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RESEARCH ARTICLE

TO THE THEORY OF MASS EXCHANGE IN TWO-PHASE FLOWS

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ABSTRACT

The work is devoted to the construction of the basic equations of hydromechanics of two-phase flows with external mass transfer. The flow of a two-phase liquid is regarded as a continuum consisting of a large number of different groups of particles. The derivation of the phenomenological equations of motion is given taking into account both the external attached (or detachable) mass and the phase transitions within the medium. By applying fundamental conservation equations, the equations of mass, momentum and energy transfer for individual phases and the medium as a whole are obtained. It is shown that from the obtained systems of equations, in the absence of sources (sinks) of mass, momentum and energy, as a particular case, the known equations of hydromechanics of two-phase flows follow. The obtained equations of motion are valid for describing the component of the mixture and the medium with any physical and mechanical properties. For closure them, one can use thermodynamic and rheological equations of state, as well as expressions for the heat flux, interphase forces, the mass of phase transitions, and heat transfer between phases.

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INTRODUCTION

The problems of hydromechanics of two-phase (heterogeneous, multiphase, weighted) flows are extremely important for various technical and technological problems: hydro and heat power engineering; oil, gas and chemical industries; hydraulic engineering and water management; drilling, environmental protection and agriculture, etc. The range of problems of this topic is extremely wide and includes the flows of such systems as "liquid (or gas) - solid particles", "liquid-gas bubbles", "gas-liquid droplets", between which mass, momentum and energy can be exchanged. The study of hydromechanics of two-phase flows is a complex problem and develops in several directions, each of which has its own specifics and characteristics, both from the point of view of the theoretical description and the experimental study [1-7]. Their analysis showed that the basic equations of motion of two-phase flows are established only without taking into account external mass transfer (ie, for flows of constant mass). In the presence of external mass transfer (ie, attached or detachable mass), the hydrodynamic parameters of the flow can significantly change. Such flows are often encountered in problems: distribution (or combined) oil and gas, and water pipes (channels); blowing (or suction) while controlling the boundary layer; Injection and separation systems; scattering and drainage pipes (channels); sedimentation tanks of continuous action and with hydromechanical cleaning, etc. Common to these problems is that the flow in their flow part occurs with a change in mass (ie, with the addition or detachment of mass along the path). Their account is necessary for a complex solution of the problems of hydrodynamics of two-phase flows.

Basic equations of motion

To construct the basic equations of motion, we consider a two-phase medium as a continuum (macrosystem) consisting of a carrier (a liquid or a gas) and a sink (solid particles, bubbles or droplets) of phases whose masses and mixtures as a whole continuously change due to disconnection from them or attachment to them he new masses of both phases. Under these conditions, to construct the basic equations of hydromechanics (consisting of the equations of continuity, dynamics and energy) of two-phase flows with mass transfer, we select an arbitrary volume of the mixture V (t) bounded by the surface S (t).

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Equation of continuity

It is a mathematical formulation of the law on the conservation of matter, which in the presence of external sources (or sinks) of mass and phase transitions is formulated as follows: the total time derivative of the mass of the medium (phase) in an arbitrary volume is equal to its change as a result of the addition (or detachment) of particles and phase transitions. The above for the i-th phase of the mixture is written in the integral form:

$$\int_{V(t)} \partial_t(\rho_i\varphi_i)\, dV + \int_{S(t)} \rho_i\varphi_i U_{i\Pi} ds = \int_{V(t)}(q_{*i} + (-1)^i æ)\, dV. \qquad (1)$$

where $\partial_t = \frac{\partial}{dt}$; $i = 1.2$ here and below, the subscript 1 refers to the carrier (fluid or gas), and 2 refers to the incompatible (solid particles, drops or bubbles) phase of the mixture;ρ_i, φ_i −true density ($\rho_i = m_i/V_i$, where m_i, V_i −mass and volume$i - th\, phase)\, and the volume concentration of the i - th\, phase (\varphi_i = V_i/V$ where V is the volume of the mixture); U_in is the projection of the velocity vector of the ith phase U ⃗_i on the direction of the outer normal n ⃗; q _ (* i) is the specific mass of the i-th phase added (or detachable, for this q_ (* i) <0); æ is the specific mass of the phase transition from the carrier phase to the incommensurable phase. Under the condition of motion with continuously differentiable characteristics and applying the known Gauss-Ostrogradsky formula to the second integral on the left-hand side of (1), in view of the arbitrariness of the chosen volume, we obtain the continuity equation for the i-th phase of the mixture in the differential form

