Available online at http://www.journalcra.com

International Journal of Current Research
Vol. 8, Issue, 12, pp.44248-44253, December, 2016

INTERNATIONAL JOURNAL OF CURRENT RESEARCH

ISSN: 0975-833X

RESEARCH ARTICLE

SPECIALLY STRUCTURED N × 2 FLOW SHOP SCHEDULING PROBLEM WITH JOBS IN A STRING OF DISJOINT JOB BLOCKS INCLUDING TRANSPORTATION TIME

*[1]Deepak Gupta, [2]Shashi Bala and [3]Pradeep Bishnoi

[1]Department of Mathematics, M.M.University, Mullana, Ambala, India
[2]Department of Mathematics, M. P. College for Women, Mandi Dabwali, India
[3]Department of Mathematics, G.C. Dabwali (Sirsa), India

ARTICLE INFO

Article History:
Received 18th September, 2016
Received in revised form
10th October, 2016
Accepted 18th November, 2016
Published online 30th December, 2016

Key words:
Disjoint Job Blocks, Elapsed time, Equivalent job, Jobs in a String, Processing time, Specially Structured Flow Shop Scheduling, Transportation Time, Utilization Time

ABSTRACT

This paper provides a simple heuristic algorithm to minimize utilization time for specially structured n-job and 2-machine flow shop scheduling problem with jobs in a string of disjoint job blocks in which the processing times are associated with probabilities including transportation time. Usually in machine scheduling models it is assumed that the jobs are delivered instantaneously from one point to another without taking into account the transportation time involved therein. In this paper we study machine scheduling problems having jobs in a string of disjoint job blocks and by taking into account the explicit transportation considerations. Also, the processing times are not random but bear well defined relationship to one another. In flow shop scheduling the emphasis is on minimization of idle time/elapsed time but minimization of elapsed time may not always lead to minimization of utilization time. Here, the objective is to find an algorithm to minimize the utilization time of machines. The proposed algorithm is validated with the help of a numerical example.

Citation: Deepak Gupta, Shashi Bala and Pradeep Bishnoi. 2016. "Specially structured n × 2 flow shop scheduling problem with jobs in a string of disjoint job blocks including transportation time", *International Journal of Current Research*, 8, (12), 44248-44253.

INTRODUCTION

In today's manufacturing and distribution systems, scheduling have significant role to meet customer requirements as quickly as possible while maximizing the profits. Scheduling means the allocation of resources to tasks over given time with the objective to optimize some performance measure(s). In a flow shop scheduling problem n-jobs are processed on m-machines and the processing order i.e. the order in which various machines are required for completing the job is given. Johnson (1954) developed an algorithm for two stage production schedule for minimizing the makespan. In this model an optimal sequence of jobs is found with each job to be processed by the two machines, say M_1, M_2 in the prescribed order M_1M_2. The study to minimize the makespan for two and three stage flow shop scheduling problem was further developed by Jackson (1956), Bellman (1956), Mitten (1959), Cambell (1970), Baker (1974) and many other researchers. Gupta, J.N.D. (1975) developed an algorithm to find the optimal sequence for specially structured flow shop scheduling problem.

Corresponding author: Deepak Gupta
Department of Mathematics, M.M.University, Mullana, Ambala, India

Gupta, D., Sharma, S., and Bala, S. (2012) considered specially structured two stage flow shop problem to minimize the rental cost of machines under pre-defined rental policy in which the probabilities have been associated with processing time. The practical importance of scheduling models depends upon various factors such as job transportation time, weight of jobs, setup time, breakdown time, job block etc. In this paper we study two stage specially structured flow shop scheduling problem with explicit transportation considerations and having jobs in a string of disjoint job blocks. There are many practical situations where the transportation time are significant and cannot be simply neglected. However, most of the published literature on sequencing and scheduling up to the year 1980 does not take into consideration the transportation time i.e. the moving time for a job from one machine to another machine during the processing of jobs. The earliest scheduling paper that explicitly considers the transportation factor is probably the one by Maggu and Dass (1980). In this paper they consider a two machine flow shop makespan problem with unlimited buffer spaces on both machines in which there are a sufficient number of transporters so that whenever a job is completed on the first machine so that it can be transported with a job dependent transportation time, to the second machine immediately. Maggu *et al*. (1981) studied the same problem with additional constraint that some jobs must be scheduled consecutively. Kise (1991) considered a similar scheduling problem but with only one transporter with a capacity of one i.e. it can transport only one job at a time. Langston (1987) gave heuristic solution to minimize makespan for a k-station flow shop problem where each station has a number of machines that can be used to process jobs, and there is only one transporter with a capacity to transport one job with transportation times dependent on the physical locations of the starting and destination machines. Chung *et al*. (2001) studied machine scheduling problems with explicit transportation considerations. They considered models for two types of transportation situations. The first situation investigated transporting a semi-finished job from one machine to another for further processing and the second situation considered the case of delivering a finished job to the customer or warehouse. Gupta, D., Sharma, S., and Bala, S. (2012) gave a heuristic algorithm to minimize the utilization time and rental cost of machines for n×2 specially structured flow shop problem involving transportation time.

