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RESEARCH ARTICLE

FUZZY EOQ MODEL FOR CONTROLLABLE DETERIORATION RATE AND TIME DEPENDENT DEMAND AND INVENTORY HOLDING COST

^{1,*}Palani, R. and ²Maragatham, M.

¹Research Scholar, PG and Research Department of Mathematics, Periyar E.V.R. College,
Tiruchirappalli - 620023, Tamil Nadu, India

²Associate Professor, PG and Research Department of Mathematics, Periyar E.V.R. College,
Tiruchirappalli - 620023, Tamil Nadu, India

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ABSTRACT

In this paper, we have developed a deterministic fuzzy inventory model for deteriorating items in which demand rate quadratic and holding cost is linear function of time. During deterioration period, deterioration rate can be controlled using preservation technology (PT). The ordering cost, deterioration cost and purchase cost are assumed as triangular fuzzy number. The purpose of our study is to find an optimal replenishment cycle and order quantity so that the total inventory cost per unit time is minimum. An exponential distribution is used to represent the distribution of time to deterioration. The derived model is illustrated by a numerical example.

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INTRODUCTION

Inventory may be considered as an accumulation of a product that would be used to satisfy future demands for that product. An optimal replenishment policy is dependent on ordering cost, inventory carrying cost and shortage cost. An important problem confronting a supply manager in any modern organization is the control and maintenance of inventories of deteriorating items. Fortunately, the rate of deterioration is too small for items like steel, toys, glassware, hardware, etc. There is little requirement for considering deterioration in the determination of economic lot size. So in this paper, an inventory model is developed for deteriorating items by considering the fact that using the preservation technology the retailer can reduce the deterioration rate by which he can reduce the economic losses, improve the customer service level and increase business competitiveness. In reality, the demand and holding cost for physical goods may be time dependent. Time also plays an important role in the inventory system. So, in this paper we consider that demand and holding cost are time dependent. Recently, Mishra and Singh (2016) developed a deteriorating inventory model with partial backlogging when demand and deterioration rate is constant. Vinod kumar Mishra (2013) developed an inventory model of instantaneous deteriorating items with controllable deterioration rate for time dependent demand and holding cost. Vinod kumar Mishra (2014) developed deteriorating inventory model with controllable deterioration rate for time-dependent demand and time-varying holding cost. Parmar Kirtan and Gothi (2015) developed EOQ model with constant deterioration rate and time dependent demand and IHC. Leea and Dye (2012) formulated a deteriorating inventory model with stock-dependent demand by allowing preservation technology cost as a decision variable in conjunction with replacement policy. Dye and Hsieh (2012) presented an extended model of Hsu *et al.* (2010) by assuming that the preservation technology cost is a function of the length of replenishment cycle. Jagadeeswari and Chenniappan, (2014) developed an order level inventory model for deteriorating items with time-quadratic demand and partial backlogging. Sarala Pareek and Garima Sharma (2014) developed an inventory model with Weibull distribution deteriorating item with exponential declining demand and partial backlogging.

*Corresponding author: Palani, R.,

Research Scholar, PG and Research Department of Mathematics, Periyar E.V.R. College, Tiruchirappalli - 620023, Tamil Nadu, India

