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RESEARCH ARTICLE

SOME EQUIVALENT CONDITIONS ON k-ORTHOGONAL MATRICES

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ABSTRACT

Some equivalent conditions on k-Orthogonal matrices are given.

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INTRODUCTION

Let C_{nxn} be the space of nxn complex matrices. For a matrix A $\in C_{nxn}$, A^T, A^* and A^{-1} denote transpose, conjugate transpose and inverse of the matrix A respectively. Let k be a fixed product of disjoint transpositions in S_n the set of all permutations on $\{1,2,3,......n\}$ hence, involutary and let K be the permutation matrix associated with k. The concept of k-Orthogonal matrices is introduced as a generalization of k-real and k-hermitian (Hill and Water, 1992) and orthogonal matrices clearly K satisfies the following properties $K^2=I$ and $K=K^T=K^*$.

2.Definitions

Defn 2.1

A Matrix $A\in C_{nxn}$ is said to be k-orthogonal if $AKA^TK=KA^TKA=I$
Ie; $KA^TK=A^{-1}$

Example

$A=\begin{pmatrix}0 & 1\\ -1 & 0\end{pmatrix}$ is k-orthogonal

Definition 2.2

A matrix $A\in C_{nxn}$ is said to be k-Symmetric if $A=KA^TK$.

Definition 2.3

A matrix $A\in C_{nxn}$ is said to be involutary if $A^2=I$.

3. Some equivalent conditions on k-Orthogonal matrices.

Therom:3.1

If A is k-Orthogonal then both AA^T and A^TA are k-Orthogonal.

$(AA^T)^{-1} = (A^T)^{-1}A^{-1}$
$=\left(A^{-1}\right)^T A^{-1}$ $(\because KA^TK=A^{-1})$
$=(KA^TK)^T(KA^TK)$
$=KAKKA^TK$
$=KAA^TK$
$(AA^T)^{-1}=K(AA^T)^TK.$
$\therefore$ AA^T is k-orthogonal.

A similar proof may be given for A^TA.

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Therom:3.2

Any two of the following imply the other
(i)A is k-orthogonal
(ii)A is symmetric
(iii)KA or AK is involutary.

Proof: (i) and (ii) $\Rightarrow$ (iii)
$KA^TK=A^{-1}$
$KAK=A^{-1} \rightarrow$ (1) $\because A=A^T$
Post multiplying (1) by A we get
$KAKA=A^{-1}A$
$(KA)^2 = I$
Pre multiplying (1) by A we get
$AK\ AK = AA^{-1}$
$(AK)^2 = I$
(ii) and (iii) $\Rightarrow$ (i)
$(KA)^2 = I$
$KAKA=I$
$KAK=A^{-1}$
$KA^TK=A^{-1}$
$\because$ A is k-orthogonal (since A is symmetric)
A similar proof may be given when we assume $(AK)^2=I$.
(iii) and (i) $\Rightarrow$ (ii)
$(KA)^2=I$
$KAKA=I$
$KAK=A^{-1}$
$KAK=KA^TK$
$A = A^T$
A is symmetric.

Therom:3.3

Any two of the following imply the other.
(i)A is k-orthogonal.
(ii)A is k-Symmetric.
(iii)A is involutary.

Proof : (i) and (ii) $\Rightarrow$ (iii)
$KA^TK=A^{-1}$
$A=A^{-1}$ ($\therefore$ A is k-Symmetric)
$A^2=I$ $\therefore$ A is involutary.
(iii) and (i) $\rightarrow$(ii)
$A^2=I \Rightarrow AA= I$
$A=A^{-1}$
$A=KA^TK$
$\therefore$ A is k- Symmetric.

Remark:3.4 For any matrix A ,A commutes with K iff A^T commutes with K.

$KA=AK \Leftrightarrow (KA)^T = (AK)^T$
$\Leftrightarrow A^TK = KA^T$

Therom:3.5

Any two the following imply the other
(i)A is orthogonal
(ii)A is k-orthogonal
(iii)AK=KA

Proof: (i) and (ii) $\Rightarrow$(iii)
$KA^TK=A^{-1}$
$KA^{-1}K=A^{-1}$
$(KA^{-1}K)^{-1}=(A^{-1})^{-1}$
$KAK=A \Rightarrow AK=KA$
(ii) and (iii) $\Rightarrow$(i)
$KA^TK=A^{-1}$
$A^T=A^{-1}$ (by remark 3.4)
(iii)and (i) $\Rightarrow$(ii)
$A^T=A^{-1} \Rightarrow KKA^T=A^{-1}$
$\Rightarrow KA^TK=A^{-1}$. (by remark 3.4)

Remark 3.6:

If a matrix A is non singular then by Cayley-Hamilton therom we call find a polynomial P(t) such that $A^{-1}=P(A)$.