Maggu and Das (1977) studied n-job, 2-machine flow job-shop scheduling problem in which some jobs must be processed consecutively either by technological constraints or by externally imposed policy and thus gave the basic concept of equivalent job block in job sequencing. In equivalent job block a group or block of distinct and finite number of jobs is to be replaced by a single job to be considered as equivalent to the group of jobs and it is assumed that no other job is to be processed between any two jobs included in the block. Singh and Gupta (2005) gave an algorithm to minimize the rental cost of machines for two stage flow shop problem including job block criteria. Gupta *et al*. (2012) considered minimization of rental cost for specially structured two stage flow shop scheduling problem including transportation time, job block criteria and weightage of jobs. The concept of string of job blocks is significant when the jobs are to be processed as set of two job blocks. The string of disjoint job blocks involves the processing of jobs as a set of two job blocks having no job in common such that in one job block the order of jobs is fixed and in second job block the order of jobs is arbitrary. Heydari (2003) considered flow shop scheduling problem with processing of jobs in two disjoint job blocks having one job block in which the order of jobs is fixed and other block in which order of jobs is arbitrary. Singh, Kumar and Gupta (2006) dealt with two stage flow-shop scheduling problem having jobs in a string of disjoint job blocks with processing time and set up time both associated with probabilities. Gupta, Sharma and Gulati, N. (2011) investigated three stage flow shop scheduling problem in which processing time, set up time each associated with probabilities along with jobs in a string of disjoint job blocks.

In this paper we study two stage specially structured flow shop scheduling problem with jobs in a string of disjoint job blocks including transportation time and the objective is to obtain an optimal sequence of jobs to minimize the utilization time of machines. An algorithm is proposed to optimize the utilization time and is validated by a numerical example. The remaining paper is organized as follows: In section 2 we give the practical situation. In sections 3 and 4 we introduce the notations used and assumptions underlying the scheduling problem, respectively. In section 5 we give the definitions used and in section 6 we study problem formulation. In sections 7 and 8 of this paper we deal with the proposed algorithm and numerical illustration respectively. Finally, we give the conclusion in section 9 and is followed by references.

Practical Situation

Two machine specially structured flowshop scheduling problem has been taken up as there are many practical and experimental situations where the processing times of jobs are not random but follow well defined structural relationship to one another. Most machine scheduling models assume that either there are an infinite number of transporters for delivering jobs or jobs are delivered instantaneously from one location to another without transportation time involved. In many production and distribution units, semi-finished tasks are transferred from one machine to another for completion of processing through various modes such as automated guided vehicles and conveyors, and finished jobs are delivered to consumers or storehouses by vehicles such as trains or trucks. Machine scheduling models that take into account the job transportation time are certainly more practical than those scheduling models that do not take these factors into consideration. The concept of job block in machine scheduling is required when some jobs are to be processed consecutively and is thus also essential to make a decision regarding the cost of providing priority in service to the customer and cost of giving service with non priority. Processing of jobs in a string of disjoint job blocks can be seen in case of steel manufacturing units where certain jobs such as heating and molding are to be processed as a fixed job block and other jobs such as cutting, grinding, chroming etc. can be processed in a block disjoint from the first block.

