



COMPREHENSIVE INSIGHT TO FUZZY LOGIC LEARNING AND IMPROVED FUZZY LOGICAL
RELATIONSHIPS FOR FORECASTING

¹Devendra Tayal, Charu Gupta^{2,*}, and Suyash Gupta³

¹Associate Professor & Head of Department, Indian Gandhi Institute of Technology, Kashmere Gate, New Delhi

²Assistant Professor, Bhagwan Parshuram Institute of Technology, Delhi

³Team leader, DishTV India Ltd. ZEE networks, Noida

ARTICLE INFO

Article History:

Received 23th January, 2012
Received in revised form
24th February, 2012
Accepted 18th March, 2012
Published online 30th April, 2012

Key words:

Fuzzy logic,
Fuzzy rules,
Forecasting,
Fuzzy logical relationships.

ABSTRACT

It is an era where there are constant advancements in the area of Information Technology. Technologies such as, internet, database management system, bar code readers or information systems in general have created countless databases of scientific, administrative and commercial nature. Information retrieval is not enough for decision making. Thus, it becomes important for us to develop automatic and intelligent tools for analyzing, interpreting and co-relating data in order to develop and select strategies in the context of the application. In this paper, we provide an insight to fuzzy learning, the rules governing fuzzy logic and study the application of fuzzy forecasting. We also discuss the fuzzy logical relationships and propose an improvement in the fuzzy relationships.

Copy Right, IJCR, 2012, Academic Journals. All rights reserved.

INTRODUCTION

The idea of fuzzy was born in 1965 when Lofti A. Zadeh, a well- respected professor in the department of electrical engineering and computer science at University of California, Berkeley, believed that all real world problems could be solved with efficient and analytical methods. He began to feel that traditional system analysis techniques were too precise for many complex real world problems. In a paper written in 1961, he mentioned that a different kind of mathematics is needed. In his words, "We need a radically different kind of mathematics, the mathematics of fuzzy or cloudy quantities which are not described in terms of probability distributions". Initially, the concept of fuzzy sets encountered sharp criticism from the academic community. Some rejected it because of the name, without knowing the content in detail, some rejected it because of the theory's emphasis on imprecision. But Zadeh continued to broaden the foundation of fuzzy sets in a wide variety of fields- ranging from psychology, sociology, philosophy, and economics. Owing to this, fuzzy logic grew in both wide and comprehensive way and established the foundation of fuzzy logic technology and led to the development of application of this technology in the following years [8]. In this paper, we provide a comprehensive insight to fuzzy learning and the various rules governing the fuzzy logic. We also study the use of fuzzy logical relationships in forecasting. In section II we discuss a brief insight to fuzzy logic, section III discusses the fuzzy rules and structure of

fuzzy rules, section IV gives the types of fuzzy rules- fuzzy mapping and fuzzy implication. Section V discusses the use of fuzzy logical relationships in forecasting. Finally in section VI we conclude with a summary.

Insight into fuzzy logic

Fuzzy logic is a multivalued logic that allows us to model imprecise concepts, imprecise dependencies (Ross, (1995)). Fuzzy logic has rapidly become one of the most successful of today's technologies for developing sophisticated control systems. Fuzzy logic poses the ability to mimic the human mind to effectively employ modes of reasoning that are approximate rather than exact that has an ability to generate precise solutions from certain or approximate information. While other approaches require accurate equations to model real-world behaviours, fuzzy design can accommodate the ambiguities of real-world human language and logic. It provides both an intuitive method for describing systems in human terms and automates the conversion of those system specifications into effective models. This imprecision is represented by a fuzzy set that has a smooth boundary. The core techniques of this logic rely on four basic concepts [8]:

1. *Fuzzy sets*: sets with smooth boundaries;
2. *Linguistic variables*: variables whose values are both qualitatively and quantitatively described by a fuzzy set;
3. *Possibility distributions*: constraints on the value of a linguistic variable imposed by assigning it a fuzzy set;

*Corresponding author: charu_2287@yahoo.com

4. *Fuzzy rules*: a knowledge representation scheme for describing a functional mapping or a logic formula that generalizes an implication in two-valued logic.

The fourth concept is important because it is the basic for most industrial applications of fuzzy logic developed to date, including many fuzzy logic control systems. The details and the intricacies of the fuzzy rules, types of fuzzy rules and fuzzy sets are explained in the subsequent section that are necessary to develop the concept of the logical relationships used in the proposed methodology.