$$\partial_t(\rho_i\varphi_i) + \nabla\left(\rho_i\varphi_i\vec{U}_i\right) = q_{*i} + (-1)^i æ \qquad (2)$$

Where ∇-nabla (the Hamiltonian operator).

Summing (1) or (2) over i, we can obtain a differential equation of continuity for a two-phase mixture as a whole

$$\partial_t\rho + \nabla\left(\rho\vec{U}\right) = q_*, \qquad (3)$$

whereρ and U ⃗ are the density and velocity vector of a two-phase medium ($\rho = \sum\rho_i\varphi_i$; $\vec{U} = \sum(\rho_i\varphi_i\vec{U}_i)/\rho$); q_*- is the specific mass of the mixture ($q_* < 0$) added (or detachable, with ($q_* = \sum q_{*i}$)).

Equations of dynamics: These include equations expressing the vector measures of mechanical motion-quantity of movement (impulce of the body) and its moment with respect to a certain center (angular momentum or kinetic moment). In the presence of external mass transfer, the theorem on the change in the momentum can be formulated as follows: the total time derivative of the amount of motion of the medium in an arbitrary volume is equal to the sum of all external (mass, surface and interphase) forces applied to it, as well as the amount of motion of the attached (or detachable) mass and phase transitions per unit time. This theorem for the i-th phase of a two-phase mixture is written in integral form:

$$\int_{V(t)} \partial_t(\rho_i\varphi_i\vec{U}_i)\, dV + \int_{S(t)} (\rho_i\varphi_i\vec{U}_i)U_{i\,n} ds = \int_{V(t)} \rho_i\varphi_i\vec{F}_i\, dV + \int_{V(t)} \varphi_i\vec{\sigma}_{in} ds + + \int_{V(t)} [\vec{U}_{*i}q_{*i} + (-1)^i(\vec{R}_i + \vec{U}_æ æ)]dV, \qquad (4)$$

Where $\vec{F}_i and \vec{R}_i$–the share vector of mass and interphase forces of the i-th phase; $\sigma_{in} = \vec{\sigma}_i\vec{n}_i, \vec{\sigma}_i$– the specific stress vector of the surface forces of the i-th phase;$\vec{U}_{*i}, \vec{U}_æ$ - the velocity vector of the attached (or detachable) mass of the i-th phase and the phase transitions (the carrier phase into the one being carried).In the region of continuous motions, from (4) taking into account (2) and the Gauss-Ostrogradsky formula, we obtain the equation of dynamics for the i-th phase of the mixture in the differential form

$$\rho_i\varphi_i(\partial_t\vec{U}_i + (\vec{U}_i\nabla)\vec{U}_i) = \rho_i\varphi_i\vec{F}_i + \nabla(\varphi_i\vec{\sigma_i}) + (-1)^i[\vec{R}_i + (\vec{U}_æ - \vec{U}_i)æ] + (\vec{U}_{*i} - \vec{U}_i)q_* \qquad (5)$$

Summing (5) with respect to i, we obtain the differential equation of momentum for a two-phase mixture as a whole:

$$\rho(\partial_t\vec{U} + (\vec{U}_{i_i}\nabla)\vec{U}) = \rho\vec{F} + \nabla\vec{\sigma} + (\vec{U}_{*i} - \vec{U})q_*, \qquad (6)$$

where$\rho\vec{F} = \sum\rho_i\varphi_i\vec{F}_i, \vec{\sigma} = \sum\vec{\sigma_i}$.

