th July, 2010
Accepted 27th August, 2010
Published online 4th September, 2010

Key words:
Finite capacity queueing system
Control policy

ABSTRACT

Finite capacity queueing system with correlated arrival and service processes is considered with provision for additional server along with controllable arrival rate. The additional service channel is introduced according to (r_1, R_1) control policy. In this policy if the system size reaches R_1 additional server is introduced and the introduced server is withdrawn if the system size becomes less than r_1 ($< R_1$). In a similar manner the arrival rate is controlled according to (r_2, R_2) policy, where $r_1 < R_1 < r_2 < R_2$. The steady state probabilities and the performance measures are derived. Numerical results are presented.

INTRODUCTION

Models with dependencies between inter-arrival and service time have been studied by several authors due to their wide applications in communication networks where communication nodes are connected with finite capacity transmission links. Boxma and Perry, 2001; Cidon *et al.*, 1991; have considered queueing models with dependence between service and inter-arrival times. Hadidi, 1985; Hadidi, 1981; Conolly and Choo, 1979 have analysed the M/M/1 queue where the service time and the preceding inter-arrival time have a bivariate exponential density with a positive correlation. Queueing systems with the number of channels and arrival rate depending on the system size are well known. In the human server production system the speed of the server and the rate in which jobs arrive at the system depend on the amount of work present (Bekker *et al.,* 2004; Asmussen, 2003). In packet-switched communication system, feedback information on the buffer state provides the basis for the transmission control protocol to carefully regulate the transmission rate of internet flows.

These considerations led us to study interdependent queueing system where the arrival rate and the number of additional servers depend on the amount of work present. Srinivasa Rao (2000) and Srinivasan and Thiagarajan (2006) have considered respectively M/M/1 and M/M/1/K interdependent queueing models with controllable arrival rates and obtained the average system size and average waiting time in the system under steady state conditions. This paper presents finite capacity intercorrelated queueing system with simultaneous provision for controllable arrival rate and the introduction or withdrawal of additional server depending on the load of the system. This system is most suitable in banks, tax office, transportation centres, hospitals, etc., in order to meet the peak hour services.

MODEL DESCRIPTION

Consider a queueing system with N limited room capacity. There are four non-negative integers r_1, R_1, r_2, R_2 with $r_1 < R_1 < r_2 < R_2$ called switchover levels or thresholds. Let n be the load of the system. If $n \le r_1$, the system operates with single server having service rate μ_1. One additional service channel with service rate μ' is open when the number in the system reaches R_1 and the system operates with two servers having total service rate $\mu_2 = \mu_1 + \mu'$ for $n \ge R_1$. If $n \le r_2$ the arrival rate is λ_0 and when it increases to R_2 arrival rate is reduced to λ_1 and the system continues with the same arrival rate as long as $n \ge R_2$. The system keeps the current mode in the cases $r_1 < n < R_1$ and $r_2 < n < R_2$. In other words, if t is an arbitrary time and n, with $r_1 < n < R_1$ or $r_2 < n < R_2$, is the number of customers at the moment $t + 0$ then the mode of the system in force at $t + 0$ is same as that was in $t - 0$. The switchover time between modes is assumed to be negligible. The model under study is described diagrammatically in the following figure below.

Assume that the arrival process and the service process of the system are correlated and follow a bivariate Poisson process with mean dependent rate ε. Define the steady state probabilities as Let n denote the number of customers in the system. Different states of the system are as defined below (Table 1):

