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RESEARCH ARTICLE

ON CLASSES OF GREEN'S FUZZY GENARALIZED INVERSE SEMIGROUP

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ABSTRACT

In the field of research, the study about fuzziness in the classes of regular semigroups becomes very important in the recent days. The fast and intensive development of the subject fuzzy relations has been continued from the twentieth century. In logical algebra using fuzzy tools many definitions related to fuzzy algebraic structure are established recently. In the paper "On Green's Fuzzy Orthodox semigroup" (G Hariprakash, 2016) and "On Green's fuzzy generalized inverse semigroup" G Hariprakash, 2017) characterizes orthodox semigroups and generalized inverse semigroups using fuzzy property. This work is a follow up of these studies. Mainly the work concentrated on classes of Green's Fuzzy Orthodox semigroups and classes of Green's Fuzzy Generalized inverse semigroups. In particular the study is about the quotient classes of Green's Fuzzy Orthodox semigroups and quotient classes of Green's Fuzzy Generalized inverse sEmigroups. The text includes two lammas in Green's Fuzzy Generalized inverse semigroup and two lemmas in Green's Fuzzy Orthodox semigroup. The conclusion of the work is to find out a necessary and sufficient condition for quotient classes of Green's Fuzzy Regular semigroup to be a Green's Fuzzy Orthodox semigroup. This work endeavors the opening of a new area of study Green's Fuzzy Idempotent Separating, Krenal of Green's Fuzzy relations and other related notions.

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INTRODUCTION

Definition 1.1; Idompotent: An element 'e' in asemigroup S is called an idempotent if e.e=e.The set of idempotent in S is denoted by E(S)

Definition 1.2. Regular semigroup:-A semigroup S is regular if all its elements are regular. (A.H. Clifford and G.B. Preston, 1967). That is for any $a \in S$ there exists $x \in S$, such that; axa =a

Definition 1.3: Inverse semigroup:-A semigroup S is called an inverse semigroup if every element 'a' in S posses a unique inverse.(A.H. Clifford and G.B. Preston,1967)

Definition 1.4: Fuzzy congruence:-A fuzzy compatible similarity relation on a semigroup is called a fuzzy congruence (J.P Kim and, D.R Bae, 1997).

Definition 1.5: Fuzzy Orthodox semigroup:-An Orthodox semigroup S is a regular semigroup in which the set of its idempotents form a semigroup (S. Madhavan,1978). If a membership function is defined on S, it is called a Fuzzy Orthodox semigroup (G Hariprakash, 2016). Definition 1.6; Fuzzy Generalized inverse semigroup:-An orthodox semigroup S is a Generalized Inverse semigroup if the set E(S) of its idempotent which is band is normal (Madhavan, 1978). That is for e,f,g, h $\in$E (S);efgh=egfh. (Hariprakash, 2017). A Generalized inverse semigroup in which a membership function is defined is called a fuzzy generalized inverse semigroup (Hariprakash, 2017) this study $\hat{\mathcal{L}}$ and $\hat{\mathcal{R}}$ denotes Fuzzy Green's Relations (Green, 1951), (Hariprakash, 2016]).

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$\hat{\mathcal{L}}$-fuzzy generalized inverse semigroup

Definition 1.7 ; If $\hat{\mathcal{L}}$ is a fuzzy congruence on a orthodox semigroup S,and if S/$\hat{\mathcal{L}}$ itself is an orthodox semigroup then S/$\hat{\mathcal{L}}$ is called an $\hat{\mathcal{L}}$-fuzzy orthodox semigroup.

Lemma 2.1. Let $S/\hat{\mathcal{L}}$ be a $\hat{\mathcal{L}}$-fuzzy generalized inverse semigroup. If $\hat{\mathcal{L}}_e \in E_{S/}\hat{\mathcal{L}}$ and $a \in S$, for every $a' \in V(a)$, the elements $\hat{\mathcal{L}}_{a'ea}$ and $\hat{\mathcal{L}}_{aea'}$ are both idempotent.

Proof.

$(\hat{\mathcal{L}}_{a'ea})^2 = \hat{\mathcal{L}}_{a'eaa'ea}$
$= \hat{\mathcal{L}}_{a'aa'eaa'ea}$
$= \hat{\mathcal{L}}_{a'aa'aa'eea}$
$= \hat{\mathcal{L}}_{a'aa'ea}$
$= \hat{\mathcal{L}}_{a'ea}$
Hence, $\hat{\mathcal{L}}_{a'ea}$ is an idempotent in $S/\hat{\mathcal{L}}$.

