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RESEARCH ARTICLE

GENERAL FORMULA FOR $(1\ ,\ 2)$-FIBONACCI SEQUENCE

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ABSTRACT

In this communication, we establish the general formula for $(1\ ,\ 2)$-Fibonacci sequence. Also, we prove some theorems using the recurrence relation for $(1\ ,\ 2)$-Fibonacci sequence and the properties of the matrices.

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INTRODUCTION

Number theory, is the study of the set of positive whole numbers $(1,2,3,........)$ which are often called the set of natural numbers. We will especially want to study the relationships between different sorts of numbers. The main goal of number theory is to discover interesting and unexpected relationships between different sorts of numbers and to prove that these relationships are true. The theory of numbers offers a rich variety of fascinating properties. In this context one may refer (Ivan Niven, Fifth edition; David M. Burton, Sixth edition; Andre Weil, 1987; Carmichael, 1959; Brother U.Alfered, 1963; Butcher, 1978; Connell, 1959; Hendy, 1978; Stolarsky, 1977). In this communication, we find the general formula for $(1\ ,\ 2)$-Fibonacci sequence by representing it in the matrix of order 3. Also, we prove some theorems using the recurrence relation for $(1\ ,\ 2)$-Fibonacci sequence and the properties of the matrices.

Method of analysis

The $(1\ ,\ 2)$-Fibonacci sequence satisfy $F_0 = 0$ and $F_1 = 1$ which are $0,1,1,3,5,11,\ 21,43,85,.............$. The properties of these numbers are summarized in the form $F_n = \frac{1}{3}\left[(-1)^{n+1} + 2^n\right]$.

The $(1\ ,\ 2)$-Fibonacci sequence matrix is given by $R = \begin{pmatrix} 1 & 2 & 0 \\ 0 & 1 & 2 \\ 0 & 1 & 0 \end{pmatrix}$ which can be generalized as

$$R\begin{pmatrix} F_n \\ F_{n-1} \\ F_{n-2} \end{pmatrix} = \begin{pmatrix} F_{n+1} \\ F_n \\ F_{n-1} \end{pmatrix}$$

The recurrence relation for $(1\ ,\ 2)$-Fibonacci sequence is given by $F_n = F_{n-1} + 2F_{n-2}$.

Theorem: 1

Let R be a matrix $\begin{pmatrix} 1 & 2 & 0 \\ 0 & 1 & 2 \\ 0 & 1 & 0 \end{pmatrix}$, then

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$$R^n = \begin{pmatrix} F_2 & F_{n+2}-1 & 2F_{n+1}-2 \\ F_0 & F_{n+1} & 2F_n \\ F_0 & F_n & 2F_{n-1} \end{pmatrix}$$

Proof

Let us prove the theorem by using the principle of mathematical induction on n.

We have, $$R = \begin{pmatrix} 1 & 2 & 0 \\ 0 & 1 & 2 \\ 0 & 1 & 0 \end{pmatrix} \quad (1)$$

We know that,

$$F_0 = 0\ , F_1 = 1,\ F_2 = 1,\ F_3\ = 3$$

Substituting the above values in (1), we get

$$R = \begin{pmatrix} F_2 & F_3-1 & 2F_2-2 \\ F_0 & F_2 & 2F_1 \\ F_0 & F_1 & 2F_0 \end{pmatrix}$$

Therefore, the result is true for $n = 1$.

Assume that the result is true for $n = k$.

That is $$R^k = \begin{pmatrix} F_2 & F_{k+2}-1 & 2F_{k+1}-2 \\ F_0 & F_{k+1} & 2F_k \\ F_0 & F_k & 2F_{k-1} \end{pmatrix}$$

To prove, the result is true for $n = k+1$.

$$R^{k+1} = R^k R = \begin{pmatrix} F_2 & F_{k+2}+2F_{k+1}-1 & 2F_{k+2}-2 \\ F_0 & 2F_0+F_{k+1}+2F_k & 2F_{k+1} \\ F_0 & 2F_0+F_k+2F_{k-1} & 2F_k \end{pmatrix}$$

$$= \begin{pmatrix} F_2 & F_{k+3}-1 & 2F_{k+2}-2 \\ F_0 & F_{k+2} & 2F_{k+1} \\ F_0 & F_{k+1} & 2F_k \end{pmatrix}$$

Hence, we conclude that

$$R^n = \begin{pmatrix} F_2 & F_{n+2}-1 & 2F_{n+1}-2 \\ F_0 & F_{n+1} & 2F_n \\ F_0 & F_n & 2F_{n-1} \end{pmatrix}$$

Corollary

$$Det\left(R^n\right) = (-1)^{n+1} 2^n$$

Theorem: 2

Let R be a matrix $\begin{pmatrix} 1 & 2 & 0 \\ 0 & 1 & 2 \\ 0 & 1 & 0 \end{pmatrix}$, then $R^n \begin{pmatrix} F_s \\ F_{s-1} \\ F_{s-2} \end{pmatrix} = \begin{pmatrix} F_{n+s} \\ F_{n+s-1} \\ F_{n+s-2} \end{pmatrix}$

Proof

Let us prove the theorem by using the principle of mathematical induction on n.

We have,

$$R \begin{pmatrix} F_s \\ F_{s-1} \\ F_{s-2} \end{pmatrix} = \begin{pmatrix} F_{s+1} \\ F_s \\ F_{s-1} \end{pmatrix}$$

Therefore, the result is true for $n = 1$.