Notations

The following notations have been used throughout the paper:

σ: Sequence of n- jobs obtained by applying Johnson's algorithm.
σ_k:Sequence of jobs obtained by applying the proposed algorithm, k = 1, 2, 3, ------.
M_j: Machine j, j= 1, 2.
a_{ij}: Processing time of i^{th} job on machine M_j.
$T_{i,1\to2}$: Transportation time of i^{th} job from first machine to second machine.
p_{ij}: Probability associated to the processing time a_{ij}.
A_{ij}: Expected processing time of i^{th} job on machine M_j.
$t_{ij}(\sigma_k)$: Completion time of i^{th} job of sequence σ_k on machine M_j.
T (σ_k): Total elapsed time for jobs 1, 2, --------, n for sequence σ_k.
I_{ij} (σ_k): Idle time of machine M_j for job i in the sequence σ_k.
U_j (σ_k): Utilization time for which machine M_j is required for sequence σ_k.
A_{ij} (σ_k): Expected processing time of i^{th} job on machine M_j for sequence σ_k.
α: Fix order job block.
β: Job block with arbitrary order.
β_k: Job block with jobs in an optimal order obtained by applying the proposed algorithm, k = 1, 2, 3, ------.
S: String of job blocks α and β i.e. S = (α, β)
S′: Optimal string of job blocks α and β_k.

Assumptions

The assumptions for the proposed algorithm are stated below:

- Jobs are independent to each other and are processed thorough two machines M_1 and M_2 in order M_1 M_2.
- Machine breakdown is not taken into account.
- Processing of a job on a machine once started cannot be stopped unless the processing is completed i.e. pre-emption is not allowed.
- Processing times for fictitious machines G and H satisfy the structural conditions

  $A'_{i1} \geq A'_{j2}$ or $A'_{i1} \leq A'_{j2}$ for each job i and j; where $A'_{i1} = A_{i1} + T_{i,1\to2}$ and

  $A'_{j2} = A_{j2} + T_{j,1\to2}$

- Each job has to undergo two operations and each job is processed through each of the machine once and only once.
- Only one machine of each type is available.
- Each machine can perform only one task at a time.
- A job is not available to the next machine until and unless processing on the current machine is completed.
- The independency of processing times of jobs on the schedule is maintained.

  $\sum_{i=1}^{n} p_{ij} = 1,\ 0 \leq p_{ij} \leq 1.$

- Jobs i_1, i_2, ---------------, i_h are to be processed as a job block (i_1, i_2, ----------------, i_h) showing priority of job i_1 over i_2 etc. in that order in case of a fixed order job block.

Definition

(1). Completion time of i^{th} job on machine M_j is given by,

$t_{ij} = max\ (t_{i-1,j}\ ,\ t_{i,j-1}) + T_{i,1\to2} + A_{ij}$; $j \geq 2$,

where A_{ij} = Expected processing time of i^{th} job on machine M_j.

(2). Utilization time U_2 of 2nd machine for sequence σ_k is given by

$U_2\ (\sigma_k) = T\ (\sigma_k) - A_{11}\ (\sigma_k) - T_{1,1\to2}$

Problem Formulation

Let n- jobs (i = 1, 2, ------, n) be processed on two machines M_j (j = 1, 2) in the order M_1M_2. Let a_{ij} be the processing time of i^{th} job on j^{th} machine with probabilities p_{ij} such that $0 \leq p_{ij} \leq 1$ and $\sum_{i=1}^{n} p_{ij} = 1$. Let $T_{i,1\to2}$ be the transportation time of i^{th} job from machine M_1 to machine M_2. Let A_{ij} be the expected processing time of i^{th} job on j^{th} machine. The mathematical model of the problem in matrix form can be stated as:

Table 1.

Jobs	Machine M_1		Transportation time	Machine M_2	
i	a_{i1}	p_{i1}	$T_{i,1\to2}$	a_{i2}	p_{i2}
1	a_{11}	p_{11}	$T_{1,1\to2}$	a_{12}	p_{12}
2	a_{21}	p_{21}	$T_{2,1\to2}$	a_{22}	p_{22}
3	a_{31}	p_{31}	$T_{3,1\to2}$	a_{32}	p_{32}
-	-	-		-	-
n	a_{n1}	p_{n1}	$T_{n,1\to2}$	a_{n2}	p_{n2}

Let the jobs be processed as a string of disjoint job blocks as S = (α, β) with job block α consisting of s jobs in fixed order and β consisting of þ jobs in which order of jobs is arbitrary such that s + þ = n and $\alpha \cap \beta = \emptyset$ i.e. the two job blocks α and β form a disjoint set in the sense that the two blocks have no job in common. Our aim is to find job block β_k with jobs in an optimal order from the arbitrary order job block and hence find an optimal string S' of job blocks α and β_k i.e. to find a sequence σ_k of jobs which minimizes the utilization times of machines given that the jobs are to be processed as a string of disjoint job blocks, S = (α, β).