$\hat{\mathcal{R}}$-fuzzy generalized inverse semigroup

Similarly, we can show that $\hat{\mathcal{L}}_{aea'}$ is an idempotent in $S/\hat{\mathcal{L}}$.

Lemma 2.2. Let $S/\hat{\mathcal{R}}$ be a $\hat{\mathcal{R}}$-fuzzy genaralized inverse semigroup. If $\hat{\mathcal{R}}_e \in E_{(S/\hat{\mathcal{R}})}$ and $a \in S$, for every $\hat{\mathcal{R}}_{a'} \in V(Ra)$ the element $\hat{\mathcal{R}}_{a'ea}$ and $\hat{\mathcal{R}}_{aea'}$ belongs to $E_{(S/\hat{\mathcal{R}})}$.

Lemma 2.3. Let $S/\hat{\mathcal{L}}$ be a $\hat{\mathcal{L}}$-fuzzy orthodox semigroup. If $a \in S$ and $\hat{\mathcal{L}}_d \in V(\hat{\mathcal{L}}_a)$; $V(\hat{\mathcal{L}}_a) = E\hat{\mathcal{L}}_{a^{-1}a} * \hat{\mathcal{L}}_{a-1} * E\hat{\mathcal{L}}_{aa^{-1}}$.

Let $x \in E\hat{\mathcal{L}}_{a^{-1}a} * \hat{\mathcal{L}}_{a-1} * E\hat{\mathcal{L}}_{aa^{-1}}$.

Then

$x = \hat{\mathcal{L}}_{a'a} * \hat{\mathcal{L}}_{a'} * \hat{\mathcal{L}}_{aa'}$
$= \hat{\mathcal{L}}_{a'aa'aa'}$
$= \hat{\mathcal{L}}_{aa'aa'}$
$= \hat{\mathcal{L}}_{a'} \in V(\hat{\mathcal{L}}_a)$.
$\therefore E\hat{\mathcal{L}}_{a^{-1}a} * \hat{\mathcal{L}}_{a-1} * E\hat{\mathcal{L}}_{aa^{-1}} \subseteq V(\hat{\mathcal{L}}_a)$.(1)

Consider $\hat{\mathcal{L}}_{a''} \in V(\hat{\mathcal{L}}_a)$. Then

$\hat{\mathcal{L}}_{a''} = \hat{\mathcal{L}}_{a''}\hat{\mathcal{L}}_a\hat{\mathcal{L}}_{a''}$
$= \hat{\mathcal{L}}_{a''aa''}$
$= \hat{\mathcal{L}}_{a''aa''aa''}$
$= \hat{\mathcal{L}}_{a''} * \hat{\mathcal{L}}_{a''} * \hat{\mathcal{L}}_{aa''}$

$\in E\hat{\mathcal{L}}_{a^{-1}a} * \hat{\mathcal{L}}_{a-1} * E\hat{\mathcal{L}}_{aa^{-1}}$.
$\therefore V(\hat{\mathcal{L}}_{a'}) \subseteq E\hat{\mathcal{L}}_{a^{-1}a} * \hat{\mathcal{L}}_{a-1} * E\hat{\mathcal{L}}_{aa^{-1}}$. (2)

From (1) and (2),

$V(\hat{\mathcal{L}}_a) = E\hat{\mathcal{L}}_{a^{-1}a} * \hat{\mathcal{L}}_{a-1} * E\hat{\mathcal{L}}_{aa^{-1}}$.

Lemma 2.4 Let $S/\hat{\mathcal{R}}$ be a $\hat{\mathcal{R}}$ - fuzzy orthodox semigroup. If $a \in S$ and $a' \in V(a)$,

$V(\hat{\mathcal{R}}_a) = E\hat{\mathcal{R}}_{a^{-1}a} * \hat{\mathcal{R}}_{a-1} * E\hat{\mathcal{R}}_{aa^{-1}}$.

Proof: Result follows from lemma 2.3 using the property of $\hat{\mathcal{R}}_a$.

Result 2.5. If $x'_1 \in V(x)$ and $x'_2 \in V(x)$, $E_{xx'_1} = E_{xx'_2}$ and $E_{x'_1x} = E_{x'_2x}$.

Theorem 2.6. A regular semigroup $S/\hat{\mathcal{L}}$ is a $\hat{\mathcal{L}}$- fuzzy orthodox if and only if for all $a, b \in S$,

$V(\hat{\mathcal{L}}_a) \cap V(\hat{\mathcal{L}}_b) \neq \theta \Rightarrow V(\hat{\mathcal{L}}_a) = V(\hat{\mathcal{L}}_b)$.

