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RESEARCH ARTICLE

ON QUASI SIMPLE TERNARY Γ-SEMIRING

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ABSTRACT

In this paper, we give some characterizations of quasi-simple ternary Γ-semirings.
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1. INTRODUCTION

Algebraic structures play a prominent role in mathematics with wide applications in many disciplines such as computer sciences, information sciences, engineering, physics etc. The theory of ternary algebraic system was introduced by Lehmer [Lehmer, 1932] in 1932, but earlier such structures was studied by Kasner [1904] who give the idea of n-ary algebras. Lehmer investigated certain algebraic systems called triplexes which turn out to be commutative ternary groups. Ternary semigroups are universal algebras with one associative ternary operation. The notion of ternary Γ-semiring was introduced by M. Sajani Lavanya, D.Madhusudhana Rao and V. Syam Julius Rajendra [2015] in the year 2015**,** who is credited with example of a ternary Γ-semiring which cannot reduce to Γ-semiring.

2. Preliminaries

Definition 2.1 [9]: Let T and Γ be two additive commutative semigroups. T is said to be a *Ternary* **Γ**-*semiring* if there exist a mapping from T ×Γ× T ×Γ× T to T which maps $(x_1, \alpha, x_2, \beta, x_3) \rightarrow [x_1\alpha x_2\beta x_3]$ satisfying the conditions:

i) $[[a\alpha b\beta c]\gamma d\delta e] = [a\alpha[b\beta c\gamma d]\delta e] = [a\alpha b\beta[c\gamma d\delta e]]$
ii) $[(a + b)c\beta d] = [a\alpha c\beta d] + [b\alpha c\beta d]$
iii) $[a\alpha\,(b + c)\beta d] = [a\alpha b\beta d] + [a\alpha c\beta d]$
iv) $[a\alpha b\beta(c + d)] = [a\alpha b\beta c] + [a\alpha b\beta d]$ for all $a, b, c, d \in$ T and $\alpha, \beta, \gamma, \delta \in \Gamma$.

Note 2.2 [9]: For the convenience we write $x_1\alpha x_2\beta x_3$ instead of $[x_1\alpha x_2\beta x_3]$

Note 2.3 [9]: Let T be a ternary Γ-semiring. If A,B and C are three subsets of T , we shall denote the set AΓBΓC = $\{\Sigma a\alpha b\beta c : a \in A, b \in B, c \in C, \alpha, \beta \in \Gamma\}$.

Note 2.4 [9]: Let T be a ternary Γ-semiring. If A,B are two subsets of T , we shall denote the set A + B = $\{a + b : a \in A, b \in B\}$ and 2A = $\{ a + a : a \in A\}$.

Example 2.5 [9]: Let T be the set of all $m \times n$ matrices over the set of all non-negative rational numbers and Γ be the set of all $n \times m$ matrices over the set of all non-negative rational numbers. Define A + B = usual matrix addition and AαBβC = usual matrix product of A, α, B, β and C; for all A, B, C ∈ T and for all $\alpha, \beta \in \Gamma$. Then T is a ternary Γ-semiring with matrix addition and matrix multiplication as the ternary operation.

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Definition 2.6 [9]: An element 0of a ternary Γ-semiring T is said to be an ***absorbing zero*** of T provided $0 + x = x = x + 0$and $0\alpha a\beta b = a\alpha 0\beta b = a\alpha b\beta 0 = 0 \ \forall\ a, b, x \in$ T and $\alpha, \beta \in \Gamma$.

Definition 2.7 [9]: A ternary Γ-semiring T is said to be ***commutative ternary Γ-semiring*** provided $a\Gamma b\Gamma c = b\Gamma c\Gamma a = c\Gamma a\Gamma b = b\Gamma a\Gamma c = c\Gamma b\Gamma a = a\Gamma c\Gamma b$for all $a, b, c \in$ T.