Fuzzy rules

The fuzzy *if-then* rules (in short, fuzzy rules) are by far the most visible due to its wide range of successful applications. Fuzzy rules have been applied to many disciplines such as control systems, decision making, pattern recognition and system modelling. Fuzzy *if-then* rules also play an important role in industrial applications, robotics, process control & medical imaging. With the proposed work, the fuzzy rules are extended in the application area of forecasting. Fuzzy rule inference can be understood as an interpolation scheme as it enables the fusion of multiple fuzzy rules when their conditions are satisfied to a degree [7]. From a logical viewpoint, fuzzy rule inference is a generalization of a logical reasoning scheme called *modes ponens*. In classical logic, if a rule is true and the antecedent of the rule is true, it can be inferred that the consequent of the rule is true. This is referred to as *modes ponens*. For example, say we know that the rule R1 is true:

R1: IF the annual income of a person is greater than 100k
THEN the person is rich.

We also know that the following statement is true:
Roy's annual income is 101k

Based on *modes ponens*, classical logic can deduce that the following statement is also true:
Roy is rich.

This scheme for capturing knowledge (typically, human knowledge) is imprecise and inexact by nature. This is achieved by using linguistic variables to describe elastic conditions (i.e., condition that can be satisfied to a degree) in the "if part" of fuzzy rules. This matching degree is combined with the consequent (i.e. "then part") of the rule to form a conclusion inferred by the fuzzy rule. The higher the matching degree, the closer the inferred conclusion to the rule's consequent. The main feature of reasoning using these rules is its partial matching capability, which enables an inference to be made from a fuzzy rule even when the rule's condition is partially satisfied. A fuzzy *if-then* rule associates a *condition* described using linguistic variables and fuzzy sets to a *conclusion*. It is a scheme for capturing knowledge that involves imprecision. Thus, conclusively a fuzzy set uses fuzzy if-then rules that associate a condition described by linguistic variables to a conclusion [6]. This is nothing but a broad category of fuzzy implication rules that generalizes the logical relationship between two logic formulas involving imprecise linguistic terms that is explained in the later sections. Although a discussion on the way in which the rules are constructed has been done in the previous section, but this section deals with description of the structure of a fuzzy rule.

Structure of fuzzy rule

A fuzzy rule is the basic unit for capturing knowledge in many fuzzy systems. A fuzzy rule has two components: an if-part (also referred to as the antecedent) and a then-part (also referred to as the consequent):

IF <antecedent> *THEN* <consequent>

The antecedent describes a condition, and the consequent describes a conclusion that can be drawn when the condition holds. The basics of these two components are as follows:

The antecedent of fuzzy rules

The structure of a fuzzy rule is identical to that of a conventional rule in artificial intelligence. The difference lies in the content of the rule antecedent – the antecedent of a fuzzy rule describes an elastic condition while the antecedent of a conventional rule describes a rigid condition. For instance, consider the two rules below:

- R1: IF the annual income of a person is greater than 100k,
THEN the person is rich.
R2: IF the annual income of a person is *high*,
THEN the person is *rich*.

The rule R1 is a conventional rule because its condition is rigid. In contrast, R2 is a fuzzy rule because its condition can be satisfied to a degree for those people whose income lies in the boundary of the fuzzy set *high* representing high annual income.

The consequent of fuzzy rules

The consequent of fuzzy rules can be classified into two categories:

1. *Crisp consequent*: IF ... THEN $y = a$, where 'a' is non-fuzzy numeric value. The advantage of a fuzzy rule with crisp consequent can be processed more efficiently.
2. *Fuzzy consequent*: IF ... THEN y is A, where A is a fuzzy set. The advantage of using a rule with a fuzzy consequent is easy to understand and more suitable for capturing imprecise human expertise.

An antecedent and consequent together form a fuzzy rule but it is worthwhile to point out a difference in the fuzzy sets used in rule's antecedent and consequent which has an impact on the design of membership functions. The fuzzy sets in a rule's antecedent define a fuzzy region of the input space covered by the rule; whereas, the fuzzy sets in the rule's consequent describe the vagueness of the rule's conclusion [7,8]. In the next section, the two basic types of fuzzy rules are described. The understanding of the fundamental differences between these two is important for developing successful application. Understanding the theoretical foundation of these techniques and their relationship is one of the most important factors in developing suitable applications.

Types of fuzzy rules

There are two types of fuzzy rules: 1) *fuzzy mapping rules*, and 2) *fuzzy implication rules*. A fuzzy mapping rule describes a functional mapping relationship between inputs and an

output using linguistic values, while a fuzzy implication is a category of rules which describes a generalized logic implication relationship between the two logic formulas involving linguistic variables [6]. The foundation of a fuzzy mapping rule is a fuzzy graph, whereas a fuzzy implication rule involves a narrow sense of fuzzy logic. Fuzzy mapping rules are related to function approximation techniques in artificial and neural networks, whereas, fuzzy implication rules are related to classical two-values logic and multivalued logic. The two types of rules are discussed in the subsequent section.