***Corresponding Author:** *ebenesar.j@gmail.com*

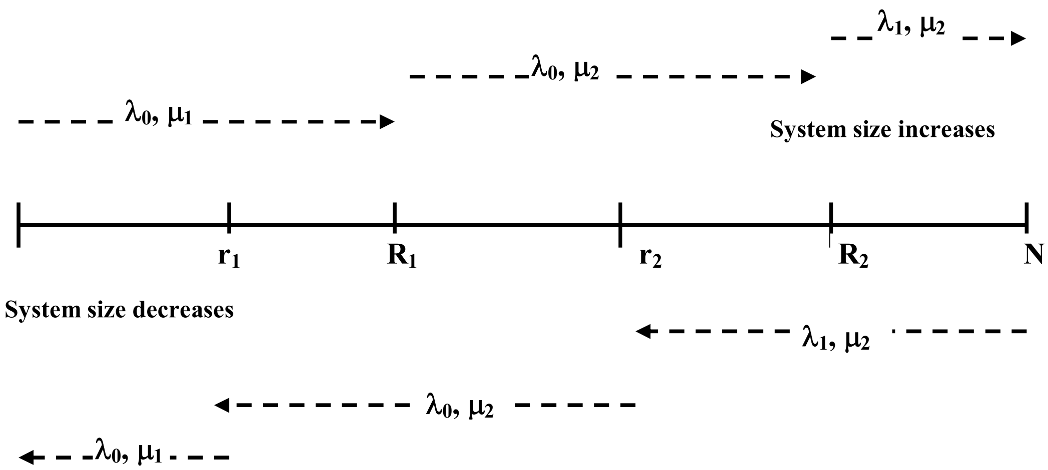

Fig. The model under study is described diagrammatically

Table. 1. Different states of the system

State	*Number of channels*	*Arrival rate*	*Range of n*	
			↑	↓
1	1	λ_0	$[0 \;\; R_1)$	$[0, r_1]$
2	2	λ_0	$[R_1 \;\; R_2)$	$(r_1, r_2]$
3	2	λ_1	$[R_2, N]$	$(r_2, N]$

Let $p_{n,i}$ be the probability that there are n customers in the system and the system is in state i. Then the equations governing the model under consideration are :

$$(\lambda_0 - \varepsilon)\, p_{0,1} = (\mu_1 - \varepsilon)\, p_{1,1} \quad (1)$$

$$(\lambda_0 + \mu_1 - 2\varepsilon)\, p_{n,1} = (\mu_1 - \varepsilon)\, p_{n+1,1} + (\lambda_0 - \varepsilon)\, p_{n-1,1} + \delta_{n r_1} (\mu_2 - \varepsilon)\, p_{n+1,2}$$
$$\text{for } n = 1, 2, \ldots, r_1 \ldots, R_1 - 2 \quad (2)$$

$$(\lambda_0 + \mu_i - 2\varepsilon)\, p_{R_i - 1, i} = (\lambda_0 - \varepsilon)\, p_{R_i - 2, i} \text{ for } i = 1, 2 \quad (3)$$

$$(\lambda_0 + \mu_2 - 2\varepsilon)\, p_{n,2} = (\mu_2 - \varepsilon)\, p_{n+1,2} + (1 - \delta_{n\, r_1 + 1})(\lambda_0 - \varepsilon)\, p_{n-1,2}$$
$$\text{for } n = r_1+1, \ldots, R_1 - 1 \quad (4)$$

$$(\lambda_0 + \mu_2 - 2\varepsilon)\, p_{n,2} = \delta_{n R_1} (\lambda_0 - \varepsilon)\, p_{n-1,1} + (\lambda_0 - \varepsilon)\, p_{n-1,2}$$
$$+ (\mu_2 - \varepsilon)\, p_{n+1,2} \quad \text{for } n = R_1 + R_1+1, \ldots, r_2 - 1 \quad (5)$$

$$(\lambda_0 + \mu_2 - 2\varepsilon)\, p_{n,2} = (\lambda_0 - \varepsilon)\, p_{n-1,2} + (\mu_2 - \varepsilon)\, p_{n+1,2} + \delta_{n r_2} (\mu_2 - \varepsilon)\, p_{n+1,3}$$
$$\text{for } n = r_2, \ldots, R_2 - 2 \quad (6)$$

$$(\lambda_1 + \mu_2 - 2\varepsilon)\, p_{n,3} = (1 - \delta_{n\, r_2 + 1})(\lambda_1 - \varepsilon)\, p_{n-1,3} + (\mu_2 - \varepsilon)\, p_{n-1,3}$$
$$\text{for } n = r_2+1, \ldots, R_2 - 1 \quad (7)$$

$$(\lambda_1 + \mu_2 - 2\varepsilon)\, p_{n,3} = (\lambda_1 - \varepsilon)\, p_{n-1,3} + (\mu_2 - \varepsilon)\, p_{n+1,3} + \delta_{n R_2} (\lambda_0 - \varepsilon)\, p_{n-1,2}$$
$$\text{for } n = R_2, \ldots, N-1 \quad (8)$$

$$(\mu_2 - 2\varepsilon)\, p_{N,3} = (\lambda_1 - \varepsilon)\, p_{N-1,3} \quad (9)$$