Proof. Suppose $S/\hat{\mathcal{L}}$ is a $\hat{\mathcal{L}}$- fuzzy orthodox semigroup and $V(\hat{\mathcal{L}}_a) \cap V(\hat{\mathcal{L}}_b) \neq \theta$. Let $\hat{\mathcal{L}}_x \in V(\hat{\mathcal{L}}_a) \cap V(\hat{\mathcal{L}}_b)$. Then

$\hat{\mathcal{L}}_x \in V(\hat{\mathcal{L}}_a)$ and $\hat{\mathcal{L}}_x \in V(\hat{\mathcal{L}}_b) \Rightarrow \hat{\mathcal{L}}_a \in V(\hat{\mathcal{L}}_x)$ and $\hat{\mathcal{L}}_b \in V(\hat{\mathcal{L}}_x)$
$\Rightarrow E(\hat{\mathcal{L}}_x\hat{\mathcal{L}}_a) = E(\hat{\mathcal{L}}_x\hat{\mathcal{L}}_b)$ and $E(\hat{\mathcal{L}}_a\hat{\mathcal{L}}_x) = E(\hat{\mathcal{L}}_b\hat{\mathcal{L}}_x)$
$\Rightarrow E\hat{\mathcal{L}}_x\hat{\mathcal{L}}_a = E\hat{\mathcal{L}}_x\hat{\mathcal{L}}_b$ and $E\hat{\mathcal{L}}_a\hat{\mathcal{L}}_x = E\hat{\mathcal{L}}_b\hat{\mathcal{L}}_x$.
$\Rightarrow E\hat{\mathcal{L}}_{xa} = E\hat{\mathcal{L}}_{xb}$ and $E\hat{\mathcal{L}}_{ax} = E\hat{\mathcal{L}}_{bx}$.

Since $\hat{\mathcal{L}}_x \in V(\hat{\mathcal{L}}_a)$, by lemma 2.3

$V(\hat{\mathcal{L}}_a) = E\hat{\mathcal{L}}_{xa} * \hat{\mathcal{L}}_x * E\hat{\mathcal{L}}_{ax}$
$= E\hat{\mathcal{L}}_{xb} * \hat{\mathcal{L}}_x * E\hat{\mathcal{L}}_{bx}$
$= V(\hat{\mathcal{L}}_b)$

Conversely, assume $S/\hat{\mathcal{L}}$ is regular and $V(\hat{\mathcal{L}}_a) = V(\hat{\mathcal{L}}_b)$. Let $\hat{\mathcal{L}}_e$ and $\hat{\mathcal{L}}_f$ be two idempotents in $S/\hat{\mathcal{L}}$. Let $\hat{\mathcal{L}}_x$ be in inverse of $\hat{\mathcal{L}}_e * \hat{\mathcal{L}}_f$. Then

$\hat{\mathcal{L}}_e * \hat{\mathcal{L}}_f * \hat{\mathcal{L}}_x * \hat{\mathcal{L}}_e * \hat{\mathcal{L}}_f = \hat{\mathcal{L}}_e * \hat{\mathcal{L}}_f$, (3)

and

$\hat{\mathcal{L}}_x * \hat{\mathcal{L}}_e * \hat{\mathcal{L}}_f * \hat{\mathcal{L}}_x = \hat{\mathcal{L}}_x$ (4)

We have

$\hat{\mathcal{L}}_{fxe} * \hat{\mathcal{L}}_{fxe} = (\hat{\mathcal{L}}_f * \hat{\mathcal{L}}_x * \hat{\mathcal{L}}_e) * (\hat{\mathcal{L}}_f * \hat{\mathcal{L}}_x * \hat{\mathcal{L}}_e)$
$= (\hat{\mathcal{L}}_f * (\hat{\mathcal{L}}_x * \hat{\mathcal{L}}_e * \hat{\mathcal{L}}_f * \hat{\mathcal{L}}_x) * \hat{\mathcal{L}}_e$
$= (\hat{\mathcal{L}}_f * \hat{\mathcal{L}}_x * \hat{\mathcal{L}}_e)$
$= \hat{\mathcal{L}}_{fxe}$

That is, $\hat{\mathcal{L}}_{fxe} \in E_{S/\hat{\mathcal{L}}}$.

