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RESEARCH ARTICLE

INTUITIONISTIC FUZZY STRONG BI-IDEALS OF NEAR-RINGS

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ABSTRACT

In this paper we introduce the notation of intuitionistic fuzzy strong bi-ideals of a near-ring and obtain a characterization of a strong bi-ideals in terms of an intuitionistic fuzzy strong bi-ideals of a near-ring. Further, we discuss the properties of intuitionistic fuzzy strong bi-ideals of a near-ring.

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1. INTRODUCTION

The notion of fuzzy subgroup was made by Rosenfeld (1971). In Liu, (1982), introduced the notion of fuzzy ideal of a ring. The notions of fuzzy sub near-ring, fuzzy ideal and fuzzy N-subgroup of a near-ring was introduced by Salah Abou-Zaid (1991) and it has been studied by several authors (Kyung Ho Kim and Young Bae Jun, 2003; Kuyng Ho Kim and Young Bae Jun, 2001; AL. Narayanan, 2001; Narayanan, 2002; Al. Narayanan and Manikantan, 2005; Saikia and Barthakur, 2003; Salah Abou-Zaid, 1991; Seung Dong Kim and Hee Sik Kim, 1996). The concept of intuitionistic fuzzy set was introduced by Atanassov (1986) as a generalisation of the notion of fuzzy set. In this paper, we introduce the notion of a intuitionistic fuzzy strong bi-ideal of a near-ring and obtain the characterization of a strong bi-ideal in terms of a intuitionistic fuzzy strong bi-ideal of a near-ring. We establish that every intuitionistic fuzzy left N-subgroup or intuitionistic fuzzy left ideal of a near-ring is a intuitionistic fuzzy strong bi-ideal of a near-ring and also we establish that every intuitionistic left permutable fuzzy right N-subgroup or intuitionistic left permutable fuzzy right ideal of a near-ring is a intuitionistic fuzzy strong bi-ideal of a near-ring.

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2. Preliminaries

Definition: 2.1

An intuitionistic fuzzy subset μ in a non empty set X is an object having the form $\mu = \{(x, A_\mu(x), B_\mu(x)) / x\in X\}$, where the functions $A_\mu : X\to(0,1)$ and $B_\mu : X\to(0,1)$ denote the degree of membership and the degree of non membership of each element $x\in X$ to the set μ, respectively, and $0 \le A_\mu(x) + B_\mu(x) \le 1$ for all $x\in X$. For the sake of simplicity, we shall use the symbol $\mu = (A_\mu , B_\mu)$ for the intuitionistic fuzzy subset $\mu = \{(x,A_\mu(x), B_\mu(x)) / x\in X\}$.

Definition: 2.2

An intuitionistic fuzzy subset $\mu = (A_\mu , B_\mu)$ of a group (G,+) is said to be a intuitionistic fuzzy subgroup of G if for all $x,y\in N$,

(i) $A_\mu(x + y) \ge \min\{A_\mu(x) , A_\mu(y)\}$
(ii) $A_\mu(-x) = A_\mu(x)$, Or equivalently $A_\mu(x - y) \ge \min\{A_\mu(x), A_\mu(y)\}$
(iii) $B_\mu(x + y) \le \max\{B_\mu(x) , B_\mu(y)\}$
(iv) $B_\mu(-x) = B_\mu(x)$, Or equivalently $B_\mu(x - y) \le \max\{B_\mu(x), B_\mu(y)\}$

Definition: 2.3

An intuitionistic fuzzy subset $\mu = (A_\mu , B_\mu)$ of N is called an intuitionistic fuzzy subnear-ring of N if for all $x,y \in N$,

(i) $A_\mu(x - y) \geq \min\{A_\mu(x) , A_\mu(y)\}$
(ii) $A_\mu(xy) \geq \min\{A_\mu(x) , A_\mu(y)\}$
(iii) $B_\mu(x - y) \leq \max\{B_\mu(x) , B_\mu(y)\}$
(iv) $B_\mu(xy) \leq \max\{B_\mu(x) , B_\mu(y)\}$

Definition: 2.4

An intuitionistic fuzzy subset $\mu = (A_\mu, B_\mu)$ of N is said to be an intuitionistic fuzzy two-sided N-subgroup of N if

(i) μ is an intuitionistic fuzzy subgroup of (N,+),
(ii) $A_\mu(xy) \geq A_\mu(x)$, for all $x,y \in N$,
(iii) $A_\mu(xy) \geq A_\mu(y)$, for all $x,y \in N$.
(iv) $B_\mu(xy) \leq B_\mu(x)$, for all $x,y \in N$.
(v) $B_\mu(xy) \leq B_\mu(y)$, for all $x,y \in N$.

If μ satisfies (i), (ii) and (iv), then μ is called an intuitionistic fuzzy right N-subgroup of N. If μ satisfies (i), (iii) and (v), then μ is called an intuitionistic fuzzy left N-subgroup of N.

Definition: 2.5

An intuitionistic fuzzy subset $\mu = (A_\mu , B_\mu)$ of N is said to be an **intuitionistic fuzzy ideal** of N if

(i) μ is an intuitionistic fuzzy subnear-ring of N,
(ii) $A_\mu(y+x-y) = A_\mu(x)$, for all $x, y \in N$,
(iii) $A_\mu(xy) \geq A_\mu(x)$, for all $x, y \in N$,
(iv) $A_\mu(a(b+i) - ab) \geq A_\mu(i)$, for all $a, b, i, \in N$.
(v) $B_\mu(y+x-y) = B_\mu(x)$, for all $x, y \in N$,
(vi) $B_\mu(xy) \leq B_\mu(x)$, for all $x, y \in N$, .
(vii) $B_\mu(a(b+i) - ab) \leq B_\mu(i)$, for all $a, b, i, \in N$.