In the problems of dynamics, one must take into account not only the quantities of motion, but also its moment with respect to the center. The dynamic characteristic of this vector measure of mechanical motion is the moment of momentum relative to the center, which is expressed by the vector product of the radius vector of the point drawn from this center by the momentum. If the internal moments and the distributed (mass and surface) pairs of forces are absent in the medium, the theorem on the change of the angular momentum for the medium (phase) is formulated as follows: The total derivative with respect to time from the moment of the amount of motion of the medium (phase) in an arbitrary volume calculated relative to the center is equal to the sum of the moments of external (mass, surface and interphase) forces, as well as the angular momentum of the attached (or detachable) mass and phase transitions relative to the same center. This theorem for the i-th phase of the medium as a whole leads to the establishment of the symmetry of the stress tensor, $\sigma_{ij} = \sigma_{ji}$ [8]. The equation of the momentum does not contain new unknowns, but simply reduces the number of unknown components of stress tensors from nine to six. Under these conditions, the equations of the dynamics of two-phase media with external mass transfer will be written in this form

$$\rho(\partial_t\vec{U} + (\vec{U} \cdot \nabla)\vec{U}) = \rho\vec{F} + \nabla\vec{\sigma} + (\vec{U}_* - \vec{U})q_*, \sigma_{ij} = \sigma_{ji}. \qquad (7)$$

Equations of energy. During the transfer of liquid media, some types of energy are converted into others (mechanical energy into thermal energy), the regularity of which is realized on the basis of the energy balance equation derived from the energy change theorem. For a medium moving with an external mass transfer, the theorem on the change in energy is formulated as follows: he total derivative of the sum of the kinetic and internal (thermal) energy of the medium (phase) in an arbitrary volume is equal to the sum of the powers (works per unit time) of external (mass, surface and interfacial) forces applied to this volume and its surface, heat plus the kinetic and internal energy of the attached (or detachable) mass and phase transitions. We write it for the i-th phase of the mixture in integral form

$$\int_{V(t)} \partial_t(\rho_i\varphi_i E_i)\, dV + \int_{S(t)} \rho_i\varphi_i E_i U_{i\,n} ds = \int_{V(t)} \rho_i\varphi_i(\vec{F_i} \cdot \vec{U_i})dV + \int_{S(t)} \varphi_i(\sigma_{in}\vec{U_i} - -q_n^*)ds + (-1)^i \int_{V(t)}[(\overrightarrow{R_i} \cdot \vec{U_i}) + Q_i + E_{æ}æ]dV + \int_{V(t)} E_{*i}q_{*i_{æ}}dV, \qquad (8)$$

где$E_i = e_i + \frac{U_i^2}{2}$; $E_{*i} = e_{*i} + \frac{U_{*i}^2}{2}$; $E_æ = e_æ + \frac{U_æ^2}{2}$; e_i, e_*, $e_æ$ – internal energy of a mass unit of the i-th phase, external mass and phase transitions; $\vec{q}_i^*$ −the specific heat syrup vector to the i-th phase from the medium through the surface $S_{(t)}$; $\frac{U_i^2}{2}$; $\frac{U_{*i}^2}{2}$; $\frac{U_æ^2}{2}$ −kinetic energy of a unit mass of the i-th phase, external mass and phase transitions. In (8), according to the Gauss-Ostrogradsky theorem from integrals over the surface to the integral over the volume, we obtain the following differential equations of total energy for the i-th phase of the mixture.

$$\rho_i\varphi_i\,(\partial_t E_i + (\vec{U}_i \cdot \nabla)\vec{U}_i) = \rho_i\varphi_i(\vec{F_i} \cdot \vec{U_i}) + div(\vec{\sigma_i} \cdot \vec{U}_i - \vec{q_i^*})\varphi_i + (E_{*i} - E_i)q_{*i} + +(-1)^i[(\overrightarrow{R_i} \cdot \vec{U_i}) + Q_i + (E_æ - E_i)æ] \qquad (9)$$

and for a two-phase medium as a whole.