Amutha and Chandrasekaran (2012) developed an inventory model for deteriorating Products with Weibull Distribution Deterioration, Time-Varying Demand and Partial Backlogging. Kirtan Parmar and Gothi (2014) developed a deterministic inventory model for deteriorating items where time to deterioration has Exponential distribution and with time-dependent quadratic demand. Also, Gothi and Kirtan Parmar (2015) have extended above deterministic inventory model by taking two parameter Weibull distributions to represent the distribution of time to deterioration and shortages are allowed and partially backlogged. Kirtan Parmar and Gothi (2015) developed an economic production model for deteriorating items using three parameter Weibull distributions with constant production rate and time varying holding cost. The consideration of PT is important due to rapid social changes, and the fact that PT can reduce the deterioration rate significantly. By the efforts of investing in preservation technology, we can reduce the deterioration rate. So in this paper, we made the model of Mishra and Singh (2011) more realistic by considering the fact that use of preservation technology can reduce the deterioration rate significantly, which help the retailers to reduce their economic losses. In many inventory models uncertainty is due to fuzziness and fuzziness is the closed possible approach to reality. In recent years some researchers gave their attention towards a time dependent rate because the demand of newly launched products such as fashionable garments, electronic items and mobiles etc. increases with time and later it becomes constant. Deterioration is defined as damage, decay or spoilage of the items that are stored for future use always lose part of their value with passage of time, so deterioration cannot be avoided in any business scenarios. Syed (2007) developed a fuzzy inventory model without shortages using Signed distance method. They used fuzzy triangular number for both ordering cost and holding cost. Maragatham. M, discussed the A fuzzy model with changing deterioration rate. Jhuma Bhowmick (6) discussed the fuzzy inventory model for deteriorating items with time-varying demand and shortages. Umap (2010) formed a fuzzy EOQ model for deteriorating items with two warehouses. Palani and Maragatham, (2016), developed EOQ model for controllable deterioration rate and time dependent demand and Inventory holding cost. In this paper, the same inventory model used in fuzzy environment in real life for uncertainty. The triangular fuzzy numbers are used in this paper. This model is solved by analytically to determine the optimal cycle time and the derived model is illustrated by a numerical example. This paper, some of the parameters are considered as triangular fuzzy numbers for fuzzy cases. For defuzzification of the total cost function and optimum order quantity graded mean representation method is used.

2. Preliminaries

2.1 Basic Definitions

2.1.1 Fuzzy Set

If X is a collection of objects denoted generically by x , then a fuzzy set \tilde{A} in X is defined as a set of ordered pairs $\tilde{A} = \{(x; \mu_{\tilde{A}}(x))/x \in X\}$, where $\mu_{\tilde{A}}(x)$ is called the membership function for the fuzzy set \tilde{A} . The membership function maps each element of X to a membership grade between 0 and 1 (included).

2.1.2 α - Cut

The set of elements that belong to the fuzzy set \tilde{A} at least to the degree of α is called the α level set or α - cut (*i. e.*), $\tilde{A}^{(\alpha)} = \{x \in X: \mu_{\tilde{A}}(x) \geq \alpha\}$.

2.2 Fuzzy Number

Fuzzy numbers are of great important in fuzzy systems.

2.2.1 Fuzzy Number

A fuzzy subset \tilde{A} of the real line R with membership function $\mu_{\tilde{A}}: R \rightarrow [0, 1]$ is called a fuzzy number if

- i. \tilde{A} is normal, (*i. e.*), there exist an element x_0 such that $\mu_{\tilde{A}}(x_0) = 1$.
- ii. \tilde{A} is fuzzy convex, (*i. e.*), $\mu_{\tilde{A}}[\lambda x_1 + (1 - \lambda)x_2] \geq \mu_{\tilde{A}}(x_1) \wedge \mu_{\tilde{A}}(x_2)$, $x_1, x_2 \in R, \forall \lambda \in [0, 1]$.
- iii. $\mu_{\tilde{A}}$ is upper continuous.
- iv. $\text{supp } \tilde{A}$ is bounded,

where $\text{supp } \tilde{A} = \{x \in R: \mu_{\tilde{A}}(x) > 0\}$.

2.2.2 Generalized Fuzzy Number

Any fuzzy subset of the real line R , whose membership function $\mu_{\tilde{A}}$ satisfies the following conditions, is a generalized fuzzy number

- i. $\mu_{\tilde{A}}(x)$ is a continuous mapping from R to the closed interval $[0, 1]$.
- ii. $\mu_{\tilde{A}}(x) = 0$, $-\infty < x \leq a_1$.
- iii. $\mu_{\tilde{A}}(x) = L(x)$ is strictly increasing on $[a_1, a_2]$.
- iv. $\mu_{\tilde{A}}(x) = 1$, $a_2 \leq x \leq a_3$.
- v. $\mu_{\tilde{A}}(x) = R(x)$ is strictly decreasing on $[a_3, a_4]$.

vi. $\mu_{\tilde{A}}(x) = 0, a_4 \leq x \leq \infty$, where a_1, a_2, a_3, a_4 are real numbers.