Mathematically, the problem is stated as:

Minimize U_2 (σ_k), given that S = (α, β).

Proposed Algorithm

Step 1: Compute the expected processing times A_{ij} given by $A_{ij} = a_{ij} \times p_{ij}$.

Step 2: Define two fictitious machines G and H with processing times A'_{i1} and A'_{i2} respectively as:

$$A'_{i1} = A_{i1} + T_{i,1\to2} \text{ and}$$
$$A'_{i2} = A_{i2} + T_{i,1\to2}$$

Step 3: Make sure that the processing times obtained in step 2 satisfy the structural conditions $A'_{i1} \geq A'_{j2}$ or $A'_{i1} \leq A'_{j2}$ for each job i and j.

Step 4: Compute the processing times $A'_{\alpha1}$ and $A'_{\alpha2}$ on the guidelines of Maggu and Das (1977) for the equivalent job α (say) for the job block (r, m) as follows:

$$A'_{\alpha1} = A'_{r1} + A'_{m1} - \min\ (A'_{m1}, A'_{r2})$$
$$A'_{\alpha2} = A'_{r2} + A'_{m2} - \min\ (A'_{m1}, A'_{r2})$$

To find the processing times for a job block having three or more than three jobs we use the property that the equivalent job for job-block is associative i.e. ((i_1, i_2), i_3) = (i_1, (i_2, i_3)).

Step 5: If the structural conditions hold good then find the new job block β_k having jobs in an optimal order from the arbitrary order job block β by treating job block β as sub flow shop scheduling problem of the main problem. For finding β_k follow the following steps:

(A): Obtain the job J_1 (say) having maximum processing time on 1st machine and job J_r (say) having minimum processing time on 2nd machine. If $J_1 \neq J_r$ then put J_1 on the first position and J_r at the last position and go to 5(C) otherwise go to 5(B).

(B): Take the difference of processing time of job J_1 on M_1 from job J_2 (say) having next maximum processing time on machine M_1. Call this difference as G_1. Also take the difference of processing time of job J_r on machine M_2 from job J_{r-1} (say) having next minimum processing time on M_2. Call this difference as G_2. If $G_1 \leq G_2$ then put J_r on the last position and J_2 on the first position otherwise put J_1 on 1st position and J_{r-1} on the last position. Now follow step 5(C).

(C): Arrange the remaining (þ − 2) jobs, if any between 1st job J_1 (or J_2) & last job J_r (or J_{r-1}) in any order. Due to structural conditions we get the job blocks β_1, β_2 … β_m, where m= (þ − 2)! with jobs in optimal order and each having same elapsed time. Let $\beta_k = \beta_1$ (say). Obtain the processing times $A'_{\beta_k 1}$ and $A'_{\beta_k 2}$ for the job block β_k as defined in step 4.

Step 6: Now reduce the given problem to a new problem by replacing s-jobs by job block α with processing times $A'_{\alpha 1}$ and $A'_{\alpha 2}$ and remaining þ-jobs by a disjoint job block β_k with processing times $A'_{\beta_k 1}$ and $A'_{\beta_k 2}$. The new reduced problem can be represented as:

Table 2.

Jobs	Machine G	Machine H
i	A'_{i1}	A'_{i2}
α	$A'_{\alpha 1}$	$A'_{\alpha 2}$
β_k	$A'_{\beta_k 1}$	$A'_{\beta_k 2}$

Step 7: Check the structural conditions $A'_{i1} \geq A'_{j2}$ or $A'_{i1} \leq A'_{j2}$ for each job i and j. If the structural conditions hold good go to Step 8 to find S′ otherwise modify the problem.

Step 8: For finding optimal string S′ follow the following steps:

(***a***) Obtain the job I_1 (say) having maximum processing time on 1st machine and job I'_1 (say) having minimum processing time on 2nd machine. If $I_1 \neq I'_1$ then put I_1 on the first position and I'_1 at last position to obtain S′ otherwise go to step 8(b).

(b) Take the difference of processing time of job I_1 on M_1 from job I_2 (say) having next maximum processing time on machine M_1. Call this difference as H_1. Also take the difference of processing time of job I'_1 on machine M_2 from job I'_2 (say) having next minimum processing time on M_2. Call this difference as H_2. If $H_1 \leq H_2$ then put I'_1 on the second position and I_2 at the first position otherwise put I_1 on first position and I'_2 at the second position to obtain the optimal string S′.