If μ satisfies (i),(ii),(iii),(v) and (vi) is called an intuitionistic fuzzy right ideal of N. If μ satisfies (i), (ii), (iv) and (vii) is called an intuitionistic fuzzy left ideal of N. Let A_μ and B_μ be two intuitionistic fuzzy subsets of N. we define an intuitionistic fuzzy subset

$$(A_\mu * B_\mu)(x) = \begin{cases} \sup_{x=a(b+i)-ab} \min\{ A_\mu(a), A_\mu(b), B_\mu(i)\}; & \text{If } x = a(b+i) - ab,\ a,b,i \in N. \\ 0; & \text{otherwise.} \end{cases}$$

Definition: 2.6

An intuitionistic fuzzy subset $\mu = (A_\mu , B_\mu)$ of N is said to be an **intuitionistic fuzzy bi- ideal** of N if for all $x, y \in N$,

(i) $A_\mu(x - y) \geq \min\{A_\mu(x) , A_\mu(y)\}$
(ii) $(A_\mu \circ N \circ A_\mu) \cap (A_\mu \circ N) * A_\mu \subseteq A_\mu$
(iii) $B_\mu(x - y) \leq \max\{B_\mu(x) , B_\mu(y)\}$
(iv) $(B_\mu \circ N \circ B_\mu) \cup (B_\mu \circ N) * B_\mu \supseteq B_\mu$

3. Intuitionistic Fuzzy Strong Bi-ideals of Near-Rings

Definition: 3.1

An intuitionistic fuzzy bi-ideal $\mu = (A_\mu , B_\mu)$ of N is called an intuitionistic fuzzy strong bi-ideal of N, if (i) $N \circ A_\mu \circ A_\mu \subseteq A_\mu$ (ii) $N \circ B_\mu \circ B_\mu \supseteq B_\mu$

+	0	a	b	c
0	0	a	b	c
a	a	0	c	b
b	b	c	0	a
c	c	b	a	0

•	0	a	b	c
0	0	0	0	0
a	0	0	a	0
b	0	0	b	0
c	0	0	c	0

Example: 3.2

Let N={0,a,b,c} be a near-ring with two binary operations '+' and '•' is defined as follows.

Define a fuzzy subset $\mu = (A_\mu , B_\mu)$ where $A_\mu : N \rightarrow (0,1)$ by $A_\mu(0) = 0.8$, $A_\mu(a) = 0.6$, $A_\mu(b) = 0.3 = A_\mu(c)$. Then $(A_\mu \circ \mathbf{N} \circ A_\mu)(0) = 0.3$, $(A_\mu \circ \mathbf{N} \circ A_\mu)(a) = 0.3$, $(A_\mu \circ \mathbf{N} \circ A_\mu)(b) = 0.3$, $(A_\mu \circ \mathbf{N} \circ A_\mu)(c) = 0.3$, $(\mathbf{N} \circ A_\mu \circ A_\mu)(0) = 0.3$, $(\mathbf{N} \circ A_\mu \circ A_\mu)(a) = 0.3$, $(\mathbf{N} \circ A_\mu \circ A_\mu)(b) = 0.3$, $(\mathbf{N} \circ A_\mu \circ A_\mu)(c) = 0.3$ and so A_μ is a intuitionistic fuzzy strong bi-ideal of N and $B_\mu : N \rightarrow (0,1)$ by $B_\mu(0) = 0.2$, $B_\mu(a) = 0.7$, $B_\mu(b) = 0.9 = B_\mu(c)$. Then $(B_\mu \circ \mathbf{N} \circ B_\mu)(0)=0.9$, $(B_\mu \circ \mathbf{N} \circ B_\mu)(a)=0.9$, $(B_\mu \circ \mathbf{N} \circ B_\mu)(b) = 0.9$, $(B_\mu \circ \mathbf{N} \circ B_\mu)(c) = 0.9$, $(\mathbf{N} \circ B_\mu \circ B_\mu)(0) = 0.9$, $(\mathbf{N} \circ B_\mu \circ B_\mu)(a) = 0.9$, $(\mathbf{N} \circ B_\mu \circ B_\mu)(b) = 0.9$, $(\mathbf{N} \circ B_\mu \circ B_\mu)(c) = 0.9$ and so B_μ is an intuitionistic fuzzy strong bi-ideal of N. Thus $\mu = (A_\mu , B_\mu)$ is an intuitionistic fuzzy strong bi-ideal of N.

Theorem: 3.3

Let $\{\mu_i\} = \{(A\mu_i, B\mu_i) : i \in I\}$ be any family of intuitionistic fuzzy strong bi-ideals in a near-ring N. Then $\bigcap_{i \in I} \mu_i$ is an intuitionistic fuzzy strong bi-ideal of N, wher $\bigcap_{i \in I} \mu_i = \{(\bigcap_{i \in I} A\mu_i , \bigcup_{i \in I} B\mu_i)\}$.

Proof:

Let $\{ \mu_i : i \in I\}$ be any family of intuitionistic fuzzy strong bi-ideals of N.
Now for all $x,y \in N$,

$$\bigcap_{i \in I} A\mu_i(x - y) = \min \{A\mu_i(x - y) / i \in I\}$$
$$\geq \min\{\min\{A\mu_i (x), A\mu_i (y) / i \in I\}$$
(since $A\mu_i$ is an intuitionistic fuzzy subgroup of N)
$$= \min\{\bigcap_{i \in I} A\mu_i(x), \bigcap_{i \in I} A\mu_i (y) / i \in I\}$$
$$\bigcup_{i \in I} B\mu_i(x - y) = \max \{B\mu_i(x - y) / i \in I\}$$
$$\leq \max\{\max\{B\mu_i(x), B\mu_i(y) / i \in I\}$$
(since $B\mu_i$ is an intuitionistic fuzzy subgroup of N)
$$= \max\{ \bigcup_{i \in I} B\mu_i(x), \bigcup_{i \in I} B\mu_i(y) / i \in I\}$$
Therefore $\bigcap_{i \in I} \mu_i$ is an intuitionistic fuzzy subgroup of N.
To Prove: $\bigcap_{i \in I} \mu_i$ is an intuitionistic fuzzy bi-ideal of N.
Now for all $x \in N$, since $A_\mu = \bigcap_{i \in I} A\mu_i \subseteq A\mu_i$, for every $i \in I$, we have
$$((A_\mu \circ N \circ A_\mu) \cap (A_\mu \circ N) * A_\mu))(x) \leq ((A\mu_i \circ N \circ A\mu_i) \cap (A\mu_i \circ N) * A\mu_i))(x)$$
(since $A\mu_i$ is an intuitionistic fuzzy bi-ideal of N)

$\leq A\mu_i(x)$ for every $i\in I$.

It follows that

$((A_\mu \circ N \circ A_\mu) \cap (A_\mu \circ N) * A_\mu))(x) \leq \inf\{ A\mu_i(x): i\in I\}$
$= (\cap_{i\in I} A\mu_i)(x)$
$= A_\mu(x)$

Thus $(A_\mu \circ N \circ A_\mu) \cap (A_\mu \circ N) * A_\mu) \subseteq A_\mu$

So A_μ is an intuitionistic fuzzy bi-ideal of N.