$$\rho\big(\partial_t E + (\vec{U}_i \cdot \nabla)\vec{U}_i\big) = \rho(\vec{F} \cdot \vec{U}) + div(\vec{\sigma} \cdot \vec{U} - \overrightarrow{q_*}) + (E_* - E)q_*, \qquad (10)$$

where$E = \sum E_i$, $E_* = \sum E_{*i}$, $\overrightarrow{q_*} = \sum \overrightarrow{q_{*\iota}}$,

In the conditions of slowly changing flows of two-phase media with external mass transfer (ie, at velocities of the medium flow is much less than the speed of sound), one can use the internal energy equation. This equation for the medium as a whole can be obtained from a comparison of equation (10) with the kinetic energy equation [8], in the following form

$$\rho\big(\partial_t e + (\vec{U}_i \cdot \nabla)e\big) = -\nabla(\vec{q}_* + N_*) + (e_* - e)\, q_*, \qquad (11)$$

whereN_*-specific power of internal forces of the medium with external mass transfer:

$$N_* = -\vec{\sigma} div\vec{U} - 0{,}5(\vec{U} - \vec{U}_*)^2 q_* \qquad (12)$$

The set of hydromechanical equations for the i-th phase (2), (5), (9) and the medium as a whole (3), (7), (10), (11) add a generalized system of equations for two-phase flows with external heat and mass exchange and are valid for Description of the environment with favorite physical properties. The analysis showed that these equations are general, since the proposed earlier different forms of equations of two-phase media [1-7] can be obtained from these as special cases by appropriate transformations (under certain assumptions).However, the system of equations for two-phase media with external heat and mass transfer is uncertain. For its closure it is necessary to attract thermodynamic and rheological equations of state, as well as expressions for the heat flux, interphase forces and heat transfer between phases. These additional (defining) relations are established in the construction of a mathematical model of a particular studied medium.

Model of viscous media: In many cases, for a mathematical description of disperse systems (suspension, solutions, petroleum products), a model of a quasi-homogeneous medium can be effectively applied. In this case, the disperse system is regarded as a homogeneous mixture with some effective shear viscosity μ. In the case of a small concentration of suspended particles φ in the medium flows, the effective shear viscosity μ can be determined by Einstein's formula$\mu = \mu_0(1 + 2{,}5\varphi)$, where μ_0 −shear viscosity of the carrier medium (liquid). Under these conditions, the flow of the medium qualitatively coincides with a homogeneous medium in which the components of the stress tensor are connected by the components of the strain rate tensor according to Newton's law. For a viscous incompressible medium we have:

$$\sigma_{ij} = \begin{Bmatrix} -P + 2\mu\frac{\partial U_i}{\partial x_i}, & npu\ i = j \\ \mu\left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i}\right), & npu\ i \neq j \end{Bmatrix} \qquad (13)$$

e=cT, $e_* = cT_*, q_* = -\lambda \nabla T$ ……………………………………...(14)

where$P$- pressure, c -heat capacity (in an incompressible viscous medium $c = c_v = c_p$); T, T_* –temperature of the main (current) and attached (or detachable) mass of the medium; λ – кcoefficient of thermal conductivity.

Substituting expressions (13) and (14) into the system of equations of motion (3), (7), and (11) and taking into account that for an incompressible medium ρ, μ, c, λ, the constants, we can write$\nabla \vec{U} = q$;

$$\partial_t \vec{U} + (\vec{U} \cdot \nabla)\vec{U} = \vec{F} - \rho^{-1}\nabla P + \nu \nabla^2 \vec{U} + (\vec{U}_* - \vec{U}_*)q, \quad \text{……………………………………..(15)}$$

$$\partial_t T + (\vec{U} \cdot \nabla) T = a\nabla^2 T + \frac{2\nu}{c}\varepsilon_{ij}^2 + [(T_* - T) + k]\, q,$$

where$\varepsilon_{ij} = 0{,}5\left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i}\right)$, $k = \frac{\left[(\vec{U}_* - \vec{U})^2 - \frac{2P}{\rho}\right]}{2C}$; ν–coefficient of kinematic viscosity, $\nu = \mu/\rho$; a – coefficient of temperature conductivity, $a = \frac{\lambda}{\rho C}$; $q = q_*/\rho$

