RESEARCH ARTICLE

COMPREHENSIVE INSIGHT TO FUZZY LOGIC LEARNING AND IMPROVED FUZZY LOGICAL RELATIONSHIPS FOR FORECASTING

1Devendra Tayal, Charu Gupta2*, and Suyash Gupta3

1Associate Professor & Head of Department, Indian Gandhi Institute of Technology, Kashmere Gate, New Delhi
2Assistant Professor, Bhagwan Parshuram Institute of Technology, Delhi
3Team leader, DishTV India Ltd. ZEE networks, Noida

ABSTRACT

It is an era where there are constant advancements in the area of Information Technology. Technologies such as, internet, database management system, bar code readers or information systems in general have created countless databases of scientific, administrative and commercial nature. Information retrieval is not enough for decision making. Thus, it becomes important for us to develop automatic and intelligent tools for analyzing, interpreting and co-relating data in order to develop and select strategies in the context of the application. In this paper, we provide an insight to fuzzy learning, the rules governing fuzzy logic and study the application of fuzzy forecasting. We also discuss the fuzzy logical relationships and propose an improvement in the fuzzy relationships.

INTRODUCTION

The idea of fuzzy was born in 1965 when Lofti A. Zadeh, a well-respected professor in the department of electrical engineering and computer science at University of California, Berkeley, believed that all real world problems could be solved with efficient and analytical methods. He began to feel that traditional system analysis techniques were too precise for many complex real world problems. In a paper written in 1961, he mentioned that a different kind of mathematics is needed. In his words, “We need a radically different kind of mathematics, the mathematics of fuzzy or cloudy quantities which are not described in terms of probability distributions”. Initially, the concept of fuzzy sets encountered sharp criticism from the academic community. Some rejected it because of the name, without knowing the content in detail, some rejected it because of the theory’s emphasis on imprecision. But Zadeh continued to broaden the foundation of fuzzy sets in a wide variety of fields- ranging from psychology, sociology, philosophy, and economics. Owing to this, fuzzy logic grew in both wide and comprehensive way and established the foundation of fuzzy logic technology and led to the development of application of this technology in the following years [8]. In this paper, we provide a comprehensive insight to fuzzy learning and the various rules governing the fuzzy logic. We also study the use of fuzzy logical relationships in forecasting. In section II we discuss a brief insight to fuzzy logic, section III discusses the fuzzy rules and structure of fuzzy rules, section IV gives the types of fuzzy rules- fuzzy mapping and fuzzy implication. Section V discusses the use of fuzzy logical relationships in forecasting. Finally in section VI we conclude with a summary.

Insight into fuzzy logic

Fuzzy logic is a multivalued logic that allows us to model imprecise concepts, imprecise dependencies (Ross, (1995)). Fuzzy logic has rapidly become one of the most successful of today’s technologies for developing sophisticated control systems. Fuzzy logic poses the ability to mimic the human mind to effectively employ modes of reasoning that are approximate rather than exact that has an ability to generate precise solutions from certain or approximate information. While other approaches require accurate equations to model real-world behaviours, fuzzy design can accommodate the ambiguities of real-world human language and logic. It provides both an intuitive method for describing systems in human terms and automates the conversion of those system specifications into effective models. This imprecision is represented by a fuzzy set that has a smooth boundary. The core techniques of this logic rely on four basic concepts [8]:

1. Fuzzy sets: sets with smooth boundaries;
2. Linguistic variables: variables whose values are both qualitatively and quantitatively described by a fuzzy set;
3. Possibility distributions: constraints on the value of a linguistic variable imposed by assigning it a fuzzy set;

*Corresponding author: charu_2287@yahoo.com
4. Fuzzy rules: a knowledge representation scheme for describing a functional mapping or a logic formula that generalizes an implication in two-valued logic.

The fourth concept is important because it is the basis for most industrial applications of fuzzy logic developed to date, including many fuzzy logic control systems. The details and the intricacies of the fuzzy rules, types of fuzzy rules and fuzzy sets are explained in the subsequent section that are necessary to develop the concept of the logical relationships used in the proposed methodology.