2.2.3 Triangular Fuzzy Number

The fuzzy set $\tilde{A} = (a_1, a_2, a_3)$, where $a_1 \leq a_2 \leq a_3$ and defined on \mathbb{R} , is called triangular fuzzy number, if the membership function of \tilde{A} is given by

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{x - a_1}{a_2 - a_1}, & a_1 \leq x \leq a_2 \\ \frac{a_3 - x}{a_3 - a_2}, & a_2 \leq x \leq a_3 \\ 0, & \text{otherwise} \end{cases}$$

2.2.4 Operations of Triangular Fuzzy Number

Consider two triangular fuzzy numbers $\tilde{A} = (a_1, a_2, a_3), \tilde{B} = (b_1, b_2, b_3)$.

i. The addition of \tilde{A} and \tilde{B} is

$$\begin{aligned} \tilde{A} + \tilde{B} &= (a_1, a_2, a_3) + (b_1, b_2, b_3) \\ &= (a_1 + b_1, a_2 + b_2, a_3 + b_3) \end{aligned}$$

where $a_1, a_2, a_3, b_1, b_2, b_3$ are real numbers.

ii. The multiplication of \tilde{A} and \tilde{B} is

$$\tilde{A} \times \tilde{B} = (c_1, c_2, c_3),$$

where $T = \{a_1 b_1, a_1 b_3, a_3 b_1, a_3 b_3\}$, $c_1 = \min T$, $c_2 = a_2 b_2$, $c_3 = \max T$

If $a_1, a_2, a_3, b_1, b_2, b_3$ are all non zero positive real numbers, then $\tilde{A} \times \tilde{B} = (a_1 b_1, a_2 b_2, a_3 b_3)$

iii. $[-\tilde{B}] = -(b_1, b_2, b_3) = (-b_3, -b_2, -b_1)$ then the subtraction of \tilde{B} from \tilde{A} is

$$\begin{aligned} \tilde{A} + \tilde{B} &= (a_1, a_2, a_3) - (b_1, b_2, b_3) \\ &= (a_1 - b_3, a_2 - b_2, a_3 - b_1) \end{aligned}$$

where $a_1, a_2, a_3, b_1, b_2, b_3$ are real numbers.

iv. $\frac{1}{\tilde{B}} = \tilde{B}^{-1} = \left(\frac{1}{b_3}, \frac{1}{b_2}, \frac{1}{b_1}\right)$, where b_1, b_2, b_3 are all non zero positive real numbers, then division of \tilde{A} and \tilde{B} is

$$\frac{\tilde{A}}{\tilde{B}} = \frac{(a_1, a_2, a_3)}{(b_1, b_2, b_3)} = \left(\frac{a_1}{b_3}, \frac{a_2}{b_2}, \frac{a_3}{b_1}\right)$$

v. For any real number

$$k, k\tilde{A} = \begin{cases} (ka_1, ka_2, ka_3), & \text{if } k > 0 \\ (ka_3, ka_2, ka_1), & \text{if } k < 0 \end{cases}$$

2.3 Defuzzification

Defuzzification is the conversion of a fuzzy quantity to a crisp quantity. Defuzzification methods obtain the representative value of a fuzzy set.

2.3.1 Graded Mean Representation Method

Let \tilde{A} be a fuzzy number with left reference function L and right reference function R . Let L^{-1} and R^{-1} be the inverse functions of L and R respectively. The graded mean integration representation of (\tilde{A}) is defined by

$$\rho(\tilde{A}) = \frac{\frac{1}{2} \int_0^1 h [L^{-1}(h) + R^{-1}(h)] dh}{\int_0^1 h dh}$$

with $0 < h < 1$

By the above formula, the graded mean representations of triangular fuzzy number $\tilde{A} = (a_1, a_2, a_3)$ is given by $\rho(\tilde{A}) = \frac{a_1 + 4a_2 + a_3}{6}$.

