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RESEARCH ARTICLE

DYNAMICAL SYSTEMS WITH THREE ALMOST COMPLEX STRUCTURES

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ABSTRACT

In this paper we presented an analysis of Lagrange and Hamilton formulas. with Three Almost Complex Structures. We have reached important results in differential geometry that can be applied in theoretical physics.

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INTRODUCTION

The geometric study of dynamical systems is an important chapter of contemporary mathematics due to its applications in Mechanics, Theoretical Physics. If M is a differentiable manifold that corresponds to the configuration space, a dynamical system can be locally given by a system of ordinary differential equations of the form $\dot{x}^i = f^i(t; x)$, which are called equations of evolution. Globally, a dynamical system is given by a vector field X on the manifold $M \times R$ whose integral curves, $c(t)$ are given by the equations of evolution, $X \circ c(t) = \dot{c}(t)$. The theory of dynamical systems deals with the integration of such systems. One of the most important papers on the topic entitled Mechanical Equations with Two Almost Complex Structures on Symplectic Geometry It has been used in this paper using two complex structures, examined mechanical systems on symplectic geometry. In this paper, we study dynamical systems with Three Almost Complex Structures . After Introduction in Section 1, we consider Historical Background paper basic. Section 2 deals with the study Almost Complex Structures. Section 3 is devoted to study Lagrangian Dynamics .Section 4 is devoted to study Hamiltonian Dynamics.

Almost Complex Structures

Definition 2.1[http//en.wikipedia.org /wiki/almost complex structure]

Let M be a smooth manifold. An almost complex structure J on M is a linear complex structure (that is, a linear map which squares to -1) on each tangent space of the manifold, which varies smoothly on the manifold. In other words, we have a smooth tensor field J of degree (1,1) such that $J^2 = -1$ when regarded as a vector bundle isomorphism $J : T\mathcal{M} \rightarrow T\mathcal{M}$ on the tangent bundle. A manifold equipped with an almost complex structure is called an almost complex manifold.

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Integrable almost complex structures

Definition 2.2 [http//en.wikipedia.org /wiki/almost complex structure]

Every complex manifold is itself an almost complex manifold. In local holomorphic coordinates $Z = x_k + iy_k$ one can define the maps

$$J\left(\frac{\partial}{\partial x_k}\right) = \frac{\partial}{\partial y_k}\ ,\quad J\left(\frac{\partial}{\partial y_k}\right) = -\frac{\partial}{\partial x_k}$$

Proposition 2.3

Suppose that $\{x_1, x_2, x_3, x_4, x_5, x_6\}$, be a real coordinate system on $(\mathcal{M}, J)$. Then we denote by

$$\left\{\frac{\partial}{\partial x_1}, \frac{\partial}{\partial x_2}, \frac{\partial}{\partial x_3}, \frac{\partial}{\partial x_4}, \frac{\partial}{\partial x_5}, \frac{\partial}{\partial x_6}\right\}$$

$\{dx_1, dx_2, dx_3, dx_4, dx_5, dx_6\}$

$$J\left(\frac{\partial}{\partial x_1}\right) = \frac{\partial}{\partial x_2}\ ,\quad J\left(\frac{\partial}{\partial x_2}\right) = -\frac{\partial}{\partial x_1}\ ,\quad J\left(\frac{\partial}{\partial x_3}\right) = \frac{\partial}{\partial x_4}$$

$$J\left(\frac{\partial}{\partial x_4}\right) = -\frac{\partial}{\partial x_3}\ ,\quad J\left(\frac{\partial}{\partial x_5}\right) = \frac{\partial}{\partial x_6}\ ,\quad J\left(\frac{\partial}{\partial x_6}\right) = -\frac{\partial}{\partial x_5}$$

$z_1 = x_1 + ix_2\ ,\ z_2 = x_3 + ix_4\ ,\ z_3 = x_5 + ix_6$

$$J^2\left(\frac{\partial}{\partial x_1}\right) = \frac{\partial}{\partial x_2} = J\left(\frac{\partial}{\partial x_2}\right) = -\frac{\partial}{\partial x_1}$$
$$J^2\left(\frac{\partial}{\partial x_2}\right) = J\left(-\frac{\partial}{\partial x_1}\right) = -\frac{\partial}{\partial x_2}$$
$$J^2\left(\frac{\partial}{\partial x_3}\right) = J\left(\frac{\partial}{\partial x_4}\right) = -\frac{\partial}{\partial x_3}$$
$$J^2\left(\frac{\partial}{\partial x_4}\right) = J\left(-\frac{\partial}{\partial x_3}\right) = -\frac{\partial}{\partial x_4}$$
$$J^2\left(\frac{\partial}{\partial x_5}\right) = J\left(\frac{\partial}{\partial x_6}\right) = -\frac{\partial}{\partial x_5}$$
$$J^2\left(\frac{\partial}{\partial x_6}\right) = J\left(-\frac{\partial}{\partial x_5}\right) = -\frac{\partial}{\partial x_6}$$