Let $\rho_1 = \dfrac{\lambda_0 - \varepsilon}{\mu_1 - \varepsilon}$, $\rho_2 = \dfrac{\lambda_0 - \varepsilon}{\mu_2 - \varepsilon}$, $\rho_3 = \dfrac{\lambda_1 - \varepsilon}{\mu_2 - \varepsilon}$

Solving recursively the equations (1) through (19) we get the following steady state probabilities.

$$p_{n,1} = \rho_1^{\,n}\, p_{0,1} \qquad n = 1, 2, \ldots, r_1$$

For $n = r_1+1, \ldots, R_1-1$,

$$p_{n,1} = \begin{cases} (\rho_1^{n} - \rho_1^{R_1+r_1}(1-\rho_1^{n-r_1})/(\rho_1^{r_1} - \rho_1^{R_1}))\, p_{0,1}\,, & \rho_1 \neq 1 \\ p_{0,1} - ((n-r_1)/\rho_2)\, p_{r_1+1,2}\,, & \rho_1 = 1 \end{cases}$$

$$p_{r_1+1,2} = \begin{cases} (\rho_2(1-\rho_1)\,\rho_1^{R_1+r_1-1})/(\rho_1^{r_1} - \rho_1^{R_1}))\, p_{0,1}\,, & \rho_1 \neq 1 \\ (\rho_2/(R_1-r_1))\, p_{0,1}\,, & \rho_1 = 1 \end{cases}$$

For $n = r_1+2, \ldots, R_1$,

$$p_{n,2} = \begin{cases} ((1-\rho_2^{n-r_1})/(1-\rho_2))\, p_{r_1+1,2}\,, & \rho_2 \neq 1 \\ (n-r_1)\, p_{r_1+1,2}\,, & \rho_2 = 1 \end{cases}$$

For $n = R_1+1, \ldots, r_2$,

$$p_{n,2} = \begin{cases} ((\rho_2^{n-R_1} - \rho_2^{n-r_1})/(1-\rho_2))\, p_{r_1+1,2}\,, & \rho_2 \neq 1 \\ (R_1-r_1)\, p_{r_1+1,2}\,, & \rho_2 = 1 \end{cases}$$

For $n = r_2+1, \ldots, R_2-1$,

$$p_{n,2} = \begin{cases} ((\rho_2^{n-R_1} - \rho_2^{n-r_1})\, p_{r_1+1,2} - (1-\rho_2^{n-r_2})\, p_{r_2+1,3})/(1-\rho_2)\,, & \rho_2 \neq 1 \\ (R_1-r_1)\, p_{r_1+1,2} - (n-r_2)\, p_{r_2+1,3}\,, & \rho_2 = 1 \end{cases}$$

$$p_{r_2+1,3} = \begin{cases} ((\rho_2^{R_2-R_1} - \rho_2^{R_2-r_1})/(1-\rho_2^{R_2-r_2}))\, p_{r_1+1,2}\,, & \rho_2 \neq 1 \\ ((R_1-r_1)/(R_2-r_2))\, p_{r_1+1,2}\,, & \rho_2 = 1 \end{cases}$$

For $n = r_2+2, \ldots, R_2$

$$p_{n,3} = \begin{cases} ((1-\rho_3^{n-r_2})/(1-\rho_3))\, p_{r_2+1,3}\,, & \rho_3 \neq 1 \\ (n-r_2)\, p_{r_2+1,3}\,, & \rho_3 = 1 \end{cases}$$

For $n = R_2+1, \ldots, N$

$$p_{n,3} = \begin{cases} ((\rho_3^{n-R_2} - \rho_3^{n-r_2})/(1-\rho_3))\, p_{r_2+1,3}\,, & \rho_3 \neq 1 \\ (R_2-r_2)\, p_{r_2+1,3}\,, & \rho_3 = 1 \end{cases}$$

PERFORMANCE MEASURES

The probability that the system is in state 1 is

$$P(1) = \sum_{n=0}^{R_1-1} p_{n,1}$$

$$= \begin{cases} (1/(1-\rho_1) - \rho_1^{r_1+R_1}(R_1-r_1)/(\rho_1^{r_1} - \rho_1^{R_1}))\, p_{0,1}\,, & \rho_1 \neq 1 \\ (R_1+r_1+1)\, p_{0,1}/2\,, & \rho_1 = 1 \end{cases}$$