Definition 2.8 [9]: A non-empty subset S of a ternary Γ-semiring T is a ***ternary sub Γ-semiring*** if and only if S + S ⊆ S and SΓSΓS ⊆ S.

Definition 2.9 [9]: A nonempty subset A of a ternary Γ-semiring T is a ***left ternary Γ-ideal*** of T provided i) $a + b \in$ A $\Rightarrow a + b \in$ A and ii) $b, c \in$ T, $a \in$ A, $\alpha, \beta \in \Gamma \Rightarrow b\alpha c\beta a \in A$.

Definition 2.10 [9]: A nonempty subset of A of a ternary Γ-semiring T is a ***lateral ternary Γ-ideal*** of T if i) $a + b \in$ A $\Rightarrow a + b \in$ A and ii) $b, c \in$ T, $a \in$ A, $\alpha, \beta \in \Gamma \Rightarrow b\alpha a\beta c \in A$.

Definition 2.11 [9]: A nonempty subset A of a ternary Γ-semiring T is a ***right ternary Γ-ideal*** of T if i) $a + b \in$ A $\Rightarrow a + b \in$ A and ii) $b, c \in$ T, $a \in$ A, $\alpha, \beta \in \Gamma \Rightarrow a\alpha b\beta c \in A$.

Definition 2.12 [9]: A nonempty subset A of a ternary Γ-semiring T is a ***ternary Γ-ideal*** of T if and only if it is left ternary Γ-ideal, lateral ternary Γ-ideal and right ternary Γ-ideal of T.

Definition 2.13 [9]: An additive sub-semigroup Q of a ternary Γ-semiring T is called a quasi-ternary Γ-ideal of T if QΓTΓT∩(TΓQΓT+TΓTΓQΓTΓT)∩TΓTΓQ ⊆ Q.

3. Quasi Simple:

Definition 3.1: Let T be a ternary Γ-semiring and A be a non-empty sub set of T. A quasi ternary Γ-ideal $< A >_q$ denoted the ***quasi ternary Γ-ideal of T generated by A***. Since $< A >_q$ is the smallest quasi ternary Γ-ideal of T containing A.

Theorem 3.2: Let T be a ternary Γ-Semiring and A is a non-empty sub set of T, then

$< \mathbf{A} >_q = A\cup\left(T\Gamma T\Gamma A\cap\left(T\Gamma A\Gamma T + T\Gamma T\Gamma A\Gamma T\Gamma T\right)\cap A\Gamma T\Gamma T\right)$

for any $a \in$ T and so

$< a >_q = a\cup\left(T\Gamma T\Gamma a\cap\left(T\Gamma a\Gamma T + T\Gamma T\Gamma a\Gamma T\Gamma T\right)\cap a\Gamma T\Gamma T\right)$

for any $a \in$ T.

Definition 3.3: A ternary Γ-semiring T is said to be ***quasi simple*** if T has no proper quasi ternary Γ-ideals.

Example 3.4: The ternary ternary Γ-semiring T = { $-i$, i} and Γ = {$-i$,1, i} is a quasi simple.

Theorem 3.5: Let T be a ternary Γ-Semiring, then the following statements are equivalent

1.T is quasi simple

2. $\left(T\Gamma T\Gamma a\cap\left(T\Gamma a\Gamma T + T\Gamma T\Gamma a\Gamma T\Gamma T\right)\cap a\Gamma T\Gamma T\right) = T$ **for all** $a \in$ **T**

3. $< a >_q =$ **T for all** $a \in$ **T.**

Proof: (i) ⇒ (ii) : Suppose that T is a quasi simple and $a \in$ T.