Step 9: Compute the in - out table for sequence σ_k of jobs in the optimal string S′.

Step 10: Compute the total elapsed time T (σ_k).

Step 11: Calculate the utilization time U_2 of 2nd machine for optimal sequence σ_k, given by

$U_2(\sigma_k) = T(\sigma_k) - A_{11}(\sigma_k) - T_{1,1\to 2}$

Numerical Illustration

Consider 6 jobs to be processed on two machines in a string S of disjoint blocks consisting of job block α = (2, 4) with fixed order of jobs and job block β = (1, 3, 5, 6) with arbitrary order of jobs such that α ∩ β =∅. The problem is to find an optimal string S' to minimize the elapsed time and hence the utilization time of machines. The processing times associated with probabilities and transportation times of jobs are given in the following table:

Table 3.

Jobs	Machine M_1		$T_{i,1\to 2}$	Machine M_2	
i	a_{i1}	p_{i1}		a_{i2}	p_{i2}
1	32	0.2	3	14	0.2
2	67	0.1	2	8	0.1
3	44	0.2	2	17	0.2
4	30	0.2	3	7	0.2
5	24	0.1	4	6	0.1
6	36	0.2	2	16	0.2

Solution

Step 1: The expected processing times for machines M_1 and M_2 are given in the following table:

Table 4.

Jobs	Machine M_1	$T_{i,1\to 2}$	Machine M_2
i	A_{i1}		A_{i2}
1	6.4	3	2.8
2	6.7	2	0.8
3	8.8	2	3.4
4	6.0	3	1.4
5	2.4	4	0.6
6	7.2	2	3.2

Step 2: The processing times for fictitious machines G and H are given in the table below:

Table 5.

Jobs	Machine G	Machine H
i	A'_{i1}	A'_{i2}
1	9.4	5.8
2	8.7	2.8
3	10.8	5.4
4	9.0	4.4
5	6.4	4.6
6	9.2	5.2

Step 3: We have $A'_{i1} \geq A'_{j2}$ for each job i and j as computed in step 2 and so the structural conditions are satisfied.

Step 4: The processing times $A'_{\alpha 1}$ and $A'_{\alpha 2}$ for the equivalent job block α = (2, 4) are calculated as:

$A'_{\alpha 1} = A'_{r1} + A'_{m1} - \min(A'_{m1}, A'_{r2})$ (Here r=2 & m=4)
$= 8.7 + 9.0 - \min(9.0, 2.8)$
$= 17.7 - 2.8 = 14.9$

$A'_{\alpha 2} = A'_{r2} + A'_{m2} - \min(A'_{m1}, A'_{r2})$
$= 2.8 + 4.4 - \min(9.0, 2.8)$
$= 7.2 - 2.8 = 4.4$

Step 5: We have $A'_{i1} \geq A'_{j2}$ for each i and j and so using step 5 we get β_k = (3, 1, 6, 5).

Now, we know that the equivalent job for job-block is associative i.e. $((i_1, i_2), i_3) = (i_1, (i_2, i_3))$ and so we have,

β_k = (3, 1, 6, 5) = ((3, 1), 6, 5) = (α_1, 6, 5) = (α_2, 5); where α_1 = (3, 1) and α_2 = (α_1, 6).

Therefore, we obtain

$A'_{\alpha_1 1} = 10.8 + 9.4 - \min(9.4, 5.4) = 20.2 - 5.4 = 14.8$
$A'_{\alpha_1 2} = 5.4 + 5.8 - \min(9.4, 5.4) = 11.2 - 5.4 = 5.8$
$A'_{\alpha_2 1} = 14.8 + 9.2 - \min(9.2, 5.8) = 24.0 - 5.8 = 18.2$
$A'_{\alpha_2 2} = 5.8 + 5.2 - \min(9.2, 5.8) = 11.0 - 5.8 = 5.2$
$A'_{\beta_k 1} = 18.2 + 6.4 - \min(6.4, 5.2) = 24.6 - 5.2 = 19.4$
$A'_{\beta_k 2} = 5.2 + 4.6 - \min(6.4, 5.2) = 9.8 - 5.2 = 4.6$

Step 6: The reduced problem is defined below:

Table 6.