$\hat{\mathcal{L}}_{fxe} * \hat{\mathcal{L}}_{efxe} * \hat{\mathcal{L}}_{fxe} = \hat{\mathcal{L}}_{fx} * \hat{\mathcal{L}}_e * \hat{\mathcal{L}}_e * \hat{\mathcal{L}}_{fxe} * \hat{\mathcal{L}}_{fxe}$
$= \hat{\mathcal{L}}_{fx} * (\hat{\mathcal{L}}_e)^2 * (\hat{\mathcal{L}}_{fxa})^2$
$= \hat{\mathcal{L}}_{fx} * \hat{\mathcal{L}}_e * \hat{\mathcal{L}}_{fxe}$
$= \hat{\mathcal{L}}_{fxe} * \hat{\mathcal{L}}_{fxe}$
$= (\hat{\mathcal{L}}_{fxe})^2$
$= \hat{\mathcal{L}}_{fxe}$.

Also,

$\hat{\mathcal{L}}_{efxe} * \hat{\mathcal{L}}_{efxe} = \hat{\mathcal{L}}_e * \hat{\mathcal{L}}_f * \hat{\mathcal{L}}_x * \hat{\mathcal{L}}_e * \hat{\mathcal{L}}_e * \hat{\mathcal{L}}_f * \hat{\mathcal{L}}_x\hat{\mathcal{L}}_e = \hat{\mathcal{L}}_e * (\hat{\mathcal{L}}_f * \hat{\mathcal{L}}_x * \hat{\mathcal{L}}_e) * (\hat{\mathcal{L}}_f * \hat{\mathcal{L}}_x * \hat{\mathcal{L}}_e)$
$= \hat{\mathcal{L}}_e * (\hat{\mathcal{L}}_f * \hat{\mathcal{L}}_x * \hat{\mathcal{L}}_e)^2$
$= \hat{\mathcal{L}}_e. = \hat{\mathcal{L}}_{fxe}$

$= \hat{\mathcal{L}}_{efxe}$.

Hence $\hat{\mathcal{L}}_f * \hat{\mathcal{L}}_x * \hat{\mathcal{L}}_e$ and $\hat{\mathcal{L}}_e * \hat{\mathcal{L}}_f * \hat{\mathcal{L}}_x * \hat{\mathcal{L}}_e$ are indempotents in $S/\hat{\mathcal{L}}$.

That is, $\hat{\mathcal{L}}_{fxe}$ and $\hat{\mathcal{L}}_{efxe}$ are idempotents in $S/\hat{\mathcal{L}}$.
Again,

$$\hat{\mathcal{L}}_{efxe}\hat{\mathcal{L}}_{fxe}\hat{\mathcal{L}}_{efxe} = \hat{\mathcal{L}}_e * \hat{\mathcal{L}}_{fxe} * \hat{\mathcal{L}}_{efxe} * \hat{\mathcal{L}}_{efxe}$$
$$= \hat{\mathcal{L}}_e * (\hat{\mathcal{L}}_{fxe})^2 * \hat{\mathcal{L}}_{efxe}$$
$$= \hat{\mathcal{L}}_e * \hat{\mathcal{L}}_{fxe} * \hat{\mathcal{L}}_{efxe}$$
$$= \hat{\mathcal{L}}_{efxe} * \hat{\mathcal{L}}_{efxe}$$
$$= \hat{\mathcal{L}}_{efxe}.$$

Also, we get $\hat{\mathcal{L}}_{fxe} * \hat{\mathcal{L}}_{efxe} * \hat{\mathcal{L}}_{fxe} = \hat{\mathcal{L}}_{fxe}$. Hence $\hat{\mathcal{L}}_{efxe}$. is the inverse of $\hat{\mathcal{L}}_{efxe}$. That is, $\hat{\mathcal{L}}_{fxe} \in V(\hat{\mathcal{L}}_{efxe})$. Since $\hat{\mathcal{L}}_{fxe}$ is an idempotent $\hat{\mathcal{L}}_{fxe}$ is an inverse of itself. That is $\hat{\mathcal{L}}_{fxe} \in V(\hat{\mathcal{L}}_{fxe})$.
That is, $\hat{\mathcal{L}}_{fxe} \in V(\hat{\mathcal{L}}_{efxe}) \cap V(\hat{\mathcal{L}}_{fxe}) \Rightarrow V(\hat{\mathcal{L}}_{fxe}) \cap V(\hat{\mathcal{L}}_{efxe}) \neq \theta$