Fuzzy rules

The fuzzy if-then rules (in short, fuzzy rules) are by far the most visible due to its wide range of successful applications. Fuzzy rules have been applied to many disciplines such as control systems, decision making, pattern recognition and system modelling. Fuzzy if-then rules also play an important role in industrial applications, robotics, process control & medical imaging. With the proposed work, the fuzzy rules are extended in the application area of forecasting. Fuzzy rule inference can be understood as an interpolation scheme as it enables the fusion of multiple fuzzy rules when their conditions are satisfied to a degree [7]. From a logical viewpoint, fuzzy rule inference is a generalization of a logical reasoning scheme called *modus ponens*. In classical logic, if a rule is true and the antecedent of the rule is true, it can be inferred that the consequent of the rule is true. This is referred to as *modus ponens*. For example, say we know that the rule R1 is true:

R1: IF the annual income of a person is greater than 100k THEN the person is rich.

We also know that the following statement is true:

Roy’s annual income is 101k

Based on *modus ponens*, classical logic can deduce that the following statement is also true:

Roy is rich.

This scheme for capturing knowledge (typically, human knowledge) is imprecise and inexact by nature. This is achieved by using linguistic variables to describe elastic conditions (i.e., condition that can be satisfied to a degree) in the “if part” of fuzzy rules. This matching degree is combined with the consequent (i.e., “then part”) of the rule to form a conclusion inferred by the fuzzy rule. The higher the matching degree, the closer the inferred conclusion to the rule’s consequent. The main feature of reasoning using these rules is its partial matching capability, which enables an inference to be made from a fuzzy rule even when the rule’s condition is partially satisfied. A fuzzy if-then rule associates a condition described using linguistic variables and fuzzy sets to a conclusion. It is a scheme for capturing knowledge that involves imprecision. Thus, conclusively a fuzzy set uses fuzzy if-then rules that associate a condition described by linguistic variables to a conclusion [6]. This is nothing but a broad category of fuzzy implication rules that generalizes the logical relationship between two logic formulas involving imprecise linguistic terms that is explained in the later sections. Although a discussion on the way in which the rules are constructed has been done in the previous section, but this section deals with description of the structure of a fuzzy rule.

Structure of fuzzy rule

A fuzzy rule is the basic unit for capturing knowledge in many fuzzy systems. A fuzzy rule has two components: an if-part (also referred to as the antecedent) and a then-part (also referred to as the consequent):

\[
\text{IF } \text{<antecedent>} \text{ THEN } \text{<consequent>}
\]

The antecedent describes a condition, and the consequent describes a conclusion that can be drawn when the condition holds. The basics of these two components are as follows:

The antecedent of fuzzy rules

The structure of a fuzzy rule is identical to that of a conventional rule in artificial intelligence. The difference lies in the content of the rule antecedent – the antecedent of a fuzzy rule describes an elastic condition while the antecedent of a conventional rule describes a rigid condition. For instance, consider the two rules below:

R1: IF the annual income of a person is greater than 100k, THEN the person is rich.

R2: IF the annual income of a person is high, THEN the person is rich.

The rule R1 is a conventional rule because its condition is rigid. In contrast, R2 is a fuzzy rule because its condition can be satisfied to a degree for those people whose income lies in the boundary of the fuzzy set high representing high annual income.

The consequent of fuzzy rules

The consequent of fuzzy rules can be classified into two categories:

1. **Crisp consequent**: IF … THEN y = a, where ‘a’ is non-fuzzy numeric value. The advantage of a fuzzy rule with crisp consequent can be processed more efficiently.

2. **Fuzzy consequent**: IF … THEN y is A, where A is a fuzzy set. The advantage of using a rule with a fuzzy consequent is easy to understand and more suitable for capturing imprecise human expertise.

An antecedent and consequent together form a fuzzy rule but it is worthwhile to point out a difference in the fuzzy sets used in rule’s antecedent and consequent which has an impact on the design of membership functions. The fuzzy sets in a rule’s antecedent define a fuzzy region of the input space covered by the rule; whereas, the fuzzy sets in the rule’s consequent describe the vagueness of the rule’s conclusion [7,8]. In the next section, the two basic types of fuzzy rules are described. The understanding of the fundamental differences between these two is important for developing successful application. Understanding the theoretical foundation of these techniques and their relationship is one of the most important factors in developing suitable applications.