3. Assumptions and Notations

The mathematical model is based on the following notations and assumptions.

Notations

$R(t)$: Quadratic demand rate.

\tilde{A} : Fuzzy ordering cost per order.

\tilde{C}_d : Fuzzy deterioration cost per unit per unit time.

C_h : Inventory holding cost per unit per unit of time.

$m(\xi)$: Reduced deterioration rate due to use of preservation technology.

θ : Deterioration rate.

τ_p : Resultant deterioration rate, $\tau_p = (\theta - m(\xi))$.

\tilde{P}_c : Fuzzy purchase cost per unit.

Q : Order quantity in one cycle.

t_d : The time from which the deterioration start in the inventory.

t_1 : Length of cycle time (decision variable).

TC : Total cost per unit time.

$\tilde{TC}_{dG}(t_1)$: The defuzzified value of $\tilde{TC}(t_1)$ by Graded mean representation method.

$Q(t)$: The instantaneous state of the positive inventory level at time t .

\tilde{Q}_{dG} : The defuzzified value of \tilde{Q} by Graded mean representation method

Assumptions

The model is derived under the following assumptions.

1. The inventory system deals with single item.
2. The annual demand rate is a function of time and it is $R(t) = a + bt + ct^2$ ($a, b, c > 0$).
3. Preservation technology is used for controlling the deterioration rate.
4. Holding cost is linear function of time and it is $C_h = h + rt$ ($h, r > 0$).
5. The lead time is zero.
6. Time horizon is finite.
7. The deterioration rate is constant.
8. No repair or replacement of the deteriorated items takes place during a given cycle.
9. Total inventory cost is a real, continuous function which is convex to the origin.

4. Mathematical Model and Analysis

Here, the replenishment policy of a deteriorating item with partial backlogging is considered. The objective of the inventory problem is to determine the optimal order quantity and the length of ordering cycle so as to keep the total relevant cost as low as possible, where the preservation technology is used to control the deterioration rate. The behavior of inventory system at any time is shown in figure 1.

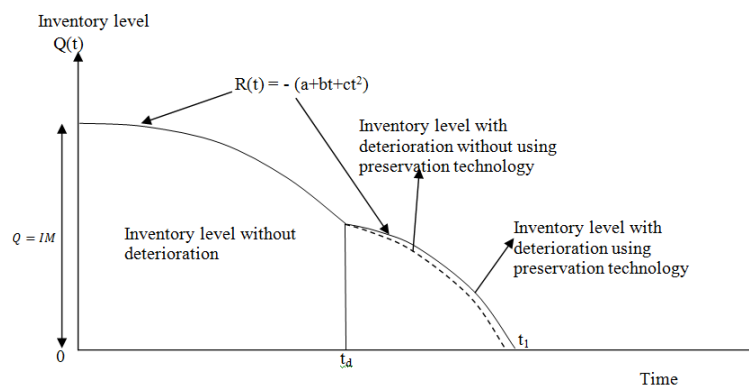


Figure 1. Graphical representation of the inventory system

Replenishment is made at time $t = 0$ and the inventory level is at its maximum level Q . During the period $(0, t_d)$ the inventory level is decreasing and at time t_1 the inventory reaches zero level.

The pictorial representation is shown in the figure 1.