Proposition 2.4

The dual form J^* of the above J is as follows

$$J^{*2}(dx_1) = J^*(dx_2) = -dx_1$$
$$J^{*2}(dx_2) = J^*(-dx_1) = -dx_2$$
$$J^{*2}(dx_3) = J^*(dx_4) = -dx_3$$
$$J^{*2}(dx_4) = J^*(-dx_3) = -dx_4$$
$$J^{*2}(dx_5) = J^*(dx_6) = -dx_4$$
$$J^{*2}(dx_6) = J^*(-dx_5) = -dx_6$$

Theorem 2.5 [Mehmet Tekkoyun, 2009] Let $\mathcal{M}$ be m-real dimensional configuration manifold .A tensor field J on $T^*\mathcal{M}$ is called an almost complex structure on $T^*\mathcal{M}$ if at every point p of $T^*\mathcal{M}$, J is endomorphism of the tangent space $T_p{}^*(\mathcal{M})$ such that $J^2 = -1$ are complex is $J^{*2} = J^* \circ J^* = -1$ is called structures are complex manifold

Lagrangian Dynamical Systems

Definition 3.1. A Lagrangian function for a Hamiltonian vector field X on $\mathcal{M}$ is a smooth function $L : T\mathcal{M} \rightarrow R$ such that

$$i_X\Phi_L = dE_L \qquad (1)$$

Let ξ be the vector field by

$$\xi = X_1 \frac{\partial}{\partial x_1} + X_2 \frac{\partial}{\partial x_2} + X_3 \frac{\partial}{\partial x_3} + X_4 \frac{\partial}{\partial x_4} + X_5 \frac{\partial}{\partial x_5} + X_6 \frac{\partial}{\partial x_6} \tag{2}$$

And

$X_1 = \dot{x}_1$, $X_2 = \dot{x}_2$, $X_3 = \dot{x}_3$, $X_4 = \dot{x}_4$, $X_5 = \dot{x}_5$, $X_6 = \dot{x}_6$

$$U = J(\xi) = X_1 \frac{\partial}{\partial x_1} - X_2 \frac{\partial}{\partial x_2} + X_3 \frac{\partial}{\partial x_3} - X_4 \frac{\partial}{\partial x_4} + X_5 \frac{\partial}{\partial x_5} - X_6 \frac{\partial}{\partial x_6}$$

Let that Liouville Vector field on complex manifold $(\mathcal{M}, U)$

Kinetic energy given $T: T\mathcal{M} \to \mathcal{M}$

$$T = \frac{1}{2} m_i (\dot{x}_1^2 + \dot{x}_2^2 + \dot{x}_3^2 + \dot{x}_4^2 + \dot{x}_5^2 + \dot{x}_6^2)$$

Potential energy $P: T\mathcal{M} \to \mathcal{M}$

$$P = m_i g h$$

The Lagrangian function (energy function)

$$L = T - P$$

$$E_L^J = U_{G_1}(L) - L$$

Is vertical derivation (differentiation) d_J is defined

$$d_{G_J} = \left[i_{G_J}, d\right] = i_{G_J} d - d i_J$$

$\phi_{\mathrm{L}} = d d_1 L$ such that

$$d_J = \frac{\partial}{\partial x_2} dx_1 - \frac{\partial}{\partial x_1} dx_2 + \frac{\partial}{\partial x_4} dx_3 - \frac{\partial}{\partial x_3} dx_4 + \frac{\partial}{\partial x_6} dx_5 - \frac{\partial}{\partial x_5} dx_6 \tag{3}$$

Defined by operator $d_J: A(\mathcal{M}) \to \wedge^1 \mathcal{M}$

$$d_J L = \left(\frac{\partial}{\partial x_2} dx_1 - \frac{\partial}{\partial x_1} dx_2 + \frac{\partial}{\partial x_4} dx_3 - \frac{\partial}{\partial x_3} dx_4 + \frac{\partial}{\partial x_6} dx_5 - \frac{\partial}{\partial x_5} dx_6 \right) L$$

$$d_J L = \frac{\partial L}{\partial x_2} dx_1 - \frac{\partial L}{\partial x_1} dx_2 + \frac{\partial L}{\partial x_4} dx_3 - \frac{\partial L}{\partial x_3} dx_4 + \frac{\partial L}{\partial x_6} dx_5 - \frac{\partial L}{\partial x_5} dx_6 \tag{4}$$

That

$$\phi_{\mathrm{L}} = -d\left(d_{G_1}\right) = -d\left(\frac{\partial}{\partial x_2} dx_1 - \frac{\partial}{\partial x_1} dx_2 + \frac{\partial}{\partial x_4} dx_3 - \frac{\partial}{\partial x_3} dx_4 + \frac{\partial}{\partial x_6} dx_5 - \frac{\partial}{\partial x_5} dx_6 \right)$$