Then, by hypothesis, $V(\hat{\mathcal{L}}_{fxe}) = V(\hat{\mathcal{L}}_{efxe})$. Again,

$$\hat{\mathcal{L}}_{ef} * \hat{\mathcal{L}}_{fxe} * \hat{\mathcal{L}}_{ef} = \hat{\mathcal{L}}_e * \hat{\mathcal{L}}_f * \hat{\mathcal{L}}_f * \hat{\mathcal{L}}_x * \hat{\mathcal{L}}_e * \hat{\mathcal{L}}_e * \hat{\mathcal{L}}_f$$
$$= \hat{\mathcal{L}}_e * \hat{\mathcal{L}}_f * \hat{\mathcal{L}}_x \hat{\mathcal{L}}_e * \hat{\mathcal{L}}_f$$
$$= \hat{\mathcal{L}}_{ef} * \hat{\mathcal{L}}_f * \hat{\mathcal{L}}_x * \hat{\mathcal{L}}_e * \hat{\mathcal{L}}_f$$
$$= \hat{\mathcal{L}}_{ef} * \hat{\mathcal{L}}_x * \hat{\mathcal{L}}_{ef}$$
$$= \hat{\mathcal{L}}_{ef},$$
and

$$\hat{\mathcal{L}}_{fxe} * \hat{\mathcal{L}}_{ef} * \hat{\mathcal{L}}_{fxe} = \hat{\mathcal{L}}_f * \hat{\mathcal{L}}_x * \hat{\mathcal{L}}_e * \hat{\mathcal{L}}_e * \hat{\mathcal{L}}_f * \hat{\mathcal{L}}_f * \hat{\mathcal{L}}_x * \hat{\mathcal{L}}_e$$
$$= \hat{\mathcal{L}}_f * \hat{\mathcal{L}}_x * \hat{\mathcal{L}}_e * \hat{\mathcal{L}}_f * \hat{\mathcal{L}}_x * \hat{\mathcal{L}}_e$$
$$= \hat{\mathcal{L}}_{fxe} * \hat{\mathcal{L}}_{fxe}$$
$$= \hat{\mathcal{L}}_{fxe}.$$

We have $\hat{\mathcal{L}}_{ef} \in V(\hat{\mathcal{L}}_{fxe}) \Rightarrow \hat{\mathcal{L}}_{ef} \in V(\hat{\mathcal{L}}_{efxe})$. Therefore,

$$\hat{\mathcal{L}}_e * \hat{\mathcal{L}}_f = \hat{\mathcal{L}}_{ef} = \hat{\mathcal{L}}_{ef} * \hat{\mathcal{L}}_{efxe} * \hat{\mathcal{L}}_{ef} = \hat{\mathcal{L}}_e * \hat{\mathcal{L}}_f * \hat{\mathcal{L}}_e * \hat{\mathcal{L}}_f * \hat{\mathcal{L}}_x * \hat{\mathcal{L}}_e * \hat{\mathcal{L}}_e * \hat{\mathcal{L}}_f$$
$$= \hat{\mathcal{L}}_e * \hat{\mathcal{L}}_f * \hat{\mathcal{L}}_e * \hat{\mathcal{L}}_f * \hat{\mathcal{L}}_x * \hat{\mathcal{L}}_e * \hat{\mathcal{L}}_f$$
$$= \hat{\mathcal{L}}_e * \hat{\mathcal{L}}_f * \hat{\mathcal{L}}_e * \hat{\mathcal{L}}_f$$
$$= (\hat{\mathcal{L}}_e * \hat{\mathcal{L}}_f)^2$$

That is, $\hat{\mathcal{L}}_e * \hat{\mathcal{L}}_f = (\hat{\mathcal{L}}_e * \hat{\mathcal{L}}_f)^2$. That is, the product of two idempotents in $S/\hat{\mathcal{L}}$ is an indempotent. Given $S/\hat{\mathcal{L}}$ is regular. So $S/\hat{\mathcal{L}}$ is a $\hat{\mathcal{L}}$-fuzzy orthodox semigroup.

Theorem 2.7. A regular semigroup $S/\hat{\mathcal{R}}$ is a $\hat{\mathcal{R}}$-fuzzy orthodox if and only if for all $a, b \in S, V(\hat{\mathcal{R}}_a) \cap \hat{\mathcal{R}}_a \neq \theta \Rightarrow V(\hat{\mathcal{R}}_a) = V(\hat{\mathcal{R}}_b)$.

The result follows from theorem 4.3.13 using the property of $\hat{\mathcal{R}}_a$.

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