Now for all $x\in N$, since $B_\mu = \cup_{i\in I} B\mu_i \supseteq B\mu_i$ for some $i\in I$, we have

$((B_\mu \circ N \circ B_\mu) \cup (B_\mu \circ N) * B_\mu))(x)$
$\geq ((B\mu_i \circ N \circ B\mu_i) \cup (B\mu_i \circ N) * B\mu_i))(x)$
(since $B\mu_i$ is an intuitionistic fuzzy bi-ideal of N)
$\geq B\mu_i(x)$ for some $i\in I$

It follows that

$((B_\mu \circ N \circ B_\mu) \cup (B_\mu \circ N) * B_\mu))(x) \geq \sup\{ B\mu_i(x): i\in I\}$
$= (\cup_{i\in J} B\mu_i(x))$
$= B_\mu(x)$

Thus $(B_\mu \circ N \circ B_\mu) \cup (B_\mu \circ N) * B_\mu) \supseteq B_\mu$

So B_μ is an intuitionistic fuzzy bi-ideal of N.

Thus $\cap_{i\in I} \mu_i$ is an intuitionistic fuzzy bi-ideal of N

Next we prove: $\cap_{i\in I} \mu_i$ is an intuitionistic fuzzy strong bi-ideal of N.

Now for all $x\in N$, since $A_\mu = \cap_{i\in I} A\mu_i \subseteq A\mu_i$, for every $i\in I$, we have

$(\mathbf{N} \circ A_\mu \circ A_\mu)(x) \leq (\mathbf{N} \circ A\mu_i \circ A\mu_i)(x)$
$\leq A\mu_i(x)$ for every $i\in I$
(since $A\mu_i$ is an intuitionistic fuzzy strong bi-ideal of N)

It follows that, $(\mathbf{N} \circ A_\mu \circ A_\mu)(x) \leq \inf\{ A\mu_i(x) : i\in I\}$
$= (\cap_{i\in J} A\mu_i(x))$
$= A_\mu(x)$

Thus $\mathbf{N} \circ A_\mu \circ A_\mu \subseteq A_\mu$. So A_μ is an intuitionistic fuzzy strong bi-ideal of N.

Now for all $x\in N$, since $B_\mu = \cup_{i\in I} B\mu_i \supseteq B\mu_i$, for some $i\in I$, we have

$(\mathbf{N} \circ B_\mu \circ B_\mu)(x) \geq (\mathbf{N} \circ B\mu_i \circ B\mu_i)(x)$
$\geq B\mu_i(x)$ for every $i\in I$
(since $B\mu_i$ is an intuitionistic fuzzy strong bi-ideal of N)

It follows that, $(\mathbf{N} \circ B_\mu \circ B_\mu)(x) \geq \sup\{B\mu_i(x) : i\in I\}$
$= (\cup_{i\in J} B\mu_i(x))$
$= B_\mu(x)$

Thus $\mathbf{N} \circ B_\mu \circ B_\mu \supseteq B_\mu$. So B_μ is an intuitionistic fuzzy strong bi-ideal of N.

Thus $\cap_{i\in I} \mu_i$ is an intuitionistic fuzzy strong bi-ideal of N

Theorem: 3.4

Every left permutable intuitionistic fuzzy right N-subgroup of N is an intuitionistic fuzzy strong bi-ideal of N.

Proof:

Let $\mu = (A_\mu, B_\mu)$ be a left permutable intuitionistic fuzzy right N-subgroup of N.

To prove: μ is an intuitionistic fuzzy strong bi-ideal of N.

First we prove: μ is an intuitionistic fuzzy bi-ideal of N.

Choose $a,b,c,x,y,i,b_1,b_2,x_1,x_2,y_1, y_2$ in N such that $a = bc = x(y+i) - xy$, $b = b_1 b_2$, $x = x_1 x_2$ and $y = y_1 y_2$. Then

$(A_\mu \circ \mathbf{N} \circ A_\mu) \cap ((A_\mu \circ \mathbf{N}) * A_\mu))(a) = \min\{(A_\mu \circ \mathbf{N} \circ A_\mu)(a), ((A_\mu \circ \mathbf{N}) * A_\mu)(a)\}$

$= \min\{\sup_{a=bc} \min(A_\mu \circ \mathbf{N})(b), A_\mu(c)\}, ((A_\mu \circ \mathbf{N}) * A_\mu)(x(y+i)-xy)\}$

$= \min\{\sup_{a=bc} \min\{\sup_{b=b_1b_2} \min\{A_\mu(b_1), \mathbf{N}(b_2)\}, A_\mu(c)\}, ((A_\mu \circ \mathbf{N}) * A_\mu)(x(y+i)-xy)\}$

(since $\mathbf{N}(z) = 1$, for all $z\in N$)

$= \min\{\sup_{a=bc} \min\{\sup_{b=b_1b_2} \{A_\mu(b_1), A_\mu(c)\}, ((A_\mu \circ \mathbf{N}) * A_\mu)(x(y+i)-xy)\}$

(Since A_μ is an intuitionistic fuzzy right N-subgroup of N, $A_\mu(bc) = A_\mu(b_1b_2c) = A_\mu(b_1(b_2c)) \geq A_\mu(b_1)$)

$\leq \min\{\sup_{a=bc} \min\{A_\mu(bc), \mathbf{N}(c)\}, \mathbf{N}(x(y+i)-xy)\}$

$= \min\{\sup_{a=bc} \min\{A_\mu(bc), \mathbf{N}(x(y+i)-xy)\} = A_\mu(bc) = A_\mu(a)$

Thus $(A_\mu \circ \mathbf{N} \circ A_\mu) \cap ((A_\mu \circ \mathbf{N}) * A_\mu)) \subseteq A_\mu$.

Hence A_μ is an intuitionistic fuzzy bi-ideal of N.