Now, assume that the result is true for $n = r-1$.

That is

$$R^{r-1} \begin{pmatrix} F_s \\ F_{s-1} \\ F_{s-2} \end{pmatrix} = \begin{pmatrix} F_{s+r-1} \\ F_{s+r-2} \\ F_{s+r-3} \end{pmatrix}$$

To prove, the result is true for $n = r$.

$$R^r \begin{pmatrix} F_s \\ F_{s-1} \\ F_{s-2} \end{pmatrix} = \begin{pmatrix} F_{s+r-1}+2F_{s+r-2} \\ F_{s+r-2}+2F_{s+r-3} \\ F_{s+r-2} \end{pmatrix} = \begin{pmatrix} F_{r+s} \\ F_{r+s-1} \\ F_{r+s-2} \end{pmatrix}$$

Hence, we conclude that

$$R^n \begin{pmatrix} F_s \\ F_{s-1} \\ F_{s-2} \end{pmatrix} = \begin{pmatrix} F_{n+s} \\ F_{n+s-1} \\ F_{n+s-2} \end{pmatrix}$$

Theorem: 3

Let $R = \begin{pmatrix} 1 & 2 & 0 \\ 0 & 1 & 2 \\ 0 & 1 & 0 \end{pmatrix}$ be the matrix for $(1,2)$-Fibonacci sequence, then $F_n = \frac{1}{3}\left[(-1)^{n+1} + 2^n\right]$

Proof

Given $R = \begin{pmatrix} 1 & 2 & 0 \\ 0 & 1 & 2 \\ 0 & 1 & 0 \end{pmatrix}$

The characteristic equation of R is given by

$$\left|R - \lambda I\right| = 0$$

Therefore, the Eigen values of R pointed out by

$$\lambda_1 = 1,\ \lambda_2 = -1, \lambda_3 = 2$$

The Eigen vectors of R are given by $(R - \lambda I)X = 0$

Hence, the Eigen vectors of R corresponding to the Eigen values of R are obtained as

$$\begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 1 & -1 & 1 \\ 4 & 2 & 1 \end{pmatrix}$$

Now, the diagonal matrix of R is given by,

$$\text{Diag } [\lambda_1, \lambda_2, \lambda_3] = D = P^{-1}RP$$

where,

$$P = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}^T = \begin{pmatrix} 1 & 1 & 4 \\ 0 & -1 & 2 \\ 0 & 1 & 1 \end{pmatrix}$$

Hence,

$$D = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 2 \end{pmatrix}$$

In general,

$$D^n = \begin{pmatrix} 1 & 0 & 0 \\ 0 & (-1)^n & 0 \\ 0 & 0 & 2^n \end{pmatrix}$$

Using the properties of similar matrices, we can write

$$D^n = P^{-1}R^nP$$

where n is any positive integer. Furthermore, we can write

$$R^n = PD^nP^{-1}$$

$$R^n = \frac{1}{3}\begin{pmatrix} 3 & -3 + (-1)^{n+1} + 2^{n+2} & -6 + 2(-1)^n + 2^{n+2} \\ 0 & (-1) + 2^{n+1} & 2(-1)^{n+1} + 2^{n+1} \\ 0 & (-1)^{n+1} + 2^n & 2(-1)^n + 2^n \end{pmatrix} \quad (2)$$

By theorem 1, we have

$$R^n = \begin{pmatrix} F_2 & F_{n+2} - 1 & 2F_{n+1} - 2 \\ F_0 & F_{n+1} & 2F_n \\ F_0 & F_n & 2F_{n-1} \end{pmatrix} \quad (3)$$

From (2) and (3), we get

$$\begin{pmatrix} F_2 & F_{n+2} - 1 & 2F_{n+1} - 2 \\ F_0 & F_{n+1} & 2F_n \\ F_0 & F_n & 2F_{n-1} \end{pmatrix} = \frac{1}{3}\begin{pmatrix} 3 & -3 + (-1)^{n+1} + 2^{n+2} & -6 + 2(-1)^n + 2^{n+2} \\ 0 & (-1)^n + 2^{n+1} & 2(-1)^{n+1} + 2^{n+1} \\ 0 & (-1)^{n+1} + 2^n & 2(-1)^n + 2^n \end{pmatrix}$$

Equating the $(3\ ,\ 2)$ entry on both sides, we get

$$F_n = \frac{1}{3}\left[(-1)^{n+1} + 2^n\right]$$

Theorem: 4

Let S be a matrix $\begin{pmatrix} 1 & 4 & 4 \\ 0 & 3 & 2 \\ 0 & 1 & 2 \end{pmatrix}$, then

$$S^n = \begin{pmatrix} F_2 & F_{2n+4} - 1 & 2F_{2n+3} - 2 \\ F_0 & F_{2n+3} & 2F_{2n+2} \\ F_0 & F_{2n+2} & 2F_{2n+1} \end{pmatrix}$$

Corollary

$$Det\left(S^n\right) = (-1)^{2n+1}\, 2^{2n+2}$$

Conclusion

In this paper, we evaluate the general formula for $(1,2)$-Fibonacci sequence and also we prove some theorems by using various properties of matrices. In this manner, one may prove some other theorems for other sequences.

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