$$\begin{aligned}
\phi_{\mathrm{L}} = &- \frac{\partial^2 L}{\partial x_1 \partial x_2} dx_1 \wedge dx_1 + \frac{\partial^2 L}{\partial x_1 \partial x_1} dx_1 \wedge dx_2 - \frac{\partial^2 L}{\partial x_1 \partial x_4} dx_1 \wedge dx_3 + \frac{\partial^2 L}{\partial x_1 \partial x_3} dx_1 \wedge dx_4 - \frac{\partial^2 L}{\partial x_1 \partial x_6} dx_1 \wedge dx_5 + \frac{\partial^2 L}{\partial x_1 \partial x_5} dx_1 \wedge \\
& dx_6 - - \frac{\partial^2 L}{\partial x_2 \partial x_2} dx_2 \wedge dx_1 + \frac{\partial^2 L}{\partial x_2 \partial x_1} dx_2 \wedge dx_2 - \frac{\partial^2 L}{\partial x_2 \partial x_4} dx_2 \wedge dx_3 + \frac{\partial^2 L}{\partial x_2 \partial x_3} dx_2 \wedge dx_4 - \frac{\partial^2 L}{\partial x_2 \partial x_6} dx_2 \wedge dx_5 + \frac{\partial^2 L}{\partial x_2 \partial x_5} dx_2 \wedge dx_6 - \\
& \frac{\partial^2 L}{\partial x_3 \partial x_2} dx_3 \wedge dx_1 + \frac{\partial^2 L}{\partial x_3 \partial x_1} dx_3 \wedge dx_2 - \frac{\partial^2 L}{\partial x_3 \partial x_4} dx_3 \wedge dx_3 + \frac{\partial^2 L}{\partial x_3 \partial x_3} dx_3 \wedge dx_4 - \frac{\partial^2 L}{\partial x_3 \partial x_6} dx_3 \wedge dx_5 + \frac{\partial^2 L}{\partial x_3 \partial x_5} dx_3 \wedge dx_6 \\
& - \frac{\partial^2 L}{\partial x_4 \partial x_2} dx_4 \wedge dx_1 + \frac{\partial^2 L}{\partial x_4 \partial x_1} dx_4 \wedge dx_2 - \frac{\partial^2 L}{\partial x_4 \partial x_4} dx_4 \wedge dx_3 + \frac{\partial^2 L}{\partial x_4 \partial x_3} dx_4 \wedge dx_4 - \frac{\partial^2 L}{\partial x_4 \partial x_6} dx_4 \wedge dx_5 + \frac{\partial^2 L}{\partial x_4 \partial x_5} dx_4 \wedge dx_6 \\
& - \frac{\partial^2 L}{\partial x_5 \partial x_2} dx_5 \wedge dx_1 + \frac{\partial^2 L}{\partial x_5 \partial x_1} dx_5 \wedge dx_2 - \frac{\partial^2 L}{\partial x_5 \partial x_4} dx_5 \wedge dx_3 + \frac{\partial^2 L}{\partial x_5 \partial x_3} dx_5 \wedge dx_4 - \frac{\partial^2 L}{\partial x_5 \partial x_6} dx_5 \wedge dx_5 + \frac{\partial^2 L}{\partial x_5 \partial x_5} dx_5 \wedge dx_6 \\
& - \frac{\partial^2 L}{\partial x_6 \partial x_2} dx_6 \wedge dx_1 + \frac{\partial^2 L}{\partial x_6 \partial x_1} dx_6 \wedge dx_2 - \frac{\partial^2 L}{\partial x_6 \partial x_4} dx_6 \wedge dx_3 + \frac{\partial^2 L}{\partial x_6 \partial x_3} dx_6 \wedge dx_4 - \frac{\partial^2 L}{\partial x_6 \partial x_6} dx_6 \wedge dx_5 + \frac{\partial^2 L}{\partial x_6 \partial x_5} dx_6 \wedge dx_6
\end{aligned}$$