As described above, the inventory level decreases owing to demand rate as well as deterioration during $(0, t_1)$. Hence, the differential equation representing the inventory status is given by

$$\frac{dQ(t)}{dt} = -(a + bt + ct^2) \quad (0 \leq t \leq t_d) \tag{1}$$

$$\frac{dQ(t)}{dt} + \tau_p Q(t) = -(a + bt + ct^2) \quad (t_d \leq t \leq t_1) \tag{2}$$

The boundary conditions are $Q(0) = Q$ and $Q(t_1) = 0$ (3)

Using the boundary condition $Q(0) = Q$ the solution of the equation (1) is

$$\Rightarrow Q(t) = Q - \left(at + \frac{bt^2}{2} + \frac{ct^3}{3} \right) \quad (0 \leq t \leq t_d) \tag{4}$$

Similarly, the solution of equation (2) is given by

$$e^{(\theta - m(\xi))t} Q(t) = - \int (a + bt + ct^2) e^{(\theta - m(\xi))t} dt$$

$$\Rightarrow e^{(\theta - m(\xi))t} Q(t) = \left\{ k - \left[at + (a\theta + b) \frac{t^2}{2} + (b\theta + c) \frac{t^3}{3} + c\theta \frac{t^4}{4} - m(\xi) \left\{ a \frac{t^2}{2} + b \frac{t^3}{3} + c \frac{t^4}{4} \right\} \right] \right\}$$

(Neglecting higher powers of θ) (where $k = at_1 + (a\theta + b) \frac{t_1^2}{2} + (b\theta + c) \frac{t_1^3}{3} + c\theta \frac{t_1^4}{4} - m(\xi) \left\{ a \frac{t_1^2}{2} + b \frac{t_1^3}{3} + c \frac{t_1^4}{4} \right\}$)

which is obtained using $Q(t_1) = 0$

$$Q(t) = k - k(\theta - m(\xi))t + a(\theta - m(\xi)) \frac{t^2}{2} + b(\theta - m(\xi)) \frac{t^3}{6} + c(\theta - m(\xi)) \frac{t^4}{12} + c(\theta - m(\xi)) \frac{t^5}{4} - at - b \frac{t^2}{2} - c \frac{t^3}{3} - m(\xi) \left\{ a(2\theta - m(\xi)) \frac{t^2}{2} + b(2\theta - m(\xi)) \frac{t^3}{3} + c(2\theta - m(\xi)) \frac{t^4}{4} \right\}$$

$(t_d \leq t \leq t_1)$ (5)

In equations (4) and (5) values of $Q(t)$ and $Q(t)$ should coincide at $t = t_d$, which implies that

$$Q - \left(at_d + \frac{bt_d^2}{2} + \frac{ct_d^3}{3} \right) = \left[k - k(\theta - m(\xi))t_d + a(\theta - m(\xi)) \frac{t_d^2}{2} + b(\theta - m(\xi)) \frac{t_d^3}{6} + c(\theta - m(\xi)) \frac{t_d^4}{12} + c(\theta - m(\xi)) \frac{t_d^5}{4} - at_d - b \frac{t_d^2}{2} - c \frac{t_d^3}{3} - m(\xi) \left\{ a(2\theta - m(\xi)) \frac{t_d^2}{2} + b(2\theta - m(\xi)) \frac{t_d^3}{3} + c(2\theta - m(\xi)) \frac{t_d^4}{4} \right\} \right]$$

$$Q = IM = \left[k - k(\theta - m(\xi))t_d + a(\theta - m(\xi)) \frac{t_d^2}{2} + b(\theta - m(\xi)) \frac{t_d^3}{6} + c(\theta - m(\xi)) \frac{t_d^4}{12} + c(\theta - m(\xi)) \frac{t_d^5}{4} - m(\xi) \left\{ a(2\theta - m(\xi)) \frac{t_d^2}{2} + b(2\theta - m(\xi)) \frac{t_d^3}{3} + c(2\theta - m(\xi)) \frac{t_d^4}{4} \right\} \right]$$

(6)

Fuzzy model

In real life situation ordering cost, deterioration cost and purchase cost for uncertainty consider in fuzzy environment system. The total cost followed in proposed inventory model (12).

Here some of the parameters are fuzzy numbers namely \tilde{A} , \tilde{C}_d and \tilde{P}_c .