Choose a, b, c, x, y, i, b_1, b_2, x_1, x_2, y_1, y_2 in N such that $a = bc = x(y+i)-xy$, $b = b_1, b_2$, $x = x_1 x_2$ and $y = y_1 y_2$. Then

$(B_\mu \circ \mathbf{N} \circ B_\mu) \cup ((B_\mu \circ \mathbf{N}) * B_\mu))(a) = \max\{(B_\mu \circ \mathbf{N} \circ B_\mu)(a), ((B_\mu \circ \mathbf{N}) * B_\mu))(a)\}$

$= \max\{\inf_{a=bc} \max(B_\mu \circ \mathbf{N})(b), B_\mu(c)\}, ((B_\mu \circ \mathbf{N}) * B_\mu)(x(y+i)-xy)\}$

$= \max\{\inf_{a=bc} \max\{\inf_{b=b_1b_2} \max\{B_\mu(b_1), \mathbf{N}(b_2)\}, B_\mu(c)\}, ((B_\mu \circ \mathbf{N}) * B_\mu)(x(y+i)-xy)\}$

(since $\mathbf{N}(z) = 0$, for all $z\in N$)

$= \max\{\inf_{a=bc} \max\{\inf_{b=b_1b_2} \{B_\mu(b_1), B_\mu(c)\}, ((B_\mu \circ \mathbf{N}) * B_\mu)(x(y+i)-xy)\}$

(Since B_μ is a intuitionistic fuzzy right N-subgroup of N, $B_\mu(bc) = B_\mu(b_1b_2c) = B_\mu(b_1(b_2c)) \leq B_\mu(b_1)$)

$\geq \max\{\inf_{a=bc} \max\{B_\mu(bc), \mathbf{N}(c)\}, \mathbf{N}(x(y+i)-xy)\}$

$= \max\{\inf_{a=bc} \max\{B_\mu(bc), \mathbf{N}(x(y+i)-xy)\} = B_\mu(bc) = B_\mu(a)$

Thus $(B_\mu \circ \mathbf{N} \circ B_\mu) \cup ((B_\mu \circ \mathbf{N}) * B_\mu))(a) \supseteq B_\mu$.

Hence B_μ is an intuitionistic fuzzy bi-ideal of N.

Thus $\mu = (A_\mu, B_\mu)$ is an intuitionistic fuzzy bi-ideal of N.

Next we prove: μ is an intuitionistic fuzzy strong bi-ideal of N.

Choose a, b,c, $b_1, b_2 \in N$ such that $a = bc$ and $b = b_1, b_2$. Then

$\mathbf{N} \circ A_\mu \circ A_\mu(a) = \sup_{a=bc} \min\{\mathbf{N} \circ A_\mu(b), A_\mu(c)\}$

$= \sup_{a=bc} \min\{\sup_{b=b_1b_2} \min\{\mathbf{N}(b_1), A_\mu(b_2)\}, A_\mu(c)\}$

$= \sup_{a=bc} \min\{\sup_{b=b_1b_2} \{A_\mu(b_2), A_\mu(c)\}$

(Since A_μ is a left permutable intuitionistic fuzzy right N-subgroup of N, $A_\mu(bc) = A_\mu((b_1b_2)c) = A_\mu((b_2b_1)c) > A_\mu(b_2)$) and $\mathbf{N}(c) \geq A_\mu(c)$

$\leq \sup_{a=bc} \min\{A_\mu(bc), \mathbf{N}(c)\}$

$= \sup_{a=bc} \min\{A_\mu(bc), 1\}$

$= \sup_{a=bc} A_\mu(bc)$

$= A_\mu(a)$

Therefore $\mathbf{N} \circ A_\mu \circ A_\mu \subseteq A_\mu$.

Hence A_μ is an intuitionistic fuzzy strong bi-ideal of N.

Choose a, b, c, $b_1, b_2 \in N$ such that $a = bc$ and $b = b_1 b_2$. Then

$(\mathbf{N} \circ B_\mu \circ B_\mu)(a) = \inf_{a=bc} \max(\mathbf{N} \circ B_\mu)(b), B_\mu(c)\}$

$= \inf_{a=bc} \max\{\inf_{b=b_1b_2} \max\{\mathbf{N}(b_1), B_\mu(b_2)\}, B_\mu(c)\}$

$= \inf_{a=bc} \max\{\inf_{b=b_1b_2} \{B_\mu(b_2), B_\mu(c)\}$

(Since B_μ is a left permutable intuitionistic fuzzy right N-subgroup of N, $B_\mu(bc) = B_\mu((b_1b_2)c) = B_\mu((b_2b_1)c) \leq B_\mu(b_2)$)

$\geq \inf_{a=bc} \max\{B_\mu(bc), \mathbf{N}(c)\}$

$\leq A\mu_i(x)$ for every $i\in I$.

It follows that

$((A_\mu \circ N \circ A_\mu) \cap (A_\mu \circ N) * A_\mu))(x) \leq \inf\{ A\mu_i(x) : i\in I\}$

$= (\bigcap_{i\in I} A\mu_i)(x)$

$= A_\mu(x)$

Thus $(A_\mu \circ N \circ A_\mu) \cap (A_\mu \circ N) * A_\mu) \subseteq A_\mu$

So A_μ is an intuitionistic fuzzy bi-ideal of N.

Now for all $x\in N$, since $B_\mu = \bigcup_{i\in I} B\mu_i \supseteq B\mu_i$ for some $i\in I$, we have

$((B_\mu \circ N \circ B_\mu) \cup (B_\mu \circ N) * B_\mu))(x)$

$\geq ((B\mu_i \circ N \circ B\mu_i) \cup (B\mu_i \circ N) * B\mu_i))(x)$

(since $B\mu_i$ is an intuitionistic fuzzy bi-ideal of N)

$\geq B\mu_i(x)$ for some $i\in I$

It follows that

$((B_\mu \circ N \circ B_\mu) \cup (B_\mu \circ N) * B_\mu))(x) \geq \sup\{ B\mu_i(x) : i\in I\}$

$= (\bigcup_{i\in J} B\mu_i(x))$

$= B_\mu(x)$

Thus $(B_\mu \circ N \circ B_\mu) \cup (B_\mu \circ N) * B_\mu) \supseteq B_\mu$

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(since $A\mu_i$ is an intuitionistic fuzzy strong bi-ideal of N)

It follows that, $(\mathbf{N} \circ A_\mu \circ A_\mu)(x) \leq \inf\{ A\mu_i(x) : i\in I\}$

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$= (\bigcup_{i\in J} B\mu_i(x))$

$= B_\mu(x)$

Thus $\mathbf{N} \circ B_\mu \circ B_\mu \supseteq B_\mu$. So B_μ is an intuitionistic fuzzy strong bi-ideal of N.