Calculate $\phi_L(\xi)$

$$\begin{aligned} i_X\phi_L = \phi_L(\xi) = \Big(&-\frac{\partial^2 L}{\partial x_1 \partial x_2} dx_1 \wedge dx_1 + \frac{\partial^2 L}{\partial x_1 \partial x_1} dx_1 \wedge dx_2 - \frac{\partial^2 L}{\partial x_1 \partial x_4} dx_1 \wedge dx_3 + \frac{\partial^2 L}{\partial x_1 \partial x_3} dx_1 \wedge dx_4 - \frac{\partial^2 L}{\partial x_1 \partial x_6} dx_1 \wedge dx_5 \\ &+ \frac{\partial^2 L}{\partial x_1 \partial x_5} dx_1 \wedge dx_6 - -\frac{\partial^2 L}{\partial x_2 \partial x_2} dx_2 \wedge dx_1 + \frac{\partial^2 L}{\partial x_2 \partial x_1} dx_2 \wedge dx_2 - \frac{\partial^2 L}{\partial x_2 \partial x_4} dx_2 \wedge dx_3 + \frac{\partial^2 L}{\partial x_2 \partial x_3} dx_2 \wedge dx_4 \\ &- \frac{\partial^2 L}{\partial x_2 \partial x_6} dx_2 \wedge dx_5 + \frac{\partial^2 L}{\partial x_2 \partial x_5} dx_2 \wedge dx_6 - \frac{\partial^2 L}{\partial x_3 \partial x_2} dx_3 \wedge dx_1 + \frac{\partial^2 L}{\partial x_3 \partial x_1} dx_3 \wedge dx_2 - \frac{\partial^2 L}{\partial x_3 \partial x_4} dx_3 \wedge dx_3 \\ &+ \frac{\partial^2 L}{\partial x_3 \partial x_3} dx_3 \wedge dx_4 - \frac{\partial^2 L}{\partial x_3 \partial x_6} dx_3 \wedge dx_5 + \frac{\partial^2 L}{\partial x_3 \partial x_5} dx_3 \wedge dx_6 - \frac{\partial^2 L}{\partial x_4 \partial x_2} dx_4 \wedge dx_1 + \frac{\partial^2 L}{\partial x_4 \partial x_1} dx_4 \wedge dx_2 \\ &- \frac{\partial^2 L}{\partial x_4 \partial x_4} dx_4 \wedge dx_3 + \frac{\partial^2 L}{\partial x_4 \partial x_3} dx_4 \wedge dx_4 - \frac{\partial^2 L}{\partial x_4 \partial x_6} dx_4 \wedge dx_5 + \frac{\partial^2 L}{\partial x_4 \partial x_5} dx_4 \wedge dx_6 - \frac{\partial^2 L}{\partial x_5 \partial x_2} dx_5 \wedge dx_1 \\ &+ \frac{\partial^2 L}{\partial x_5 \partial x_1} dx_5 \wedge dx_2 - \frac{\partial^2 L}{\partial x_5 \partial x_4} dx_5 \wedge dx_3 + \frac{\partial^2 L}{\partial x_5 \partial x_3} dx_5 \wedge dx_4 - \frac{\partial^2 L}{\partial x_5 \partial x_6} dx_5 \wedge dx_5 + \frac{\partial^2 L}{\partial x_5 \partial x_5} dx_5 \wedge dx_6 \\ &- \frac{\partial^2 L}{\partial x_6 \partial x_2} dx_6 \wedge dx_1 + \frac{\partial^2 L}{\partial x_6 \partial x_1} dx_6 \wedge dx_2 - \frac{\partial^2 L}{\partial x_6 \partial x_4} dx_6 \wedge dx_3 + \frac{\partial^2 L}{\partial x_6 \partial x_3} dx_6 \wedge dx_4 - \frac{\partial^2 L}{\partial x_6 \partial x_6} dx_6 \wedge dx_5 \\ &+ \frac{\partial^2 L}{\partial x_6 \partial x_5} dx_6 \wedge dx_6\Big)\Big(X^1 \frac{\partial}{\partial x_1} + X^2 \frac{\partial}{\partial x_2} + X^3 \frac{\partial}{\partial x_3} + X^4 \frac{\partial}{\partial x_4} + X^5 \frac{\partial}{\partial x_5} + X^6 \frac{\partial}{\partial x_6}\Big) \end{aligned} \tag{5}$$

From the energy equation we get

$$E_L = V(L) - L = X^1 \frac{\partial L}{\partial x_2} - X^2 \frac{\partial L}{\partial x_1} + X^3 \frac{\partial L}{\partial x_4} - X^4 \frac{\partial L}{\partial x_3} + X^5 \frac{\partial L}{\partial x_6} - X^6 \frac{\partial L}{\partial x_5} - L \tag{6}$$

In the equation of the energy equation we obtain

$$\begin{aligned} dE_L = &\Big(\frac{\partial}{\partial x_2} dx_1 - \frac{\partial}{\partial x_1} dx_2 + \frac{\partial}{\partial x_4} dx_3 - \frac{\partial}{\partial x_3} dx_4 + \frac{\partial}{\partial x_6} dx_5 - \frac{\partial}{\partial x_5} dx_6\Big)\Big(X^1 \frac{\partial L}{\partial x_2} - X^2 \frac{\partial L}{\partial x_1} + X^3 \frac{\partial L}{\partial x_4} - X^4 \frac{\partial L}{\partial x_3} + X^5 \frac{\partial L}{\partial x_6} \\ &- X^6 \frac{\partial L}{\partial x_5} - L\Big) \end{aligned}$$