Types of fuzzy rules

There are two types of fuzzy rules: 1) *fuzzy mapping rules*, and 2) *fuzzy implication rules*. A fuzzy mapping rule describes a functional mapping relationship between inputs and an
output using linguistic values, while a fuzzy implication is a
category of rules which describes a generalized logic
implication relationship between the two logic formulas
involving linguistic variables [6]. The foundation of a fuzzy
mapping rule is a fuzzy graph, whereas a fuzzy implication
rule involves a narrow sense of fuzzy logic. Fuzzy mapping
rules are related to function approximation techniques in
artificial and neural networks, whereas, fuzzy implication
rules are related to classical two-values logic and multivalued
logic. The two types of rules are discussed in the subsequent
section.

Fuzzy Mapping Rules

The functional relationship between a set of observable
parameters and one or multiple parameters whose values are
unknown are of interest in many real world problems [5]. For
instance, a control engineer is interested in finding an
appropriate control law, which can be viewed as a functional
relationship between observable state variables & controllable
parameters. This type of rule approximates a mapping from
the observed state to a desired control action. The reason to
approximate functions is more often due to the following
reasons:

- The mathematical structure of the function not known,
- The function is complex that finding a precise
  mathematical form is either impossible or practically
  infeasible,
- The function if not impractical, implementing a
  function in its precise mathematical form may be too
costly.

Even though human insights about the function can often be
captured using fuzzy rules, the focus of this type of fuzzy rule
inference is on exploring a trade-off between accuracy and
cost. Since a fuzzy rule approximates a small segment of the
function, the entire function is approximated by a set of fuzzy
mapping rule. The function approximation techniques can be
broadly classified in three categories: global techniques,
superimposition techniques and partition-based techniques.
The global techniques approximate a function globally using
one mathematical structure. The superimposition techniques
approximate a function by superimposing functions of a given
form. The partition-based techniques approximate a function by
partitioning the input space of the function and approximate the
function in each partitioned region separately. The
superimposition and partition-based techniques are also
referred to as local modelling techniques [6]. This is a brief
definition and types of fuzzy mapping rules. We refrain to go
into the details of these rules as they are not directly used in
developing the proposed approach.

Fuzzy Implication Rules

Fuzzy implication rules are a generalization of “implication”
in two-valued logic. Their aim is to mimic human reasoning in
its ability to reason with ideas or statements that are imprecise
by nature. The difference between the semantics of fuzzy
mapping rules and fuzzy implication rules can be seen from
the difference in their inference behaviour. Even though these
two types of rules behave the same when their antecedents are
satisfied, they behave differently when their antecedents are
not satisfied.

Any logic system has two major components:

1. A formal language for constructing statements about the
world,
2. A set of inference mechanisms for inferring additional
statements about the world from those already given.

Fuzzy logic is the most commonly used reasoning scheme in
applications of fuzzy logic (narrow sense). The subject is
complicated by the fact that there isn’t a unique definition of
fuzzy implications. An important goal of fuzzy logic is to be
able to make reasonable inference even when the condition of
an implication rule is partially satisfied. This capability is
sometimes referred to as approximate reasoning. This is
achieved in fuzzy logic by two related techniques:

1. Representing the meaning of a fuzzy implication rule
using a fuzzy relation, and
2. Obtaining an inferred conclusion by applying the
compositional rule of inference to the fuzzy
implication relation.

The basis of fuzzy implication rules is utilized in developing
the fuzzy logical relationships with which the relation between
corresponding terms is obtained. The fuzzy logical
relationships so formed are of the form of if and then rules.
For instance, the relationship between year ‘$x$’ and year ‘$y$’ is
formed as “$x \rightarrow y$”, where $x$ is called as the current state and
$y$ denotes the next state. The rules for forming these fuzzy
logical relationships can be studied in [1].