The probability that the system is in state 2 is

$$P(2) = \sum_{n=r_1+1}^{R_2-1} p_{n,2}$$

$$= \begin{cases} ((R_1 - r_1) - (R_2 - r_2)(\rho_2^{R_2-R_1} - \rho_2^{R_2-r_1}) \big/ (1-\rho_2^{R_2-r_2}))(\rho_{r_1+1,2} \big/ (1-\rho_2)), \\ \qquad \rho_2 \neq 1 \\ (R_2 - R_1 - r_1 + r_2)\, p_{0,1} \big/ 2, \qquad \rho_2 = 1 \end{cases}$$

The probability that the system is in state 3 is

$$P(3) = \sum_{n=r_2+1}^{N} p_{n,3}$$

$$= \begin{cases} ((R_2 - r_2) - (\rho_3^{N-R_2+1} - \rho_3^{N-r_2+1}) \big/ (1-\rho_3))\,(p_{r_2+1,3} \big/ (1-\rho_3)), \quad \rho_3 \neq 1 \\ (2N - R_2 - r_2 + 1)\, p_{0,1} \big/ 2, \qquad \rho_3 = 1 \end{cases}$$

The probability $p_{0,1}$ that the system is empty can be calculated from the normalizing condition $P(1) + P(2) + P(3) = 1$ as

$$p_{0,1}^{-1} = (\rho_2(1-\rho_1)\, \frac{(\rho_2^{R_2-R_1} - \rho_2^{R_2-r_1})}{1-\rho_2^{R_2-r_2}} \left(\frac{(R_2 - r_2)(\rho_3 - \rho_2)}{(1-\rho_2)(1-\rho_3)} - \frac{(\rho_3^{N-R_2+1} - \rho_3^{N-r_2+1})}{(1-\rho_3)^2} \right)$$

$$(R_1 - r_1)(\rho_2 - \rho_1))\, \frac{\rho_1^{r_1+R_1-1}}{\rho_1^{r_1} - \rho_1^{R_1}} + \frac{1}{1-\rho_1}, \quad \rho_1 \neq 1,\ \rho_2 \neq 1,\ \rho_3 \neq 1$$

$$= \frac{1}{N+1}, \quad \rho_1 = \rho_2 = \rho_3 = 1$$

We now calculate the expected number of customers in the each state and in the system

Case i : Assume $\rho_1 \neq 1, \rho_2 \neq 1, \rho_3 \neq 1$

Expected number of customers in state 1, 2 and 3 are respectively given by L_{s_1}, L_{s_2} and L_{s_3} as

$$L_{s_1} = p_{0,1} \left(\frac{\rho_1}{(1-\rho_1)^2} - \frac{\rho_1^{R_1+r_1} (R_1 - r_1)(1+\rho_1)}{2(\rho_1^{r_1} - \rho_1^{R_1})(1-\rho_1)} \left(\frac{(1+\rho_1)(R_1 + r_1)}{1-\rho_1} - 1 \right) \right)$$

$$L_{s_2} = p_{r_1+1,2}\ ((R_1 - r_1)(1 + \rho_2 + (1-\rho_2)(R_1 + r_1))$$

$$+ (R_2 - r_2)(1 + \rho_2 + (1-\rho_2)(R_2 + r_2))\ (\rho_2^{R_2-r_1} - \rho_2^{R_2-R_1}) \big/ (1-\rho_2^{R_2-r_2}))$$

$$L_{s_3} = \frac{p_{r_2+1,3}}{2(1-\rho_3)^2}\ ((R_2 - r_2)(1-\rho_3)(1 + \rho_3 + (1-\rho_3)(R_2 + r_2))$$

$$+\ 2\rho_3 (1 + N(1-\rho_3))\ (\rho_3^{N-r_2} - \rho_3^{N-R_2}))$$

$L_s = L_{s_1} + L_{s_2} + L_{s_3}$, gives the expected number of customers in the system.