$$\begin{aligned} dE_L = &X^1 \frac{\partial^2 L}{\partial x_1 \partial x_2} dx_1 + X^1 \frac{\partial^2 L}{\partial x_2 \partial x_2} dx_2 + X^1 \frac{\partial^2 L}{\partial x_3 \partial x_2} dx_3 + X^1 \frac{\partial^2 L}{\partial x_4 \partial x_2} dx_4 + X^1 \frac{\partial^2 L}{\partial x_5 \partial x_2} dx_5 + X^1 \frac{\partial^2 L}{\partial x_6 \partial x_2} dx_6 \\ &-X^2 \frac{\partial^2 L}{\partial x_1 \partial x_1} dx_1 - X^2 \frac{\partial^2 L}{\partial x_2 \partial x_1} dx_2 - X^2 \frac{\partial^2 L}{\partial x_3 \partial x_1} dx_3 - X^2 \frac{\partial^2 L}{\partial x_4 \partial x_1} dx_4 - X^2 \frac{\partial^2 L}{\partial x_5 \partial x_1} dx_5 - X^2 \frac{\partial^2 L}{\partial x_6 \partial x_1} dx_6 \\ &+X^3 \frac{\partial^2 L}{\partial x_1 \partial x_4} dx_1 + X^3 \frac{\partial^2 L}{\partial x_2 \partial x_4} dx_2 + X^3 \frac{\partial^2 L}{\partial x_3 \partial x_4} dx_3 + X^3 \frac{\partial^2 L}{\partial x_4 \partial x_4} dx_4 + X^3 \frac{\partial^2 L}{\partial x_5 \partial x_4} dx_5 + X^3 \frac{\partial^2 L}{\partial x_6 \partial x_4} dx_6 \\ &-X^4 \frac{\partial^2 L}{\partial x_1 \partial x_3} dx_1 - X^4 \frac{\partial^2 L}{\partial x_2 \partial x_3} dx_2 - X^4 \frac{\partial^2 L}{\partial x_3 \partial x_3} dx_3 - X^4 \frac{\partial^2 L}{\partial x_4 \partial x_3} dx_4 - X^4 \frac{\partial^2 L}{\partial x_5 \partial x_3} dx_5 - X^4 \frac{\partial^2 L}{\partial x_6 \partial x_3} dx_6 \\ &+X^5 \frac{\partial^2 L}{\partial x_1 \partial x_6} dx_1 + X^5 \frac{\partial^2 L}{\partial x_2 \partial x_6} dx_2 + X^5 \frac{\partial^2 L}{\partial x_3 \partial x_6} dx_3 + X^5 \frac{\partial^2 L}{\partial x_4 \partial x_6} dx_4 + X^5 \frac{\partial^2 L}{\partial x_5 \partial x_6} dx_5 + X^5 \frac{\partial^2 L}{\partial x_6 \partial x_6} dx_6 \\ &-X^6 \frac{\partial^2 L}{\partial x_1 \partial x_5} dx_1 - X^6 \frac{\partial^2 L}{\partial x_2 \partial x_5} dx_2 - X^6 \frac{\partial^2 L}{\partial x_3 \partial x_5} dx_3 - X^6 \frac{\partial^2 L}{\partial x_4 \partial x_5} dx_4 - X^6 \frac{\partial^2 L}{\partial x_5 \partial x_5} dx_5 - X^6 \frac{\partial^2 L}{\partial x_6 \partial x_5} dx_6 \\ &-\frac{\partial L}{\partial x_1} dx_1 - \frac{\partial L}{\partial x_2} dx_2 - \frac{\partial L}{\partial x_3} dx_3 - \frac{\partial L}{\partial x_4} dx_4 - \frac{\partial L}{\partial x_5} dx_5 - \frac{\partial L}{\partial x_6} dx_6 \end{aligned} \tag{7}$$

Equation of Equation (6) with Equation (7) we obtain

$$\mathrm{i_X}\phi_\mathrm{L} = dE_L$$

$$-\left(X^1\frac{\partial}{\partial x_1}+X^1\frac{\partial}{\partial x_2}+X^1\frac{\partial}{\partial x_3}+X^1\frac{\partial}{\partial x_4}+X^1\frac{\partial}{\partial x_5}dx_5+X^1\frac{\partial}{\partial x_6}\right)\left(\frac{\partial L}{\partial x_2}\right)dx_1+\frac{\partial L}{\partial x_1}dx_1$$
$$\left(X^1\frac{\partial}{\partial x_1}+X^1\frac{\partial}{\partial x_2}+X^1\frac{\partial}{\partial x_3}+X^1\frac{\partial}{\partial x_4}+X^1\frac{\partial}{\partial x_5}dx_5+X^1\frac{\partial}{\partial x_6}\right)\left(\frac{\partial L}{\partial x_1}\right)dx_2+\frac{\partial L}{\partial x_2}dx_2$$
$$-\left(X^1\frac{\partial}{\partial x_1}+X^1\frac{\partial}{\partial x_2}+X^1\frac{\partial}{\partial x_3}+X^1\frac{\partial}{\partial x_4}+X^1\frac{\partial}{\partial x_5}dx_5+X^1\frac{\partial}{\partial x_6}\right)\left(\frac{\partial L}{\partial x_3}\right)dx_3+\frac{\partial L}{\partial x_3}dx_3$$
$$\left(X^1\frac{\partial}{\partial x_1}+X^1\frac{\partial}{\partial x_2}+X^1\frac{\partial}{\partial x_3}+X^1\frac{\partial}{\partial x_4}+X^1\frac{\partial}{\partial x_5}dx_5+X^1\frac{\partial}{\partial x_6}\right)\left(\frac{\partial L}{\partial x_4}\right)dx_4+\frac{\partial L}{\partial x_4}dx_4$$
$$-\left(X^1\frac{\partial}{\partial x_1}+X^1\frac{\partial}{\partial x_2}+X^1\frac{\partial}{\partial x_3}+X^1\frac{\partial}{\partial x_4}+X^1\frac{\partial}{\partial x_5}dx_5+X^1\frac{\partial}{\partial x_6}\right)\left(\frac{\partial L}{\partial x_5}\right)dx_5+\frac{\partial L}{\partial x_5}dx_5$$
$$\left(X^1\frac{\partial}{\partial x_1}+X^1\frac{\partial}{\partial x_2}+X^1\frac{\partial}{\partial x_3}+X^1\frac{\partial}{\partial x_4}+X^1\frac{\partial}{\partial x_5}dx_5+X^1\frac{\partial}{\partial x_6}\right)\left(\frac{\partial L}{\partial x_6}\right)dx_6+\frac{\partial L}{\partial x_6}dx_6=0 \qquad (8)$$