Thus $\bigcap_{i\in I} \mu_i$ is an intuitionistic fuzzy strong bi-ideal of N

Theorem: 3.4

Every left permutable intuitionistic fuzzy right N-subgroup of N is an intuitionistic fuzzy strong bi-ideal of N.

Proof:

Let $\mu = (A_\mu, B_\mu)$ be a left permutable intuitionistic fuzzy right N-subgroup of N.

To prove: μ is an intuitionistic fuzzy strong bi-ideal of N.

First we prove: μ is an intuitionistic fuzzy bi-ideal of N.

Choose $a,b,c,x,y,i,b_1,b_2,x_1,x_2,y_1, y_2$ in N such that $a = bc = x(y+i) - xy$, $b = b_1 b_2$, $x = x_1 x_2$ and $y = y_1 y_2$. Then

$(A_\mu \circ \mathbf{N} \circ A_\mu) \cap ((A_\mu \circ \mathbf{N}) * A_\mu))(a) = \min\{(A_\mu \circ \mathbf{N} \circ A_\mu)(a), ((A_\mu \circ \mathbf{N}) * A_\mu)(a)\}$

$= \min\{\sup_{a=bc} \min(A_\mu \circ \mathbf{N})(b), A_\mu(c)\}, ((A_\mu \circ \mathbf{N}) * A_\mu)(x(y+i)-xy)\}$

$= \min\{\sup_{a=bc} \min\{\sup_{b=b_1b_2} \min\{A_\mu(b_1), \mathbf{N}(b_2)\}, A_\mu(c)\}, ((A_\mu \circ \mathbf{N}) * A_\mu)(x(y+i)-xy)\}$

(since $\mathbf{N}(z) = 1$, for all $z\in N$)

$= \min\{\sup_{a=bc} \min\{\sup_{b=b_1b_2} \{A_\mu(b_1), A_\mu(c)\}, ((A_\mu \circ \mathbf{N}) * A_\mu)(x(y+i)-xy)\}$

(Since A_μ is an intuitionistic fuzzy right N-subgroup of N, $A_\mu(bc) = A_\mu(b_1b_2c) = A_\mu(b_1(b_2c)) \geq A_\mu(b_1)$)

$\leq \min\{\sup_{a=bc} \min\{A_\mu(bc), \mathbf{N}(c)\}, \mathbf{N}(x(y+i)-xy)\}$

$= \min\{\sup_{a=bc} \min\{A_\mu(bc), \mathbf{N}(x(y+i)-xy)\} = A_\mu(bc) = A_\mu(a)$

Thus $(A_\mu \circ \mathbf{N} \circ A_\mu) \cap ((A_\mu \circ \mathbf{N}) * A_\mu)) \subseteq A_\mu$.

Hence A_μ is an intuitionistic fuzzy bi-ideal of N.

Choose $a, b, c, x, y, i, b_1, b_2, x_1, x_2, y_1, y_2$ in N such that $a = bc = x(y+i)-xy$, $b = b_1, b_2$, $x = x_1 x_2$ and $y = y_1y_2$. Then

$(B_\mu \circ \mathbf{N} \circ B_\mu) \cup ((B_\mu \circ \mathbf{N}) * B_\mu))(a) = \max\{(B_\mu \circ \mathbf{N} \circ B_\mu)(a), ((B_\mu \circ \mathbf{N}) * B_\mu))(a)\}$

$= \max\{\inf_{a=bc} \max (B_\mu \circ \mathbf{N})(b), B_\mu(c)\}, ((B_\mu \circ \mathbf{N}) * B_\mu)(x(y+i)-xy)\}$

$= \max\{\inf_{a=bc} \max\{\inf_{b=b_1b_2} \max\{B_\mu(b_1), \mathbf{N}(b_2)\}, B_\mu(c)\}, ((B_\mu \circ \mathbf{N}) * B_\mu)(x(y+i)-xy)\}$

(since $\mathbf{N}(z) = 0$, for all $z\in N$)

$= \max\{\inf_{a=bc} \max\{\inf_{b=b_1b_2} \{B_\mu(b_1), B_\mu(c)\}, ((B_\mu \circ \mathbf{N}) * B_\mu)(x(y+i)-xy)\}$

(Since B_μ is a intuitionistic fuzzy right N-subgroup of N, $B_\mu(bc) = B_\mu(b_1b_2c) = B_\mu(b_1(b_2c)) \leq B_\mu(b_1)$)

$\geq \max\{\inf_{a=bc} \max\{B_\mu(bc), \mathbf{N}(c)\}, \mathbf{N}(x(y+i)-xy)\}$

$= \max\{\inf_{a=bc} \max\{B_\mu(bc), \mathbf{N}(x(y+i)-xy)\} = B_\mu(bc) = B_\mu(a)$

Thus $(B_\mu \circ \mathbf{N} \circ B_\mu) \cup ((B_\mu \circ \mathbf{N}) * B_\mu))(a) \supseteq B_\mu$.

Hence B_μ is an intuitionistic fuzzy bi-ideal of N.

Thus $\mu = (A_\mu , B_\mu)$ is an intuitionistic fuzzy bi-ideal of N.

Next we prove: μ is an intuitionistic fuzzy strong bi-ideal of N.

Choose $a, b, c, b_1, b_2 \in N$ such that $a = bc$ and $b = b_1, b_2$. Then

$\mathbf{N} \circ A_\mu \circ A_\mu(a) = \sup_{a=bc} \min\{\mathbf{N} \circ A_\mu(b), A_\mu(c)\}$

$= \sup_{a=bc} \min\{\sup_{b=b_1b_2} \min\{\mathbf{N}(b_1), A_\mu(b_2)\}, A_\mu(c)\}$

$= \sup_{a=bc} \min\{\sup_{b=b_1b_2} \{A_\mu(b_2), A_\mu(c)\}$

(Since A_μ is a left permutable intuitionistic fuzzy right N-subgroup of N, $A_\mu(bc) = A_\mu((b_1b_2)c) = A_\mu((b_2b_1) c) > A_\mu(b_2)$) and $\mathbf{N}(c) \geq A_\mu(c)$

$\leq \sup_{a=bc} \min\{A_\mu(bc), \mathbf{N}(c)\}$

$= \sup_{a=bc} \min\{A_\mu(bc), 1\}$

$= \sup_{a=bc} A_\mu(bc)$

$= A_\mu(a)$

Therefore $\mathbf{N} \circ A_\mu \circ A_\mu \subseteq A_\mu$.