Fuzzy Mapping Rules

The functional relationship between a set of observable parameters and one or multiple parameters whose values are unknown are of interest in many real world problems [5]. For instance, a control engineer is interested in finding an appropriate control law, which can be viewed as a functional relationship between observable state variables & controllable parameters. This type of rule approximates a mapping from the observed state to a desired control action. The reason to approximate functions is more often due to the following reasons:

- The mathematical structure of the function not known,
- The function is complex that finding a precise mathematical form is either impossible or practically infeasible,
- The function if not impractical, implementing a function in its precise mathematical form may be too costly.

Even though human insights about the function can often be captured using fuzzy rules, the focus of this type of fuzzy rule inference is on exploring a trade-off between accuracy and cost. Since a fuzzy rule approximates a small segment of the function, the entire function is approximated by a set of fuzzy mapping rule. The function approximation techniques can be broadly classified in three categories: *global techniques*, *superimposition techniques* and *partition-based techniques*. The *global* techniques approximate a function globally using one mathematical structure. The *superimposition* techniques approximate a function by superimposing functions of a given form. The *partition-based* techniques approximate a function by partitioning the input space of the function and approximate the function in each partitioned region separately. The superimposition and partition-based techniques are also referred to as local modelling techniques [6]. This is a brief definition and types of fuzzy mapping rules. We refrain to go into the details of these rules as they are not directly used in developing the proposed approach.

Fuzzy Implication Rules

Fuzzy implication rules are a generalization of “implication” in two-valued logic. Their aim is to mimic human reasoning in its ability to reason with ideas or statements that are imprecise by nature. The difference between the semantics of fuzzy mapping rules and fuzzy implication rules can be seen from the difference in their inference behaviour. Even though these

two types of rules behave the same when their antecedents are satisfied, they behave differently when their antecedents are not satisfied.

Any logic system has two major components:

1. A formal language for constructing statements about the world,
2. A set of inference mechanisms for inferring additional statements about the world from those already given.

Fuzzy logic is the most commonly used reasoning scheme in applications of fuzzy logic (narrow sense). The subject is complicated by the fact that there isn't a unique definition of fuzzy implications. An important goal of fuzzy logic is to be able to make reasonable inference even when the condition of an implication rule is partially satisfied. This capability is sometimes referred to as *approximate reasoning*. This is achieved in fuzzy logic by two related techniques:

1. Representing the meaning of a fuzzy implication rule using a fuzzy relation, and
2. Obtaining an inferred conclusion by applying the compositional rule of inference to the fuzzy implication relation.

The basis of fuzzy implication rules is utilized in developing the fuzzy logical relationships with which the relation between corresponding terms is obtained. The fuzzy logical relationships so formed are of the form of *if and then* rules. For instance, the relationship between year ‘x’ and year ‘y’ is formed as “ $x \rightarrow y$ ”, where x is called as the current state and y denotes the next state. The rules for forming these fuzzy logical relationships can be studied in [1].

Forecasting using fuzzy logical relationships

Forecasting has well been studied using fuzzy time series and the various techniques can be found in [4]. Apart from fuzzy time series fuzzy implication rules are used to derive fuzzy logical relationships that can be used for forecasting low dimensional data. The fuzzy relationships are formed based on the intervals obtained using clustering [1]. The use of fuzzy logical relationships for forecasting numerical data has been studied (Chen & Chen [3] (2010); Chen & Wang [2] (2010)). A method for forecasting using low order fuzzy logical relationships has been studied (Chen *et al.* [1](2009)) which uses the principle that variation in current year is related to the trend in previous year. Assuming that there are n intervals $u_1; u_2; \dots$, and u_n , then define each fuzzy set A_i , where $1 \leq i \leq n$, as follows:

$$\begin{aligned}
 A_1 &= 1/u_1 + 0.5/u_2 + 0/u_3 + 0/u_4 + 0/u_{n-1} + 0/u_n; \\
 A_2 &= 0.5/u_1 + 1/u_2 + 0.5/u_3 + 0/u_4 + 0/u_{n-1} + 0/u_n; \\
 A_3 &= 0/u_1 + 0.5/u_2 + 1/u_3 + 0.5/u_4 + 0/u_{n-1} + 0/u_n; \\
 &\dots\dots \\
 A_n &= 0/u_1 + 0/u_2 + 0/u_3 + 0/u_4 + 0.5/u_{n-1} + 1/u_n.
 \end{aligned}$$