Be an integral curve .in local coordinates it is obtained that

Suppose that a curve

$$\alpha: \mathrm{I} \subset \mathrm{R} \rightarrow \mathrm{T}^*\mathcal{M} = R^{2n}$$

is an integral curve of the Lagrangian vector field $\mathrm{X_H}$, i.e.,

$$\mathrm{X_L}(\alpha(\mathrm{t})) = \frac{\mathrm{d}\alpha(\mathrm{t})}{\mathrm{dt}}, \quad \mathrm{t} \in \mathrm{I}.$$

In the local coordinates, if it is considered to be

$$\alpha(\mathrm{t}) = \left(\mathrm{x_1(t), x_2(t), x_2(t), x_4(t), x_5(t), x_6(t)}\right)$$

we obtain

$$\frac{\mathrm{d}\alpha(\mathrm{t})}{\mathrm{dt}} = \frac{\mathrm{dx_1}}{\mathrm{dt}}\frac{\partial}{\partial \mathrm{x_1}}+\frac{\mathrm{dx_2}}{\mathrm{dt}}\frac{\partial}{\partial \mathrm{x_2}}+\frac{\mathrm{dx_3}}{\mathrm{dt}}\frac{\partial}{\partial \mathrm{x_3}}+\frac{\mathrm{dx_4}}{\mathrm{dt}}\frac{\partial}{\partial \mathrm{x_4}}+\frac{\mathrm{dx_5}}{\mathrm{dt}}\frac{\partial}{\partial \mathrm{x_5}}+\frac{\mathrm{dx_6}}{\mathrm{dt}}\frac{\partial}{\partial \mathrm{x_6}}$$
$$X^1\frac{\partial}{\partial x_1}+X^1\frac{\partial}{\partial x_2}+X^1\frac{\partial}{\partial x_3}+X^1\frac{\partial}{\partial x_4}+X^1\frac{\partial}{\partial x_5}dx_5+X^1\frac{\partial}{\partial x_6}=\frac{\partial}{\partial \mathrm{t}} \qquad (9)$$

Taking the equation(8) = the equation (9)

$$-\frac{\partial}{\partial \mathrm{t}}\left(\frac{\partial L}{\partial x_2}\right)dx_1+\frac{\partial L}{\partial x_1}dx_1=0 \rightarrow \quad -\frac{\partial}{\partial \mathrm{t}}\left(\frac{\partial L}{\partial x_2}\right)+\frac{\partial L}{\partial x_1}=0$$
$$\frac{\partial}{\partial \mathrm{t}}\left(\frac{\partial L}{\partial x_1}\right)dx_2+\frac{\partial L}{\partial x_2}dx_2=0 \quad \rightarrow \quad \frac{\partial}{\partial \mathrm{t}}\left(\frac{\partial L}{\partial x_1}\right)+\frac{\partial L}{\partial x_2}=0$$
$$-\frac{\partial}{\partial \mathrm{t}}\left(\frac{\partial L}{\partial x_3}\right)dx_3+\frac{\partial L}{\partial x_3}dx_3=0 \rightarrow \quad -\frac{\partial}{\partial \mathrm{t}}\left(\frac{\partial L}{\partial x_3}\right)+\frac{\partial L}{\partial x_3}=0$$
$$\frac{\partial}{\partial \mathrm{t}}\left(\frac{\partial L}{\partial x_4}\right)dx_4+\frac{\partial L}{\partial x_4}dx_4=0 \quad \rightarrow \quad \frac{\partial}{\partial \mathrm{t}}\left(\frac{\partial L}{\partial x_4}\right)+\frac{\partial L}{\partial x_4}=0$$
$$-\frac{\partial}{\partial \mathrm{t}}\left(\frac{\partial L}{\partial x_5}\right)dx_5+\frac{\partial L}{\partial x_5}dx_5=0 \rightarrow -\frac{\partial}{\partial \mathrm{t}}\left(\frac{\partial L}{\partial x_5}\right)+\frac{\partial L}{\partial x_5}=0$$
$$\frac{\partial}{\partial \mathrm{t}}\left(\frac{\partial L}{\partial x_6}\right)dx_6+\frac{\partial L}{\partial x_6}dx_6=0 \quad \rightarrow \quad \frac{\partial}{\partial \mathrm{t}}\left(\frac{\partial L}{\partial x_6}\right)+\frac{\partial L}{\partial x_6}=0$$

And

$$-\frac{\partial}{\partial \mathrm{t}}\left(\frac{\partial L}{\partial x_2}\right)+\frac{\partial L}{\partial x_1}=0\,, \quad \frac{\partial}{\partial \mathrm{t}}\left(\frac{\partial L}{\partial x_1}\right)+\frac{\partial L}{\partial x_2}=0\,, \quad -\frac{\partial}{\partial \mathrm{t}}\left(\frac{\partial L}{\partial x_3}\right)+\frac{\partial L}{\partial x_3}=0$$
$$\frac{\partial}{\partial \mathrm{t}}\left(\frac{\partial L}{\partial x_4}\right)+\frac{\partial L}{\partial x_4}=0\,, \quad -\frac{\partial}{\partial \mathrm{t}}\left(\frac{\partial L}{\partial x_5}\right)+\frac{\partial L}{\partial x_5}=0\,, \quad \frac{\partial}{\partial \mathrm{t}}\left(\frac{\partial L}{\partial x_6}\right)+\frac{\partial L}{\partial x_6}=0 \qquad (10)$$