Hence A_μ is an intuitionistic fuzzy strong bi-ideal of N.

Choose $a, b, c, b_1, b_2 \in N$ such that $a = bc$ and $b = b_1 b_2$. Then

$(\mathbf{N} \circ B_\mu \circ B_\mu)(a) = \inf_{a=bc} \max(\mathbf{N} \circ B_\mu)(b), B_\mu(c)\}$

$= \inf_{a=bc} \max\{\inf_{b=b_1b_2} \max\{\mathbf{N}(b_1), B_\mu(b_2)\}, B_\mu(c)\}$

$= \inf_{a=bc} \max\{\inf_{b=b_1b_2} \{B_\mu(b_2), B_\mu(c)\}$

(Since B_μ is a left permutable intuitionistic fuzzy right N-subgroup of N, $B_\mu(bc) = B_\mu((b_1b_2)c) = B_\mu((b_2b_1) c) \leq B_\mu(b_2)$)

$\geq \inf_{a=bc} \max\{B_\mu(bc), \mathbf{N}(c)\}$

$= \inf_{a=bc} \max\{B_\mu(bc), 0\}$
$= \inf_{a=bc} B_\mu(bc) = B_\mu(a)$
Therefore $(\mathbf{N} \circ B_\mu \circ B_\mu) \supseteq B_\mu$.
Hence B_μ is an intuitionistic fuzzy strong bi-ideal of N.
Thus $\mu = (A_\mu, B_\mu)$ is an intuitionistic fuzzy strong bi-ideal of N.

Theorem: 3.5

Every intuitionistic fuzzy left N-subgroup of N is an intuitionistic fuzzy strong bi-ideal of N.

Proof:

Let $\mu = (A_\mu, B_\mu)$ be an intuitionistic fuzzy left N-subgroup of N.
To prove: μ is an intuitionistic fuzzy strong bi-ideal of N.
First we prove: μ is an intuitionistic fuzzy bi-ideal of N.
Choose $a,b,c,x,y,i,c_1,c_2,x_1,x_2,y_1, y_2$ in N such that $a = bc = x(y+i) - xy$, $c = c_1 c_2$, $x = x_1 x_2$ and $y = y_1 y_2$. Then
$(A_\mu \circ \mathbf{N} \circ A_\mu) \cap ((A_\mu \circ \mathbf{N}) * A_\mu))(a) = \min\{(A_\mu \circ (\mathbf{N} \circ A_\mu))(a), ((A_\mu \circ \mathbf{N}) * A_\mu)(a)\}$
$= \min\{\sup_{a=bc} \min\{(A_\mu(b), (\mathbf{N} \circ A_\mu)(c)\}, ((A_\mu \circ \mathbf{N}) * A_\mu)(x(y+i)-xy)\}$
$= \min\{\sup_{a=bc} \min\{A_\mu(b), \sup_{c=c_1c_2} \min\{\mathbf{N}(c_1), A_\mu(c_2)\}, ((A_\mu \circ \mathbf{N}) * A_\mu)(x(y+i)-xy)\}$
$= \min\{\sup_{a=bc} \min\{A_\mu(b), \sup_{c=c_1c_2} A_\mu(c_2)\}\}, ((A_\mu \circ \mathbf{N}) * A_\mu)(x(y+i)-xy)\}$
(Since A_μ is an intuitionistic fuzzy left N-subgroup of N, $A_\mu(bc) = A_\mu(bc_1c_2) = A_\mu((bc_1)c_2) \geq A_\mu(c_2))$
$\leq \min\{\sup_{a=bc} \min\{\mathbf{N}(b), A_\mu(bc)\}, \mathbf{N}(x(y+i)-xy)\}$
$= A_\mu(bc) = A_\mu(a)$
Thus $(A_\mu \circ \mathbf{N} \circ A_\mu) \cap ((A_\mu \circ \mathbf{N}) * A_\mu)) \subseteq A_\mu$.
Hence A_μ is an intuitionistic fuzzy bi-ideal of N.
Choose a, b, c, x, y, i, $c_1, c_2, x_1, x_2, y_1, y_2$ in N such that $a = bc = x(y+i)-xy$, $c = c_1, c_2$, $x = x_1 x_2$ and $y = y_1 y_2$. Then
$(B_\mu \circ \mathbf{N} \circ B_\mu) \cup ((B_\mu \circ \mathbf{N}) * B_\mu))(a) = \max\{(B_\mu \circ \mathbf{N} \circ B_\mu)(a), ((B_\mu \circ \mathbf{N}) * B_\mu))(a)\}$
$= \max\{\inf_{a=bc} \max\{(B_\mu(b), (\mathbf{N} \circ B_\mu)(c)\}, ((B_\mu \circ \mathrm{N}) * B_\mu)(x(y+i)-xy)\}$
$= \max\{\inf_{a=bc} \max\{B_\mu(b), \inf_{c=c_1c_2} \max\{\mathbf{N}(c_1), B_\mu(c_2)\}\}, ((B_\mu \circ \mathbf{N}) * B_\mu)(x(y+i)-xy)\}$
$= \max\{\inf_{a=bc} \max\{B_\mu(b), \inf_{c=c_1c_2} B_\mu(c_2)\}, ((B_\mu \circ \mathbf{N}) * B_\mu)(x(y+i)-xy)\}$
(Since B_μ is an intuitionistic fuzzy left N-subgroup of N, $B_\mu(bc) = B_\mu(b(c_1c_2)) = B_\mu(bc_1)c_2) \leq B_\mu(c_2)$)
$\geq \max\{\inf_{a=bc} \max\{\mathbf{N}(b), B_\mu(bc)\}, \mathbf{N}(x(y+i)-xy)\} = B_\mu(bc) = B_\mu(a)$
Thus $(B_\mu \circ \mathbf{N} \circ B_\mu) \cup ((B_\mu \circ \mathbf{N}) * B_\mu) \supseteq B_\mu$.
Hence B_μ is an intuitionistic fuzzy bi-ideal of N.
Thus $\mu = (A_\mu, B_\mu)$ is an intuitionistic fuzzy bi-ideal of N.
Next we prove: μ is an intuitionistic fuzzy strong bi-ideal of N.
Choose a, b,c, $c_1, c_2 \in$N such that a = bc and $c = c_1, c_2$. Then

$$\mathbf{N} \circ A_\mu \circ A_\mu(a) = \sup_{a=bc} \min\{\mathbf{N}(b), (A_\mu \circ A_\mu)(c)\}$$