Jobs	Machine G	Machine H
i	A'_{i1}	A'_{i2}
α	14.9	4.4
β_k	19.4	4.6

Step 7: Here, $A'_{i1} \geq A'_{j2}$ for each i and j, and thus the structural relations hold good.

Step 8: The max_i {A'_{i1}} = 19.4 is for job β_k i.e. $I_1 = \beta_k$ and min_i {A'_{i2}} = 4.4 is for job α i.e. I'_1 = α. Since $I_1 \neq I'_1$, so we put $I_1 = \beta_k$ on the first position and $I'_1 = \alpha$ on the second position.

Therefore, the optimal string S′ as per step 8 is given by S′ = (β_k, α). Hence, the optimal sequence σ_k of jobs as per string S′ is σ_k = 3 – 1 − 6 – 5 − 2 – 4.

The in-out table for optimal sequence σ_k is:

Table 7.

Jobs	Machine M_1	$T_{i,1\to2}$	Machine M_2
i	In-Out		In-Out
3	0.0 – 8.8	2	10.8 − 14.2
1	8.8 − 15.2	3	18.2 – 21.0
6	15.2 – 22.4	2	24.4 – 27.6
5	22.4 – 24.8	4	28.8 – 29.4
2	24.8 – 31.5	2	33.5 – 34.3
4	31.5 – 37.5	3	40.5 – 41.9

Therefore, the total elapsed time = T (σ_k) = 41.9 units.
Utilization time of machine $M_2 = U_2$ (σ_k) = (41.9 – 10.8) units.
= 31.1 units.

Remarks

If we solve the same problem by Johnson's (1954) method by treating job block β as sub flow shop scheduling problem of the main problem we get the new job block β′ from the job block β (disjoint from job block α) as β′ = (1, 3, 6, 5). The processing times $A'_{\beta'1}$ and $A'_{\beta'2}$ for the job block β′ on the guidelines of Maggu and Das (1977) are computed as:

We have, β′ = (1, 3, 6, 5) = ((1, 3), 6, 5) = (α′, 6, 5) = (γ, 5); where α′ = (1, 3) and γ = (α′, 6).

$A'_{\alpha'1}$ = 9.4 + 10.8 – min (10.8, 5.8) = 20.2 – 5.8 = 14.4.
$A'_{\alpha'2}$ = 5.8 + 5.4 – min (10.8, 5.8) = 11.2 – 5.8 = 5.4.
$A'_{\gamma1}$ = 14.4 + 9.2 – min (9.2, 5.4) = 23.6 – 5.4 = 18.2.
$A'_{\gamma2}$ = 5.4 + 5.2 – min (9.2, 5.4) = 10.6 – 5.4 = 5.2.
$A'_{\beta'1}$ = 18.2 + 6.4 – min (6.4, 5.2) = 24.6 – 5.2 = 19.4.
$A'_{\beta'2}$ = 5.2 + 4.6 – min (6.4, 5.2) = 9.8 – 5.2 = 4.6.

The reduced problem is defined below:

Table 8.

Jobs	Machine G	Machine H
i	A'_{i1}	A'_{i2}
α	14.9	4.4
β′	19.4	4.6

By Johnson's (1954) algorithm the optimal string S′ is given by S′ = (β′, α).

Therefore, the optimal sequence σ for the original problem corresponding to optimal string S′ is given by σ = 1 – 3 – 6 – 5 − 2 – 4.

The in – out flow table for the optimal sequence σ is:

Therefore, the total elapsed time = T (σ) = 41.9 units.

Utilization time of machine M_2 = U_2 (σ) = (41.9 − 9.4) units.
= 32.5 units.

Table 9.

Jobs	Machine M_1	$T_{i,1\to2}$	Machine M_2
i	In - Out		In - Out
1	0.0 – 6.4	3	9.4 − 12.2
3	6.4 − 15.2	2	17.2 − 20.6
6	15.2 – 22.4	2	24.4 − 27.6
5	22.4 – 24.8	4	28.8 – 29.4
2	24.8 – 31.5	2	33.5 – 34.3
4	31.5 – 37.5	3	40.5 – 41.9

Conclusion

The algorithm proposed in this paper to minimize the makespan and hence the utilization time of machines gives an optimal string of jobs having minimum utilization time irrespective of the makespan. If we apply the algorithm proposed by Johnson (1954) to find the optimal string of jobs to minimize the utilization time of machines then, we see that the minimum elapsed time may not always correspond to minimum utilization time. From table 9 we see that the utilization time of machine M_2 is U_2 (σ) = 32.5 units with makespan of 41.9 units. However, if the proposed algorithm is applied the utilization time of machine M_2 as per table 7 is U_2 (σ_k) = 31.1 units with the same makespan of 41.9 units. Hence, the proposed algorithm is more efficient as it optimizes both the makespan and the utilization time simultaneously for a specially structured two stage flow shop scheduling problem with jobs in string of disjoint job blocks including transportation time.