If the datum belongs to u_i , where $1 \leq i \leq n$, then the datum is fuzzified into A_i . The method uses the approach of establishing fuzzy logical relationships where the data,

$d_1, d_2, d_3 \dots d_n$

are fuzzified into fuzzy sets A_1, A_2, \dots, A_n based on contiguous intervals formed. For instance, if the fuzzified data of year i and $i+1$ are A_r and A_s then we can construct a fuzzy logical relationship " $A_r \rightarrow A_s$ " where " A_r " and " A_s " are the current state and the next state respectively. For the sake of brevity the details of the method are not presented here and further description can be found in Chen *et al.* [1]. To keep our exposition self-contained, the formation of fuzzy relationships [1] are reproduced in the following section.

Fuzzy Logical relationships

Principle 1:

If the fuzzy logical relationship (FLR) in fuzzy logical relationship group is

" $A_r \rightarrow A_s$ ",

where A_r, A_s is the fuzzified data of year t and $t+1$, then the forecasted data value of year $t+1$ is m_s , where m_s is the midpoint of the interval u_s .

Principle 2: If there are fuzzy logical relationship group whose current state is A_r ,

" $A_r \rightarrow A_{k1}(x_1), A_{k2}(x_2) \dots A_{kp}(x_{kp})$ ",

then the forecasted data for year $t+1$ is calculated as:

$$x_1 * m_{k1} + x_2 * m_{k2} + \dots + x_p * m_{kp} / x_1 + x_2 + \dots + x_p \quad (1)$$

where, x_i denotes the number of fuzzy logical relationships " $A_r \rightarrow A_{ki}$ " in the fuzzy logical relationship group, $1 \leq i \leq p$; m_{k1}, m_{k2}, \dots , and m_{kp} are the midpoint of intervals u_{k1}, u_{k2}, \dots , and u_{kp} respectively.

Improved fuzzy relationships

The method for establishing fuzzy logical relationships as given by Chen *et al.* [1] suffers from a drawback of repetitive number of fuzzy logical relationships. This condition becomes unsuitable during optimization during differential evolution. The condition is as follows: If the fuzzified enrollment of year t is A_j and there are the following fuzzy logical relationships in the fuzzy logical relationship group (whose current state is A_j)

$$A_j \rightarrow A_k(n), \text{ where } n \geq 2$$

then the forecasted enrollment of year $t + 1$ is calculated as

$$\text{follows: } \frac{(n-1)m_k}{n} \quad (2)$$

where n denotes the number of fuzzy logical relationships " $A_j \rightarrow A_k$ " in the fuzzy logical relationship group, m_k is the midpoint of the intervals u_k and the maximum membership values of the fuzzy sets A_k occur at the intervals u_k .

Conclusion and Future Work

Fuzzy logic has always been a very interesting area with researchers all over the world using it in various real time applications. In this paper, we attempt to provide a comprehensive introduction to fuzzy logic and use of fuzzy logical relationship in forecasting. We provide a modification in the fuzzy relationships that can be used for forecasting numerical data. The proposed modification in the relationship computation can provide fruitful results with clustering to be used for forecasting.

REFERENCES

1. Chen, S.M., Wang, N.Y. and Pan, J.S., (2009) Forecasting enrollments using automatic clustering techniques and fuzzy logical relationships. *Expert Systems with Applications*, 36(8).
2. Chen, S.M. and Wang, N.Y., (2010) Fuzzy Forecasting Based on Fuzzy-Trend Logical Relationship Groups. *IEEE Transactions on systems, man, and cybernetics—Part B: cybernetics*, 40(5).
3. Chen, S. M. and Chen, C.D., (2010) Handling forecasting problems based on high-order fuzzy logical relationships. *Expert Systems with Applications*, 38(4), 3857-3864.
4. Tayal D., Sonawani S., Ansari G., Gupta C. (2011) Fuzzy Time Series Forecasting of Low Dimensional Numerical Data, *International Journal of Engineering research and applications*, 2(1), 132-135.
5. Wu and W., (1986) Fuzzy reasoning and fuzzy relation equations. *Fuzzy sets and Systems*, 20, 67-68.
6. Yen, J. and Langari, R., *Fuzzy Logic: Intelligence, Control, and Information*, Prentice Hall, (1999).
7. Yen, J., (1999). Fuzzy logic - A modern perspective. *IEEE Transactions on knowledge and data engineering*, 11, 153-165.
8. Zadeh, L.A., (1975) The concept of linguistic variable and its application to a approximate reasoning. *Information Sciences*, 8, 199-249.