Therom:3.7

Any two of the following imply the other
(i)A is k-orthogonal
(ii)A^{-1}=P(A) where P(A) is a polynomial in A
(iii)A^*=P(KAK)

Proof: (i) and (ii) $\Rightarrow$(iii)

Since A is k-Orthogonal and non singular by remark (3.6), it is possible to find a polynomial P(t) such that $A^{-1}=P(A)$
Let $P(A) = \alpha_0 A^n + \alpha_1 A^{n-1}+\ldots\ldots+ \alpha_n I$, $\alpha_0 \neq 0$
But $KA^TK = A^{-1} = P(A)$
$A^T= K(\alpha_0 A^n + \alpha_1 A^{n-1}+\ldots\ldots+ \alpha_n I)K$
$= \alpha_0 KA^nK + \alpha_1 KA^{n-1}K + \ldots..+ \alpha_n KIK$
$= \alpha_0 (KAK)^n + \alpha_1 (KAK)^{n-1} + \ldots..+ \alpha_n I$
$A^T= P(KAK)$

(ii) and (iii) $\Rightarrow$(i)
$A^T= P(KAK)$
$KA^TK= K(\alpha_0(KAK)^n + \alpha_1(KAK)^{n-1} + \ldots\ldots.+ \alpha_n I)K$
$= K(\alpha_0 KA^nK + \alpha_1 KA^{n-1}K +\ldots\ldots..+ \alpha_n KIK) K$
$=\alpha_0 A^n + \alpha_1 A^{n-1} +\ldots\ldots+ \alpha_n I =P(A)$
$KA^TK = A^{-1}$
$\therefore$ A is k-orthogonal

(iii) and (i) $\Rightarrow$(ii)
since A is k-Orthogonal and non singular by remark (3.6), it is possible to find a polynomial q(t) such that $A^{-1}=q(A)$
Let $q(A) = \beta_0 A^m + \beta_1 A^{m-1}+\ldots\ldots+ \beta_m I$, $\beta_0 \neq 0$
$KA^TK=A^{-1}$
But $A^T = P(KAK) \Rightarrow KP(KAK)K = A^{-1}=q(A)$
$K(\alpha_0(KAK)^n + \alpha_1(KAK)^{n-1} + \ldots\ldots.+ \alpha_n I)K = \beta_0 A^m + \beta_1 A^{m-1}+\ldots\ldots+ \beta_m I$
$K(\alpha_0 KA^nK + \alpha_1 KA^{n-1}K +\ldots\ldots..+ \alpha_n I) K = \beta_0 A^m + \beta_1 A^{m-1}+\ldots\ldots+ \beta_m I$
$\therefore \alpha_0 A^n + \alpha_1 A^{n-1} +\ldots\ldots...+ \alpha_n I = \beta_0 A^m + \beta_1 A^{m-1}+\ldots\ldots+ \beta_m I$
Since two polynomials in A are equal . we must have m = n and $\alpha_i = \beta_i$ for all i .
$\therefore$ P(A) =q(A)

Hence $A^{-1} = P(A)$

Definition 3.8: (Peter Lanncaster, 2009)

The matrices A and B from C_{nxn} are said to be similar if there exists a nonsingular matrix $T \in C_{nxn}$ such that $A = T^{-1}BT$

Therom 3.9:

Let A be k-orthogonal. Let B is similar to A such that $B=C^{-1}AC$. If a matrix C is

k-orthogonal then B is k-orthogonal.

Proof:

$KB^TK=K(C^{-1}AC)^T K$
$=K C^T A^T (C^{-1})^T K$
$= K C^T A^T (K C^TK)^T K$
$= K C^T A^T K C^TK^T K$
$= K C^T A^T K C$
$=K(K C^{-1}K) (K A^{-1}K) K C$
$= C^{-1} A^{-1} C$
$= (C^{-1} A C)^{-1}$
$=B^{-1}$

REFERENCES

Anna Lee, 1976. Secondary symmetric, secondary skew symmetric secondary orthogonal matrices, period. *Math. Hungary*, 7, 63-70

Hill R.D. and Water S.R. 1992. On k-real and k-hermitian matrices, *Linear Alg. Appl*.,169(1992),17-29.

Krishnamoorthy, S, K. Gunasekaran and Bhuvaneswari, G. 2011. On k-orthogonal matrices, *International Journal of Mathematical sciences and Engineering Applications,* ISSN 0973-9424, Vol.5. No.V (Sep.) PP 235-243

Peter Lanncaster, 2009. Theory of matrices, Academic Press, New york and London.

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