Let $\tilde{A} = (A_1, A_2, A_3)$, $\tilde{C}_d = (C_{d_1}, C_{d_2}, C_{d_3})$ and $\tilde{P}_c = (P_{c_1}, P_{c_2}, P_{c_3})$

The total cost comprises of following costs

1) The fuzzy ordering cost $OC = \tilde{A}$

(7)

2) The fuzzy deterioration cost during the period (t_d, t_1)

$$\tilde{DC} = \tilde{C}_d \left\{ Q - \left[a(t_1 - t_d) + \frac{b}{2}(t_1^2 - t_d^2) + \frac{c}{3}(t_1^3 - t_d^3) \right] \right\}$$

(8)

3) The inventory holding cost during the period $(0, t_1)$

$$\begin{aligned} IHC &= \int_0^{t_d} (h + rt) Q(t) dt + \int_{t_d}^{t_1} (h + rt) Q(t) dt \\ &= \int_0^{t_d} \left\{ (h + rt) \left[Q - \left(at + \frac{bt^2}{2} + \frac{ct^3}{3} \right) \right] \right\} dt \\ &\quad + \int_{t_d}^{t_1} \left\{ (h + rt) \left[k - k(\theta - m(\xi))t + a(\theta - m(\xi)) \frac{t^2}{2} + b(\theta - m(\xi)) \frac{t^3}{6} + c(\theta - m(\xi)) \frac{t^4}{12} \right. \right. \\ &\quad \left. \left. + c(\theta - m(\xi)) \frac{t^5}{4} - at - b \frac{t^2}{2} - c \frac{t^3}{3} - m(\xi) \left\{ a(2\theta - m(\xi)) \frac{t^3}{2} + b(2\theta - m(\xi)) \frac{t^4}{3} + c(2\theta - m(\xi)) \frac{t^5}{4} \right\} \right] \right\} dt \\ \Rightarrow IHC &= \left\{ \begin{aligned} &h \left[Qt_d - \left(\frac{at_d^2}{2} + \frac{bt_d^3}{6} + \frac{ct_d^4}{12} \right) \right] + r \left[\frac{Qt_d^2}{2} - \left(\frac{at_d^3}{3} + \frac{bt_d^4}{8} + \frac{ct_d^5}{15} \right) \right] \\ &+ hk(t_1 - t_d) + [rk - hk(\theta - m(\xi))] \left[\frac{t_1^2 - t_d^2}{2} \right] + ha(\theta - m(\xi)) \left[\frac{t_1^3 - t_d^3}{6} \right] + hb(\theta - m(\xi)) \left[\frac{t_1^4 - t_d^4}{24} \right] \\ &+ hc(\theta - m(\xi)) \left[\frac{t_1^5 - t_d^5}{60} \right] + hc(\theta - m(\xi)) \left[\frac{t_1^6 - t_d^6}{24} \right] - m(\xi) \left\{ \begin{aligned} &ah(2\theta - m(\xi)) \left[\frac{t_1^4 - t_d^4}{9} \right] \\ &+ bh(2\theta - m(\xi)) \left[\frac{t_1^5 - t_d^5}{15} \right] \\ &+ ch(2\theta - m(\xi)) \left[\frac{t_1^6 - t_d^6}{24} \right] \end{aligned} \right\} \\ &+ ra(\theta - m(\xi)) \left[\frac{t_1^4 - t_d^4}{9} \right] + rb(\theta - m(\xi)) \left[\frac{t_1^5 - t_d^5}{30} \right] \\ &+ rc(\theta - m(\xi)) \left[\frac{t_1^6 - t_d^6}{72} \right] + rc(\theta - m(\xi)) \left[\frac{t_1^7 - t_d^7}{29} \right] - m(\xi) \left\{ \begin{aligned} &ra(2\theta - m(\xi)) \left[\frac{t_1^5 - t_d^5}{10} \right] \\ &+ rb(2\theta - m(\xi)) \left[\frac{t_1^6 - t_d^6}{19} \right] \\ &+ rc(2\theta - m(\xi)) \left[\frac{t_1^7 - t_d^7}{29} \right] \end{aligned} \right\} \\ &- \left\{ ha \left(\frac{t_1^2 - t_d^2}{2} \right) + hb \left(\frac{t_1^3 - t_d^3}{6} \right) + hc \left(\frac{t_1^4 - t_d^4}{12} \right) + ra \left(\frac{t_1^3 - t_d^3}{3} \right) + rb \left(\frac{t_1^4 - t_d^4}{8} \right) + rc \left(\frac{t_1^5 - t_d^5}{15} \right) \right\} \end{aligned} \right\} \end{aligned}$$