Hence the triple $(\mathcal{M}\,, \phi_\mathrm{L}, \xi)$ is shown to be a Lagrangian mechanical system which are deduced by means of an almost real structure J and using of basis $\left\{\frac{\partial}{\partial \mathrm{x_i}} : \mathrm{i} = 1,2,3,4,5,6\right\}$ on the distributions $\mathcal{M}$

Hamiltonian Dynamical Systems

Definition 3.1 [Zeki Kasap, 2015]. A Hamiltonian function for a Hamiltonian vector field X on $\mathcal{M}$ is a smooth function $H : \mathcal{M} \to R$ such that

$$i_{X_H}\omega = dH \tag{11}$$

Definition 3. 2[Zeki, 2016]. A Hamiltonian system is a triple $(M;\omega;H)$, where $(\omega;H)$ is a Symplectic manifold and $H \in C^{\infty}(M)$ is a function, called the Hamiltonian function.

Suppose that an almost real structure, a Liouville form and 1-form on $T^*\mathcal{M}$ are shown by Φ^*, λ and ω, respectively. Then we have

$$\omega = \frac{1}{2}(x_1dx_1 - x_2dx_2 + x_3dx_3 - x_4dx_4 + x_5dx_5 - x_6dx_6) \tag{12}$$

and

$$\lambda = \frac{1}{2}\big(x_1J^*(dx_1) + x_2J^*(dx_2) + x_3J^*(dx_3) + x_4J^*(dx_4) + x_5J^*(dx_5) + x_6J^*(dx_6) + x_7J^*(dx_7) + x_8J^*(dx_8)\big) \tag{13}$$

We substitute equation (12) in equation (13) we get

$$\lambda = \Phi^*(\omega) = \frac{1}{2}[-x_1dx_2 + x_2dx_1 - x_3dx_4 + x_4dx_3 - x_5dx_6 + x_6dx_5]$$

differential of λ

$$\phi = -d\lambda =$$
$$= -d\frac{1}{2}[-x_1dx_2 + x_2dx_1 - x_3dx_4 + x_4dx_3 - x_5dx_6 + x_6dx_5]$$

It is known that if ϕ is a closed 2- form on $T^*\mathcal{M}$, then ϕ_H is also a symplectic structure on $T^*\mathcal{M}$.

$$\phi = dx_2 \wedge dx_1 + dx_4 \wedge dx_3 + dx_6 \wedge dx_5 \tag{14}$$

If Hamiltonian vector field X_H associated with Hamiltonian energy H is given by

$$X_H = X^1\frac{\partial}{\partial x_1} + X^2\frac{\partial}{\partial x_2} + X^3\frac{\partial}{\partial x_3} + X^4\frac{\partial}{\partial x_4} + X^5\frac{\partial}{\partial x_5} + X^6\frac{\partial}{\partial x_6}$$

Calculates a value X_H and ϕ

$$i_{X_H}\phi = \phi(X_H) = (dx_2 \wedge dx_1 + dx_4 \wedge dx_3 + dx_6 \wedge dx_5)\left(X^1\frac{\partial}{\partial x_1} + X^2\frac{\partial}{\partial x_2} + X^3\frac{\partial}{\partial x_3} + X^4\frac{\partial}{\partial x_4} + X^5\frac{\partial}{\partial x_5} + X^6\frac{\partial}{\partial x_6}\right)$$

$$i_{X_H}\phi = -X^1dx_2 + X^2dx_1 - X^3dx_4 + X^4dx_3 - X^5dx_6 + X^6dx_5 \tag{15}$$

So we find that

$$X^1 = \frac{\partial}{\partial x_2}\ ,\quad X^2 = -\frac{\partial}{\partial x_1}\ ,X^3 = \frac{\partial}{\partial x_4}\ ,X^4 = -\frac{\partial}{\partial x_3}\ ,X^5 = \frac{\partial}{\partial x_6}\ ,X^6 = -\frac{\partial}{\partial x_5}$$

Moreover, the differential of Hamiltonian energy is written as follows:

$$dH = -\frac{\partial H}{\partial x_2}\frac{\partial}{\partial x_1} + \frac{\partial H}{\partial x_1}\frac{\partial}{\partial x_2} - \frac{\partial H}{\partial x_4}\frac{\partial}{\partial x_3} + \frac{\partial H}{\partial x_3}\frac{\partial}{\partial x_4} - \frac{\partial H}{\partial x_6}\frac{\partial}{\partial x_5} + \frac{\partial H}{\partial x_5}\frac{\partial}{\partial x_6} \tag{16}$$

Suppose that a curve

$$\alpha: I \subset R \to T^*\mathcal{M} = R^{2n}$$

is an integral curve of the Hamiltonian vector field X_H, i.e.,

$$X_H(\alpha(t)) = \frac{d\alpha(t)}{dt}, \quad t \in I.$$

In the local coordinates, if it is considered to be

$$\alpha(t) = (x_1(t), x_2(t), x_2(t), x_4(t), x_5(t), x_6(t))$$

we obtain

$$\frac{d\alpha(t)}{dt} = \frac{dx_1}{dt}\frac{\partial}{\partial x_1} + \frac{dx_2}{dt}\frac{\partial}{\partial x_2} + \frac{dx_3}{dt}\frac{\partial}{\partial x_3} + \frac{dx_4}{dt}\frac{\partial}{\partial x_4} + \frac{dx_5}{dt}\frac{\partial}{\partial x_5} + \frac{dx_6}{dt}\frac{\partial}{\partial x_6} \tag{17}$$