Case ii : Assume $\rho_1 = \rho_2 = \rho_3 = 1$. Then,

$$L_{s_1} = p_{0,1}(R_1^2 + r_1^2 + R_1 r_1 - 1)/6$$

$$L_{s_2} = p_{0,1}(R_2^2 + r_2^2 + R_2 r_2 - (R_1^2 + R_1 r_1 + r_1^2))/6$$

$$L_{s_3} = p_{0,1}(1 + 3N(N+1) - (R_2^2 + r_2^2 + R_2 r_2))/6$$

and $L_s = N/2$

Numerical Results

For selected parameters presented in Table 1, expected system size L_s and probabilities that the system is in the state 1, state 2 and state 3 are calculated and presented in Tables 2, 3 and 4 by varying ε, λ_0 and μ_1.

Table 1. Input parameters

λ_0	λ_1	μ_1	μ_2	ε	r_1	R_1	r_2	R_2	N
14	2	8	10	0.5	4	8	12	16	20

Table 2 Performance measures by varying correlation coefficient

ε	L_s	P(1)	P(2)	P(3)
0	13.5492	0.0290	0.6184	0.3316
0.2	13.4635	0.0278	0.6205	0.3317
0.4	13.3919	0.0265	0.6225	0.3317
0.6	13.3343	0.0253	0.6245	0.3317
0.8	13.2906	0.0240	0.6266	0.3317

Table 3 Performance measures by varying faster arrival rate

λ_0	L_s	P(1)	P(2)	P(3)
13	15.3390	0.0461	0.6411	0.2705
15	12.6922	0.0146	0.5928	0.3835
17	12.4293	0.0050	0.5264	0.4662
19	12.4538	0.0019	0.4681	0.5292
21	12.4969	0.0008	0.4201	0.5788
23	12.5266	0.0003	0.3805	0.6190
25	12.5438	0.0002	0.3476	0.6521

Table 4 Performance measures by varying service rate of first server

μ_1	μ_2	L_s	P(1)	P(2)	P(3)
2	4	9.4383	0.0000	0.1675	0.8325
3	5	10.7460	0.0000	0.2502	0.7497
4	6	11.4832	0.0002	0.3332	0.6665
5	7	11.8931	0.0009	0.4154	0.5831
6	8	12.1139	0.0034	0.4950	0.4995
7	9	12.3680	0.0101	0.5676	0.4157
8	10	13.3614	0.0259	0.6235	0.3317
9	11	17.4429	0.0564	0.6444	0.2480

The table values reveal the following :

(i) Correlation co-efficient between arrival and service rates has insignificant effect on L_s, P(1), P(2), P(3) due to the assumption of the model under consideration.

(ii) Increase in faster arrival rate, decreases the system size to a certain level due to additional server and increases after that. This aspect may be considered for the further study in order to obtain the optimum arrival rate.

(iii) Inspite of the increase in the rate of the first server, the system size increases when the rate of the second server remains constant. However the probabilities of the system is in state 1 and state 2 increase and state 3 decreases.

REFERENCES

Asmussen, S., 2003, Applied Probability and Queues, Second Edition, Springer, New York.

Bekker, R., S.C. Borst, O.J. Boxma and O. Kelle, (2004), Queues with Workload – Dependent Arrival and Service Rates, Queueing Systems 46, 537-556.

Boxma, O.J. and D. Perry, 2001. A Queueing Model with Dependence Between Service and Inter-Arrival Times, European J. Oper. Res., 128, 611-624.

Cidon, I., R. Guerin, A. Khamisy and M. Sidi, 1991, On Queues with Inter-Arrival Times Proportional to Service Times, Tech. Rept. EE Pub., 811, Technion.

Conolly, B. and Q.H. Choo, 1979. The Waiting Process for a Generalised Correlated Queue with Exponential Demand and Service, SIAM J. Appl. Math. 37, 263-275.

Hadidi, N., 1981. Queues with Partial Correlation, SIAM, J. Appl. Math. 40, 467-475.

Hadidi, N., 1985. Further Results on Queues with Partial Correlation, Oper. Res. 33, 203-209.

Srinivasa Rao, K., T. Shobha and P. Srinivasa Rao, 2000. The M/M/1/∞ Interdependent Queueing Model with Controllable Arrival Rate, Opsearch, 37, 14-24.

Srinivasan, A. and M. Thiagarajan, 2006. M/M/1/K Interdependent Queueing Model with Controllable arrival Rates, International Journal of Management and Systems, 22, 23-34.

****