(9)

Thus, the order size during total interval $(0, t_1)$ is given by

$$Q = IM$$

4) Purchase cost per cycle

$$\tilde{PC} = \tilde{P}_c Q$$

$$\tilde{PC} = \tilde{P}_c \left\{ \left[k - k(\theta - m(\xi))t_d + a(\theta - m(\xi)) \frac{t_d^2}{2} + b(\theta - m(\xi)) \frac{t_d^3}{6} + c(\theta - m(\xi)) \frac{t_d^4}{12} + c(\theta - m(\xi)) \frac{t_d^5}{4} - m(\xi) \left\{ a(2\theta - m(\xi)) \frac{t_d^3}{2} + b(2\theta - m(\xi)) \frac{t_d^4}{3} + c(2\theta - m(\xi)) \frac{t_d^5}{4} \right\} \right] \right\}$$

(10)

Hence the fuzzy total cost per unit time is given by

$$\tilde{TC}(t_1) = \frac{1}{t_1} (\tilde{OC} + \tilde{DC} + IHC + \tilde{PC})$$

$$\begin{aligned}
 \overline{TC}(t_1) = \frac{1}{t_1} & \left\{ \begin{aligned}
 & A + \overline{C}_d \left\{ Q - \left[a(t_1 - t_d) + \frac{b}{2}(t_1^2 - t_d^2) + \frac{c}{3}(t_1^3 - t_d^3) \right] \right\} \\
 & + h \left[Qt_d - \left(\frac{at_d^2}{2} + \frac{bt_d^3}{6} + \frac{ct_d^4}{12} \right) \right] + r \left[\frac{Qt_d^2}{2} - \left(\frac{at_d^3}{3} + \frac{bt_d^4}{8} + \frac{ct_d^5}{15} \right) \right] \\
 & + hk(t_1 - t_d) + [rk - hk(\theta - m(\xi))] \left[\frac{t_1^2 - t_d^2}{2} \right] + ha(\theta - m(\xi)) \left[\frac{t_1^3 - t_d^3}{6} \right] \\
 & + hb(\theta - m(\xi)) \left[\frac{t_1^4 - t_d^4}{24} \right] \\
 & + hc(\theta - m(\xi)) \left[\frac{t_1^5 - t_d^5}{60} \right] + hc(\theta - m(\xi)) \left[\frac{t_1^6 - t_d^6}{24} \right] \\
 & - m(\xi) \left\{ \begin{aligned}
 & ah(2\theta - m(\xi)) \left[\frac{t_1^4 - t_d^4}{8} \right] \\
 & + bh(2\theta - m(\xi)) \left[\frac{t_1^5 - t_d^5}{15} \right] \\
 & + ch(2\theta - m(\xi)) \left[\frac{t_1^6 - t_d^6}{24} \right]
 \end{aligned} \right\} \\
 & + ra(\theta - m(\xi)) \left[\frac{t_1^4 - t_d^4}{8} \right] + rb(\theta - m(\xi)) \left[\frac{t_1^5 - t_d^5}{30} \right] \\
 & + rc(\theta - m(\xi)) \left[\frac{t_1^6 - t_d^6}{72} \right] + rc(\theta - m(\xi)) \left[\frac{t_1^7 - t_d^7}{28} \right] \\
 & - m(\xi) \left\{ \begin{aligned}
 & ra(2\theta - m(\xi)) \left[\frac{t_1^5 - t_d^5}{10} \right] \\
 & + rb(2\theta - m(\xi)) \left[\frac{t_1^6 - t_d^6}{18} \right] \\
 & + rc(2\theta - m(\xi)) \left[\frac{t_1^7 - t_d^7}{28} \right]
 \end{aligned} \right\} \\
 & - \left\{ \begin{aligned}
 & ha \left(\frac{t_1^2 - t_d^2}{2} \right) + hb \left(\frac{t_1^3 - t_d^3}{6} \right) + hc \left(\frac{t_1^4 - t_d^4}{12} \right) + ra \left(\frac{t_1^3 - t_d^3}{3} \right) \\
 & + rb \left(\frac{t_1^4 - t_d^4}{8} \right) + rc \left(\frac{t_1^5 - t_d^5}{15} \right)
 \end{aligned} \right\}
 \end{aligned} \right\} \\
 + \overline{P}_c & \left\{ \begin{aligned}
 & [k - k(\theta - m(\xi))]t_d + a(\theta - m(\xi)) \frac{t_d^2}{2} + b(\theta - m(\xi)) \frac{t_d^3}{6} + c(\theta - m(\xi)) \frac{t_d^4}{12} \\
 & + c(\theta - m(\xi)) \frac{t_d^5}{4} - m(\xi) \left\{ a(2\theta - m(\xi)) \frac{t_d^3}{2} + b(2\theta - m(\xi)) \frac{t_d^4}{3} + c(2\theta - m(\xi)) \frac{t_d^5}{4} \right\}
 \end{aligned} \right\}
 \end{aligned} \tag{11}$$