$= \sup_{a=bc} \min\{\mathbf{N}(b), \sup_{c=c_1c_2} \min\{A_\mu(c_1), A_\mu(c_2)\}$
$= \sup_{a=bc} \min\{1, \sup_{c=c_1c_2} \min\{A_\mu(c_1), A_\mu(c_2)\}$
(Since A_μ is an intuitionistic fuzzy left N-subgroup of N, $A_\mu(bc) = A_\mu(bc_1c_2) = A_\mu((bc_1)c_2) > A_\mu(c_2))$
$\leq \sup_{a=bc} \min\{\mathbf{N}(c_1), A_\mu(bc)\}$
$= \sup_{a=bc} \min\{1, A_\mu(bc)\}$
$= A_\mu(bc)$
$= A_\mu(a)$
Therefore $\mathbf{N} \circ A_\mu \circ A_\mu \subseteq A_\mu$.
Hence A_μ is an intuitionistic fuzzy strong bi-ideal of N.
Choose a, b,c, $c_1, c_2 \in$N such that a = bc and $c = c_1, c_2$. Then
$(\mathbf{N} \circ B_\mu \circ B_\mu)(a) = \inf_{a=bc} \max\{(\mathbf{N}(b), (B_\mu \circ B_\mu)(c)\}$
$\quad = \inf_{a=bc} \max\{0, \inf_{c=c_1c_2} \max\{B_\mu(c_1), B_\mu(c_2)\}$
$\quad = \inf_{a=bc} \max\{B_\mu(c_1), B_\mu(c_2)\}$
(Since B_μ is an intuitionistic fuzzy left N-subgroup of N, $B_\mu(bc) = B_\mu(bc_1c_2) = B_\mu((bc_1)c_2) \leq B_\mu(c_2))$
$\geq \inf_{a=bc} \max\{\mathbf{N}(c_1), B_\mu(bc)\}$
$= \inf_{a=bc} \max\{0, B_\mu(bc)\}$
$= B_\mu(bc) = B_\mu(a)$
Therefore $\mathbf{N} \circ B_\mu \circ B_\mu \supseteq B_\mu$.
Hence B_μ is an intuitionistic fuzzy strong bi-ideal of N.
Thus $\mu = (A_\mu, B_\mu)$ is an intuitionistic fuzzy strong bi-ideal of N.

Theorem: 3.6

Every left permutable intuitionistic fuzzy two-sided N-subgroup of N is an intuitionistic fuzzy strong bi-ideal of N.

Proof:

The proof is straight forward from the above Theorem 3.4 and Theorem 3.5.

Theorem: 3.7

Every left permutable intuitionistic fuzzy right ideal of N is an intuitionistic fuzzy strong bi-ideal of N.

Proof:

The proof is similar to that of Theorem 3.4.

Theorem: 3.8

Every intuitionistic fuzzy left ideal of N is an intuitionistic fuzzy strong bi-ideal of N.

Proof:

Let $\mu = (A_\mu, B_\mu)$ be an intuitionistic fuzzy left ideal of N.
To prove: μ is an intuitionistic fuzzy strong bi-ideal of N.
First we prove: μ is an intuitionistic fuzzy bi-ideal of N.
Choose a, b, c, x, y, i, $b_1, b_2, x_1, x_2, y_1, y_2$ in N such that $a = bc = x(y+i)-xy$, $b = b_1 b_2$,
$x = x_1 x_2$ and $y = y_1 y_2$. Then
$(A_\mu \circ \mathbf{N} \circ A_\mu) \cap ((A_\mu \circ \mathbf{N}) * A_\mu))(a) = \min\{(A_\mu \circ \mathbf{N} \circ A_\mu)(a), ((A_\mu \circ \mathbf{N}) * A_\mu)(a)\}$
$= \min\{\sup_{a=bc} \min(A_\mu \circ \mathbf{N})(b), A_\mu(c)\}, ((A_\mu \circ \mathbf{N}) * A_\mu)(x(y+i)-xy)\}$
$= \min\{\sup_{a=bc} \min\{(A_\mu \circ \mathbf{N})(b_1 b_2), A_\mu(c)\}, \sup_{a=x(y+i)-xy} \min((A_\mu \circ \mathbf{N})(x), (A_\mu \circ \mathbf{N})(y), A(i)\}\}$
(since $A \circ \mathbf{N} \subseteq \mathbf{N}$ and since A_μ is an intuitionistic fuzzy left ideal of N, $A_\mu(x(y+i) - xy) \geq A_\mu(i))$