Taking the equation(15) = the equation (17)

$$-\frac{\partial H}{\partial x_2}\frac{\partial}{\partial x_1} + \frac{\partial H}{\partial x_1}\frac{\partial}{\partial x_2} - \frac{\partial H}{\partial x_4}\frac{\partial}{\partial x_3} + \frac{\partial H}{\partial x_3}\frac{\partial}{\partial x_4} - \frac{\partial H}{\partial x_6}\frac{\partial}{\partial x_5} + \frac{\partial H}{\partial x_5}\frac{\partial}{\partial x_6} = \frac{dx_1}{dt}\frac{\partial}{\partial x_1} + \frac{dx_2}{dt}\frac{\partial}{\partial x_2} + \frac{dx_3}{dt}\frac{\partial}{\partial x_3} + \frac{dx_4}{dt}\frac{\partial}{\partial x_4} + \frac{dx_5}{dt}\frac{\partial}{\partial x_5} + \frac{dx_6}{dt}\frac{\partial}{\partial x_6}$$

By comparing the two sides of the equation we get the

$$-\frac{\partial H}{\partial x_2}\frac{\partial}{\partial x_1} = \frac{dx_1}{dt}\frac{\partial}{\partial x_1} \Rightarrow -\frac{\partial H}{\partial x_2} = \frac{dx_1}{dt}$$
$$\frac{\partial H}{\partial x_1}\frac{\partial}{\partial x_2} = \frac{dx_2}{dt}\frac{\partial}{\partial x_2} \Rightarrow \frac{\partial H}{\partial x_1} = \frac{dx_2}{dt}$$
$$-\frac{\partial H}{\partial x_4}\frac{\partial}{\partial x_3} = \frac{dx_3}{dt}\frac{\partial}{\partial x_3} \Rightarrow -\frac{\partial H}{\partial x_4} = \frac{dx_3}{dt}$$
$$\frac{\partial H}{\partial x_3}\frac{\partial}{\partial x_4} = \frac{dx_4}{dt}\frac{\partial}{\partial x_4} \Rightarrow \frac{\partial H}{\partial x_3} = \frac{dx_4}{dt}$$
$$-\frac{\partial H}{\partial x_6}\frac{\partial}{\partial x_5} = \frac{dx_5}{dt}\frac{\partial}{\partial x_5} \Rightarrow -\frac{\partial H}{\partial x_6} = \frac{dx_5}{dt}$$
$$\frac{\partial H}{\partial x_5}\frac{\partial}{\partial x_6} = \frac{dx_6}{dt}\frac{\partial}{\partial x_6} \Rightarrow \frac{\partial H}{\partial x_5} = \frac{dx_6}{dt}$$

Thus Hamilton's equations are

$$-\frac{\partial H}{\partial x_2} = \frac{dx_1}{dt}, \quad \frac{\partial H}{\partial x_1} = \frac{dx_2}{dt}, \quad -\frac{\partial H}{\partial x_4} = \frac{dx_3}{dt}$$

$$\frac{\partial H}{\partial x_3} = \frac{dx_4}{dt}, \quad -\frac{\partial H}{\partial x_6} = \frac{dx_5}{dt}, \quad \frac{\partial H}{\partial x_5} = \frac{dx_6}{dt} \tag{18}$$

Hence the triple $(\mathcal{M}, \phi, X_H)$ is shown to be a Hamiltonian mechanical system which are deduced by means of an almost real structure j^* and using of basis $\left\{\frac{\partial}{\partial x_i} : i = 1,2,3,4,5,6\right\}$ on the distributions $T^*\mathcal{M}$

Conclusion

Thus, equations Lagrangian of equations (10). And equations of Hamiltonian equations (18) with Three Almost Complex Structures.

REFERENCES

http//en.wikipedia.org/wiki/almost complex structure.

Mehmet Tekkoyun -Lagrangian and Hamiltonian Dynamics on Para-Kahlerian Space Form-arXiv: 0902.4522v1 [math.DS] 26 Feb 2009.

Newlander, A. and Nirenberg, L. 1957. "Complex analytic coordinates in almost complex manifolds", *Annals of Mathematics. Second Series*, 65 (3): 391–404,doi:10.2307/1970051, ISSN 0003-486X, JSTOR 1970051, MR 0088770

Oguzhan Celik, Zeki Kasap, Mechanical Equations with Two Almost Complex Structures on Symplectic Geometry, April 28, 2016.

Zeki Kasap, 2015. Conformal E Lagrange Mechanical Equations on Contact 5- Manifolds, *International Journal of Innovative Mathematical Research*, Volume 3-Lssue 5 May, pp41-48.

Zeki KASAP, 2016. Hamilton Equations on a Contact 5-Manifolds, *Elixir Adv. Math.*, 92; 38743-38748.

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