Since $\left(T\Gamma T\Gamma a\cap\left(T\Gamma a\Gamma T + T\Gamma T\Gamma a\Gamma T\Gamma T\right)\cap a\Gamma T\Gamma T\right)$ is a quasi ternary ternary Γ-ideal of T. Since T is quasi simple and hence $\left(T\Gamma T\Gamma a\cap\left(T\Gamma a\Gamma T + T\Gamma T\Gamma a\Gamma T\Gamma T\right)\cap a\Gamma T\Gamma T\right) = T$

ii) ⇒ iii): Since TΓTΓa ∩ (TΓaΓT + TΓTΓaΓTΓT) ∩ aΓTΓT ⊆ $< a >_q \Rightarrow < a >_q =$ T for all $a \in$ T

iii) ⇒ i): Let Q be a quasi ternary Γ-ideal of T and $a \in$ Q. Therefore, $< a >_q =$ T ⊆ Q ⇒ Q = T and hence T is quasi simple.

Definition 3.6: Let T be a ternary Γ-semiring with 0, TΓTΓT ≠ {0} and $|T| > 1$. Then T is said to be ***0-quasi simple*** provided T has no nonzero proper quasi ternary Γ-ideals.

Example 3.7: The ternary Γ-semiring T = { i, 0, i} where T = Γ is a 0-quasi simple.

Theorem 3.8: Let T be a ternary Γ-semiring with 0, $(T\Gamma)^2$T ≠ {0} and $|T| > 1$. Then T is a 0-quasi simple if and only if $< a >_q =$ **T for all** $a \in$ **T\{0}.**

Proof: Suppose T is a 0-quasi simple. Let $a \in$ T\{0}. Thus $< a >_q \neq$ {0} $\Rightarrow < a >_q =$ T.

Conversely, let Q be a non-zero quasi ternary Γ-ideal of T and $a \in$ Q\{0}. Then T = $< a >_q$ ⊆ Q ⊆ T. Therefore, T is a 0-quasi simple.

Theorem 3.9[9]:An additive sub semi group Q of a ternary Γ-semiring T is a quasi-ternary Γ-ideal of T if Q is the intersection of a right ternary Γ-ideal, a lateral ternary Γ-ideal, and a left ternary Γ- ideal of T

Theorem 3.10[9]: An additive sub semigroup Q of a ternary Γ-semiring T is a minimal quasi-ternary Γ-ideal of T if and only if Q is the intersection of a minimal right ternary Γ-ideal, a minimal lateral ternary Γ-ideal, and a minimal left ternary Γ-ideal of T.

The following theorem shows the relationship between minimal quasi ternary Γ-ideals and quasi-simple in ternary Γ-semiring.

Theorem 3.11: Let T be a ternary Γ-semiring and Q a quasi-ternary Γ-ideal of T. Q is a minimal quasi-ternary Γ-ideal of T if and only if Q is quasi-simple

Proof: Assume Q is a minimal quasi-ternary Γ-ideal of T and let A be a quasi-ternary Γ-ideal of Q. Then QΓQΓA ∩ AΓQΓQ ∩ (QΓAΓQ ∪ QΓQΓAΓQΓQ) ⊆ Q. It is easy to verify that QΓQΓA ∩ AΓQΓQ ∩ (QΓAΓQ ∪ QΓQΓAΓQΓQ) is a quasi-ternary Γ-ideal of T. Since Q is minimal and QΓQΓA ∩ AΓQΓQ ∩ (QΓAΓQ ∪ QΓQΓAΓQΓQ) ⊆ A ⊆ Q, QΓQΓA ∩ AΓQΓQ ∩ (QΓAΓQ ∪ QΓQΓAΓQΓQ) = A = Q. Hence Q is quasi-simple. Conversely, assume Q is quasi-simple. Let A be a quasi-ternary Γ-ideal of T such that A ⊆ Q. So A is a quasi-ternary Γ-ideal of Q, this implies A = Q. Hence Q is a minimal quasi-ternary Γ-ideal of T.