$\leq \min\{\sup_{a=bc}\min\{\mathbf{N}(b_1b_2), \mathbf{N}(c)\}, \sup_{a=x(y+i)-xy}\min\{\mathbf{N}(x), \mathbf{N}(y), A_\mu(x(y+i)-xy)\}\}$
$= A_\mu(x(y+i)-xy)$
$= A_\mu(a)$.
Thus $(A_\mu \circ \mathbf{N} \circ A_\mu) \cap ((A_\mu \circ \mathbf{N}) * A_\mu) \subseteq A_\mu$.
Hence A_μ is an intuitionistic fuzzy bi-ideal of N.
Choose a, b, c, x, y, i, b_1, b_2, x_1, x_2, y_1, y_2 in N such that a = bc = x(y+i)-xy, $b = b_1b_2$, $x = x_1 x_2$ and $y = y_1y_2$. Then
$(B_\mu \circ \mathbf{N} \circ B_\mu) \cup ((B_\mu \circ \mathbf{N}) * B_\mu))(a) = \max\{(B_\mu \circ \mathbf{N} \circ B_\mu)(a), ((B_\mu \circ \mathbf{N}) * B_\mu))(a)\}$
$= \max\{\inf_{a=bc}\max(B_\mu \circ \mathbf{N})(b), B_\mu c)\}, ((B_\mu \circ \mathbf{N}) * B_\mu)(x(y+i)-xy)\}$
$= \max\{\{\inf_{a=bc}\max(B_\mu \circ \mathbf{N})(b_1b_2), B_\mu(c)\}, \inf_{a=x(y+i)-xy}\max\{(B_\mu \circ \mathbf{N})(x), (B_\mu \circ_a \mathbf{N})(y), B_\mu(i)\}\}$
(since $B_\mu \circ \mathbf{N} \supseteq \mathbf{N}$ and since B_μ is an intuitionstic fuzzy left ideal of N, $B_\mu(x(y+i)-xy) \leq B_\mu(i)$)
$\geq \max\{\inf_{a=bc}\max\{\mathbf{N}(b_1b_2), \mathbf{N}(c)\}, \inf_{a=x(y+i)-xy}\max\{\mathbf{N}(x), \mathbf{N}(y), B_\mu(x(y+i)-xy)\}\}$
$= B_\mu(x(y+i)-xy) = B_\mu(a)$.
Therefore $(B_\mu \circ \mathbf{N} \circ B_\mu) \cup ((B_\mu \circ \mathbf{N}) * B_\mu) \supseteq B_\mu$.
Hence B_μ is an intuitionistic fuzzy bi-ideal of N.
Thus $\mu = (A_\mu, B_\mu)$ is an intuitionistic fuzzy bi-ideal of N.
Next we prove: μ is an intuitionistic fuzzy strong bi-ideal of N.
Choose a, b, c, b_1, $b_2 \in$ N such that a = bc = b(n + c) - bn. Then
$\mathbf{N} \circ A_\mu \circ A_\mu(a) = \sup_{a=bc}\min\{(\mathbf{N} \circ A_\mu)(b), A_\mu(c)\}$
$= \sup_{a=bc}\min\{\sup_{b=b_1b_2}\min\{\mathbf{N}(b_1), A_\mu(b_2)\}, A_\mu(c)\}$
$= \sup_{a=bc}\min\{\sup_{b=b_1b_2}\{A_\mu(b_2), A_\mu(c)\}$
(Since $A_\mu$is a intuitionistic fuzzy left ideal of N, $A_\mu(a) = A_\mu(bc) = A_\mu(b(n+c) - bn) > A_\mu(c)$ and $\mathbf{N}(b_2) \geq A_\mu(b_2)$)
$\leq \sup_{a=bc}\min\{\mathbf{N}(b_2), A_\mu(b(n+c) - bn)\}$
$= \sup_{a=bc} A_\mu(b(n+c) - bn)$
$= A_\mu(bc) = A_\mu(a)$
Therefore $\mathbf{N} \circ A_\mu \circ A_\mu \subseteq A_\mu$. Hence A_μ is an intuitionistic fuzzy strong bi-ideal of N.
Choose a, b, c, b_1, $b_2 \in$ N such that a = bc and $b = b_1 b_2$. Then
$\mathbf{N} \circ B_\mu \circ B_\mu(a) = \inf_{a=bc}\max\{(\mathbf{N} \circ B_\mu)(b), B_\mu(c)\}$
$= \inf_{a=bc}\max\{\inf_{b=b_1b_2}\max\{\mathbf{N}(b_1), B_\mu(b_2)\}, B_\mu(c)\}$
$= \inf_{a=bc}\max\{\inf_{b=b_1b_2}\{B_\mu(b_2), B_\mu(c)\}$
(Since A is an anti fuzzy left ideal of N, $B_\mu(a) = B_\mu(bc) = B_\mu(b(n+c) - bn) \leq B_\mu(c)$) and
$\geq \inf_{a=bc}\max\{\mathbf{N}(b_2), B_\mu(b(n+c) - bn)\}$
$= \inf_{a=bc}\max\{0, B_\mu(bc)\}$
$= B_\mu(bc) = B_\mu(a)$
Therefore $\mathbf{N} \circ B_\mu \circ B_\mu \supseteq B_\mu$. Hence B_μ is an intutionistic fuzzy strong bi-ideal of N.
Thus $\mu = (A_\mu, B_\mu)$ is an intutionistic fuzzy strong bi-ideal of N.

Theorem: 3.9

Every left permutable fuzzy ideal of N is a fuzzy strong bi-ideal of N.

Proof:

The proof is straight forward from the Theorem 3.7 and Theorem 3.8.

Theorem: 3.10

Let $\mu = (A_\mu, B_\mu)$ be any intuitionistic fuzzy strong bi-ideal of a near-ring N.Then $A_\mu(axy) \geq \min\{A_\mu(x), A_\mu(y)\}$ and $B_\mu \leq \max\{B_\mu(x), B_\mu(y)\}$ ∀a, x, y ∈N.

Proof:

Assume that (A_μ, B_μ) is an intuitionistic fuzzy strong bi-ideal of N. Then $\mathbf{N} \circ A_\mu \circ A_\mu \subseteq A_\mu$ and $\mathbf{N} \circ B_\mu \circ B_\mu \supseteq B_\mu$.
Let a, x and y be any element of N. Then
$A_\mu(axy) \geq (\mathbf{N} \circ A_\mu \circ A_\mu)(axy)$
$= \sup_{axy=pq}\min\{(\mathbf{N} \circ A_\mu)(p), A_\mu(q)\}$
$\geq \min\{(\mathbf{N} \circ A_\mu)(ax), A_\mu(y)\}$
$= \min\{\sup_{ax=z_1z_2}\min\{\mathbf{N}(z_1), A_\mu(z_2)\}, A_\mu(y)\}$
$\geq \min\{\min\{\mathbf{N}(a), A_\mu(x)\}, A_\mu(y)\}$
$= \min\{\min\{1, A_\mu(x), A_\mu(y)\}$
$= \min\{A_\mu(x), A_\mu(y)\}$
This shows that $A_\mu(axy) \geq \min\{A_\mu(x), A_\mu(y)\}$ ∀a, x, y ∈N and
$B_\mu(axy) \leq (\mathbf{N} \circ B_\mu \circ B_\mu)(axy)$
$= \inf_{axy=pq}\max\{\mathbf{N} \circ B_\mu(p), B_\mu(q)\}$
$= \max\{(\mathbf{N} \circ B_\mu)(ax), B_\mu(y)\}$
$= \max\{\inf_{ax=z_1z_2}\max\{\mathbf{N}(z_1), B_\mu(z_2)\}, B_\mu(y)\}$
$\leq \max\{\max\{\mathbf{N}(a), B_\mu x)\}, B_\mu(y)\}$
$= \max\{\max\{1, B_\mu(x), B_\mu(y)\}$
$= \max\{A_\mu(x), B_\mu(y)\}$

This shows that $B_\mu(axy) \leq \max\{B_\mu(x), B_\mu(y)\}$ ∀a, x, y ∈N

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