Then $\overline{TC}(t_1) = [TC_1(t_1), TC_2(t_2), TC_3(t_2)]$

The fuzzy total cost is defuzzified by graded mean representation method.

$$TC_{dG}(t_1) = \frac{1}{6} [TC_1(t_1) + 4TC_2(t_2) + TC_3(t_2)]$$

Our objective is to determine optimum value of t_1 to minimize TC. The values of t_1 for which

$$\frac{dTC_{dG}(t_1)}{dt_1} = 0 \quad \text{satisfying the condition}$$

$$\left(\frac{d^2TC_{dG}(t_1)}{dt_1^2} \right) > 0$$

The optimal solution of the equation (11) is obtained by using Mathematica software. This has been illustrated by the following numerical example.

5. Numerical Example

Crisp Model (Palani, 2016)

We consider the following parametric values for $A = 300, a = 10, b = 8, c = 5, h = 1,$

$r = 0.5, t_d = 1, \theta = 1.27, m(\xi) = 0.05, C_d = 5, P_c = 15.$

We obtain the optimal value of $t_1 = 1.13636$ units, $Q = 1.13283$ and minimum total cost $TC(t_1) = 214.001.$

Fuzzy Model

$$\bar{A} = (280, 300, 315), \quad \bar{C}_d = (3, 5, 8) \quad \text{and} \quad \bar{P}_c = (10, 15, 22).$$

We obtain the optimal value of $t_1 = 1.13442$ units, $Q = 1.15882$ and minimum total cost $TC_{dG}(t_1) = 212.136$.

6. Conclusions

The products with high deterioration rate are always crucial to the retailer's business. In real markets, the retailer can reduce the deterioration rate of a product by making effective capital investment in storehouse equipment. In this study, to reduce the deterioration rate during deterioration period of deteriorating items, we use the preservation technology. A solution procedure is given to find an optimal replenishment cycle and order quantity so that the total inventory cost per unit time is minimum. For fuzzy model the ordering cost, deterioration cost and purchasing cost are represented by triangular fuzzy numbers. Graded mean representation method is used for defuzzification. A numerical example has been presented to illustrate the model. This model can further be extended by taking more realistic assumptions such as probabilistic demand rate, a model allowing shortages etc.

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