Forecasting using fuzzy logical relationships

Forecasting has well been studied using fuzzy time series and
the various techniques can be found in [4]. Apart from fuzzy
time series fuzzy implication rules are used to derive fuzzy
logical relationships that can be used for forecasting low
dimensional data. The fuzzy relationships are formed based on
the intervals obtained using clustering [1]. The use of fuzzy
logical relationships for forecasting numerical data has been
studied (Chen & Chen [3] (2010); Chen & Wang [2] (2010)).
A method for forecasting using low order fuzzy logical
relationships has been studied (Chen et al. [1] (2009)) which
uses the principle that variation in current year is related to the
trend in previous year. Assuming that there are $n$ intervals $u_1; u_2; \ldots, u_n$, then define each fuzzy set $A_i$, where
$1 \leq i \leq n$, as follows:

$$A_1 = u_1 + 0.5/u_2 + 0/u_3 + 0/u_4 + 0/u_{n-1} + 0/u_n;$$
$$A_2 = 0.5/u_1 + 1/u_2 + 0.5/u_3 + 0/u_4 + 0/u_{n-1} + 0/u_n;$$
$$A_3 = 0/u_1 + 0.5/u_2 + 1/u_3 + 0.5/u_4 + 0/u_{n-1} + 0/u_n;$$
$$\ldots$$
$$A_n = 0/u_1 + 0/u_2 + 0/u_3 + 0/u_4 + 0.5/u_{n-1} + 1/u_n.$$
If the datum belongs to u, where \( 1 \leq i \leq n \), then the datum is fuzzified into \( A_i \). The method uses the approach of establishing fuzzy logical relationships where the data,

\[
d_1, d_2, d_3 \ldots d_n
\]

are fuzzified into fuzzy sets \( A_1, A_2, \ldots A_n \) based on contiguous intervals formed. For instance, if the fuzzified data of year \( i \) and \( i+1 \) are \( A_i \) and \( A_i \), then we can construct a fuzzy logical relationship “\( A_i \rightarrow A_i \)”, where “\( A_i \)” and “\( A_i \)” are the current state and the next state respectively. For the sake of brevity the details of the method are not presented here and further description can be found in Chen et al. [1]. To keep our exposition self-contained, the formation of fuzzy relationships [1] are reproduced in the following section.

**Fuzzy Logical relationships**

**Principle 1:**

If the fuzzy logical relationship (FLR) in fuzzy logical relationship group is

“\( A_i \rightarrow A_i \)”

where, \( A_i \) is the fuzzified data of year \( t \) and \( t+1 \), then the forecasted data value of year \( t+1 \) is \( m_i \), where \( m_i \) is the midpoint of the interval \( u_i \).

**Principle 2:** If there are fuzzy logical relationship group whose current state is \( A_i \),

“\( A_i \rightarrow A_1(x_1), A_2(x_2) \ldots A_p(x_p) \)”

then the forecasted data for year \( t+1 \) is calculated as:

\[
x_1 \ast m_{1} + x_2 \ast m_{2} + \ldots + x_p \ast m_{p} / x_1 + x_2 + \ldots + x_p \quad (1)
\]

where, \( x_j \) denotes the number of fuzzy logical relationships “\( A_i \rightarrow A_j \)” in the fuzzy logical relationship group, \( 1 \leq i \leq p; \; m_{1i}, m_{2i}, \ldots \), and \( m_{pi} \) are the midpoint of intervals \( u_{1i}, u_{2i}, \ldots \), and \( u_{pi} \) respectively.

**Improved fuzzy relationships**

The method for establishing fuzzy logical relationships as given by Chen et.al [1] suffers from a drawback of repetitive number of fuzzy logical relationships. This condition becomes unsuitable during optimization during differential evolution. The condition is as follows: If the fuzzified enrollment of year \( t \) is \( A_i \) and there are the following fuzzy logical relationships in the fuzzy logical relationship group (whose current state is \( A_i \)

\[
A_j \rightarrow A_i(n), \; \text{where} \; n \geq 2
\]

then the forecasted enrollment of year \( t + 1 \) is calculated as follows:

\[
\frac{(n-1)m_k}{n} \quad (2)
\]

where \( n \) denotes the number of fuzzy logical relationships “\( A_j \rightarrow A_k \)” in the fuzzy logical relationship group, \( m_k \) is the midpoint of the intervals \( u_k \) and the maximum membership values of the fuzzy sets \( A_k \) occur at the intervals \( u_k \).

**Conclusion and Future Work**

Fuzzy logic has always been a very interesting area with researchers all over the world using it in various real time applications. In this paper, we attempt to provide a comprehensive introduction to fuzzy logic and use of fuzzy logical relationship in forecasting. We provide a modification in the fuzzy relationships that can be used for forecasting numerical data. The proposed modification in the relationship computation can provide fruitful results with clustering to be used for forecasting.

**REFERENCES**