Theorem 3.12: Let T be a ternary Γ-semiring with zero and Q a nonzero quasi-ternary Γ-ideal of T. The following statements hold

(1) If Q is 0-quasi-simple, then Q is a 0-minimal quasi-ternary **Γ**-ideal of T.
(2) If Q is a 0-minimal quasi-ternary **Γ**-ideal of T and QΓQΓA ∩ AΓQΓQ ∩ (QΓAΓQ ∪ QΓQΓAΓQΓQ) ≠ {0} for all a nonzero quasi-ternary **Γ**-ideal A of Q, then Q is 0-quasi-simple.

Proof: (1) Suppose that Q is 0-quasi-simple. Let A be a nonzero quasi-ternary Γ-ideal of T such that A ⊆ Q. So A is a nonzero quasi-ternary Γ-ideal of Q, this implies A = Q. Hence Q is a 0-minimal quasi-ternary Γ-ideal of T.
(2) Suppose that Q is a 0-minimal quasi-ternary Γ-ideal of T and let A be a nonzero quasi-ternary Γ-ideal of Q. Thus QΓQΓA ∩ AΓQΓQ ∩ (QΓAΓQ ∪ QΓQΓAΓQΓQ) ⊆ A. Similar to the proof of Theorem 3.11, hence Q is 0-quasi-simple.

Example 3.13 (1) In Z_{18}, consider the ternary Γ-semiring T = $\{\overline{1},\overline{3},\overline{9}\}$, Γ = $\{\overline{1},\overline{9}\}$ under the usual addition and ternary multiplication. It is easy to see that $\{\overline{9}\}$ is a minimal quasi-ternary Γ-ideal of T. By Theorem 3.11, the ternary Γ-semiring $\{\overline{9}\}$ is quasi-simple.
(2) In Z_{18}, consider the ternary Γ-semiring T = $\{\overline{0},\overline{1},\overline{3},\overline{9}\}$, Γ = $\{\overline{1},\overline{9}\}$ under the usual addition and ternary multiplication. It is easy to see that Q = $\{\overline{0},\overline{9}\}$ is 0-quasi-simple and Q is a quasi-ternary Γ-ideal of T, by Theorem 3.12(1), Q is a 0-minimal quasi-ternary Γ-ideal of T.
(3) The converse of Theorem 3.12(1) is not true in general. In Z_{81}, consider the ternary Γ-semiring T = $\{\overline{0},\overline{3},\overline{27}\}$ where T = Γ, under the usual addition and ternary multiplication. It is easy to see that Q = $\{\overline{0},\overline{27}\}$ is a 0-minimal quasi-ternary Γ-ideal of T but Q is not 0-quasi-simple.

Lemma 3.14[9]: The intersection of arbitrary collection of quasi-ternary Γ-ideals of a ternary Γ-semiring T is a quasi-ternary Γ-ideal of T.

Theorem 3.15: Let T be a ternary Γ-semiring having proper quasi-ternary Γ-ideals. Then every proper quasi-ternary Γ-ideal of T is minimal if and only if the intersection of any two distinct proper quasi-ternary Γ-ideals is empty.

Proof: Let Q_1 and Q_2 be two distinct proper quasi-ternary **Γ**-ideals of T. Then Q_1 and Q_2 are minimal. If $Q_1 \cap Q_2 \neq \emptyset$, then by lemma 3.14, $Q_1 \cap Q_2$ is a quasi-ternary Γ-ideal of T. Since $Q_1 \cap Q_2$ is a proper subset of Q_1, a contradiction. Hence $Q_1 \cap Q_2 = \emptyset$. The converse is obvious.

Theorem 3.16: Let T be a ternary Γ-semiring with zero having nonzero proper quasi-ternary Γ-ideals. Then every nonzero proper quasi-ternary Γ-ideal of T is minimal if and only if the intersection of any two distinct proper quasi-ternary Γ-ideals is {0}.

Proof: Using the same proof of Theorem 3.15.

Conclusion

In this paper mainly we start the study of quasi-simple ternary Γ-semirings. We characterize those quasi simple ternary Γ-semirings.

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