The system (15) is the basic equation of hydromechanics of a viscous incompressible medium with external heat and mass transfer. Of these, in the absence of external sources (or sinks) of the mass $q = 0$, momentum $(\vec{U}_* - \vec{U})q = 0$ and energy $[(T_* - T) + k]q = 0$, as a particular case one can obtain the well-known Navier-Stokes equations of motion [9].

$$\nabla \vec{U} = 0,\ d_t \vec{U} + (\vec{U} \cdot \nabla)\vec{U} = \vec{F} - \rho^{-1}\nabla P + +\nu \nabla^2 \vec{U}, \quad \text{………………………………………..(16)}$$

and the heat conduction equation for a viscous incompressible medium [9].

$$\partial_t T(\vec{U} \cdot \nabla) T = a\nabla^2 T + 2\nu c^{-1}\left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i}\right)^2 \quad \text{………………………………………..(17)}$$

An analysis of the systems of hydrodynamics equations for a viscous incompressible medium (15) shows that they form a system of five equations (one equation of continuity, three equations of dynamics and one heat equation) for finding five quantities$\rho, \vec{U}(U_x,\ U_y, U_z)$ and T (other quantities$\rho, \nu, a, c, T_*, q, U_*, F$ the equations entering into the system of equations of motion (15) are given), i.e. system (15) is closed.

The conclusion

The equations of motion derived from the general conservation laws establish a relationship between the temporal and spatial changes in velocity, pressure, temperature in any point in the medium in which the process of matter transfer takes place. Their study makes it possible to construct a number of physical phenomena and solve a variety of particular problems. When solving a particular problem, it is necessary to choose from all these solutions those that satisfy certain additional conditions arising from its physical meaning. Such additional conditions are often the so-called initial and boundary conditions. The initial conditions are given only when studying nonstationary processes and are specified in the fact that for a certain instant of time $t = t_0$ (usually believe$t = 0$) should be knownа function, for example$U|_{t=0} = U(x, y, z)$, $P|_{t=0} = P(x, y, z)$ and $T|_{t=0} = T(x, y, z)$ spatial coordinates. If the viscous medium is transported with the source or sink of the mass (ie, when there is a blowing or sucking of the substance through the wall), the boundary conditions on the permeable surfaces differ from the corresponding conditions on the impermeable wall. In this case, the normal velocity component on the surface is not zero and is determined from the given mass flow through the wall $(\rho U_n)_\omega = f(s, t)$, where $f(s, t)$ –a given function characterizing the mass velocity of injection or suction of matter through the wall, the index ω refers to the parameter on the wall The slip conditions (i.e. the tangential velocity component) on the wall $(U_\tau)_\omega = \left(\frac{\lambda_* \partial U_\tau}{\partial n}\right)$, where the coefficient λ_ * is proportional to the permeability of the wall material or the characteristic pore size. The last expression has the same form as the slip condition on the wall in the theory of a rarefied gas, where λ_ * denotes the mean free path of the molecules.

As the thermal boundary condition, the wall temperature (the boundary condition of the first kind), the heat flux through the wall (the boundary condition of the second kind), the heat flux density due to thermal conductivity (the boundary condition of the third kind) and the heat exchange of the surface of the body with the environment (boundary condition fourth kind). In addition to these boundary conditions, there are others that we have to deal with when considering various practical problems.

Thus, a mathematical problem with the goal of describing reality must satisfy the following basic requirements: 1) the solution must exist, 2) the solution must be unique, 3) the solution must be stable. This means that small changes in the task should cause accordingly a small change in the solution. A task that satisfies all (three) requirements is called a correctly posed problem. To solve the boundary value problem is to find all functions satisfying a given differential equation and given boundary conditions. For various special cases of the proof of existence and uniqueness theorems for boundary value problems, there is an example of heat and mass transfer. However, in the most general case there are no proofs of these theorems. Since the solution of various hydrodynamic and thermophysical problems for the existence of a solution follows from the very formulation of the problem, it can be assumed that the existence of the solution and its uniqueness is proved for the boundary value problems under study.

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