REFERENCES

Baker, K.R. 1974. "Introduction to Sequencing and Scheduling", John Wiley and Sons, New York.

Bellman, R. 1956, "Mathematical aspects of scheduling theory, *Journal of Soc. Industrial and Applied Mathematics*, 4, pp. 168-205.

Cambell, H.A., Dudek, R.A. and Smith, M.L. 1970. "A heuristic algorithm for n-jobs, m-machines sequencing problem", *Management Science*, Vol.16, pp. 630-637.

Chung, Y.L. and Zhi, L.C. 2001. "Machine scheduling with transportation considerations", *Journal of Scheduling*, Vol. 4, pp. 3-24.

Gupta, D, Sharma, S. and Gulati, N. 2011. "n × 3 flow shop production schedule, processing time, set up time, each associated with probabilities along with jobs in a string of disjoint job-block', *Antarctica Journal of Mathematics*, Vol.8,No.5, pp. 443-457.

Gupta, D., Bala, S. and Singla, P. 2012. "A new Heuristic approach for specially structured two stage flow shop scheduling to minimize rental cost, processing time associated with probabilities including transportation time, job block criteria and job weightage", *Advances in Applied Science Research*, Vol. 3, Issue 4, pp. 2500-2507.

Gupta, D., Sharma, S. & Bala, S. 2012. "Specially Structured Two Stage Flow Shop Scheduling To Minimize the Rental Cost", *International Journal of Emerging trends in Engineering and Development*, 1(2), 206-215.

Gupta, D., Sharma, S., and Bala, S. 2012. "n×2 Specially structured flow shop scheduling with transportation time to minimizing rental cost of machines", *International Journal of Mathematical Archive*-3(2), pp. 627-635.

Gupta, J.N.D. 1975. "Optimal Schedule for specially structured flow shop," *Naval Research Logistic*, 22 (2), pp. 255-269.

Heydari, A.P.D. 2003. "On flow shop scheduling problem with processing of jobs in string of disjoint job blocks: fixed order jobs and arbitrary order jobs," JISSOR Vol. XXIV, No. 1-4, pp. 39-43.

Jackson, J. R. 1956. "An extension of Johnson's results on job lot scheduling*", Naval Research Logistics Quarterly*, 3, pp. 201-203.

Johnson, S.M. 1954. "Optimal two and three stage production schedule with setup time included", *Nav. Res. Log*. Quart.1 (1), 61-68.

Kise, H. 1991. "On an automated two-machine flowshop scheduling problem with infinite buffer", *Journal of the Operations Research Society of Japan*, 34, pp. 354-361.

Langston, M. A. 1987. "Inter-stage transportation planning in the deterministic flow-shop environment", *Operations Research*, 35, pp. 556-564.

Maggu, P. L. and Das, G. 1977. "Equivalent jobs for job blocks in job- sequencing", Opsearch, Vol. 5, pp. 293-298.

Maggu, P.L. and Das, G. 1980. "On 2 × n sequencing problem with transportation times of jobs", *Pure and Applied Mathematika Sciences,* Vol. 12, pp. 1-6.

Maggu, P.L. Das, G. and Kumar, R. 1981. "On equivalent job for job block in 2 × n sequencing problem with transportation times", *Journal of the OR Society of Japan*, 24, pp. 136-146.

Mitten, L.G. 1959. "Sequencing n jobs on two machines with arbitrary time lags", Management Science, 5, pp. 293-298.

Singh, T.P. and Gupta, D. 2005. "Minimizing rental cost in two stage flow shop, the processing time associated with probabilities including job block", *Reflection de ERA*, Vol. 1, No. 2, pp. 107-120.

Singh, T.P., Kumar, V. and Gupta, D. 2006. "n×2 flowshop scheduling problem in which processing time, Set up time each associated with probabilities along with jobs in a string of disjoint job block", *Journal of Mathematical Science, Reflection des ERA*, Vol. 1, pp. 11-20.

****