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REVIEW ARTICLE

MULTIPLIERS IN BIPOLAR-VALUED FUZZY d-ALGEBRA

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ABSTRACT

In this paper, I introduce the concept of a multipliers on bipolar-valued fuzzy d-algebra and obtain some properties and results on it.

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INTRODUCTION

The notion of d-algberas, another generalization of class of algebras, were introduced Neggers and Kim [Imai, 1966]. They studied some properties of this class of algebras. The study of multipliers has been made by various researchers in the context of rings and groups. They have studied the properties of multipliers. But the properties of multipliers on bipolar-valued fuzzy d-algebra an important class of fuzzy d-algebras containing the class BCK-algebras. So with this motivation in this paper I introduce the concept of a multipliers on a bipolar-valued fuzzy d-algebras and discuss some results of multipliers on bipolar-valued fuzzy d-algebras.

Preliminaries: In this section we describe some definitions and notions that will be used in the sequel.

Definition 2.1 **[**Akram, 2005**]** Let X be a set with binary operation $*$ and a constant 0. Then $(X.*,0)$ is called a BCK-algebra if it satisfies the following axioms

1) $\big((x*y)*(x*z)\big)*(z*y)=0$
2) $\big(x*(x*y)\big)*y=0$
3) $x*x=0$
4) $0*x=0$
5) $x*y=0$ and $y*x=0$ imply $x=y$ for all $x,y,z\in X$.

Definition 2.2 [Joseph Neggers, 1999] A d-algebra is a non-empty set X with a constant 0 and a binary operation $*$ satisfying the following axioms

1)$x*x=0$
2)$0*x=0$
3)$x*y=0$ and $y*x=0$ imply $x=y$ for all $x,y\in X$.

Example 2.3 [8] Let $X=\{0,1,2\}$ be a set with the following table

$*$	0	1	2
0	0	0	0
1	2	0	2
2	1	1	0

Then $(X,*,0)$ is a d-algebra.

Remark 2.4 It is obvious from above definitions that every BCK-algebra is a d-algebra. The following shows that converse is not true, in general.

Example 2.5 [9] Let $X=\{0,1,2,3\}$ with the binary operation $*$ defined by

$*$	0	1	2	3
0	0	0	0	0
1	1	0	0	0
2	2	2	0	0
3	3	3	1	0

Then X is a d-algebra, but it is not BCK-algebra this is because condition (2) of definition (2.1) is not satisfied as shown

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$(3*(3*2))*2 = (3*1)*2 = 3*2 = 1 \neq 0$

Definition 2.6 Let S be a non-empty subset of a d-algebra X, then S is called a subalgebra of X if $x*y \in S \forall x,y \in S$. Definition 2.7 Let X be a d-algebra and I a subset of X, then I is called an ideal of X if it satisfies the following conditions

1) $0 \in I$
2) $x*y \in I$ and $y \in I$ imply $x \in I$.

Definition 2.8 Let X be a d-algebra and I a nonempty subset of X, then I is called a d-ideal of X if it satisfies the following conditions

1)$x*y \in I$ and $y \in I$ imply $x \in I$
2)$x \in I$ and $y \in X$ imply $x*y \in I$
3)$0 \in I$.

Example 2.9 Let $X = \{0,a,b,c,d\}$ be a d-algebra with the following cayley table

$*$	0	a	b	c	d
0	0	0	0	0	0
a	a	0	a	0	a
b	b	b	0	c	c
c	c	c	a	0	c
d	c	c	a	a	0

Let $I = \{0,a,c\}$ be a subset of X, then I is a d-ideal of X.

Main Results

Definition 3.1 Let X be a d-algebra and let (f^+,f^-) be a bipolar-valued fuzzy sets on X. A self map $f^+: X \to X$ and $f^-: X \to X$ satisfying the condition $f^+(x*y) = f^+(x)*y, f^-(x*y) = x*f^-(y)$, for all $x,y \in X$, is called a multipliers on bipolar valued fuzzy d-algebra on X.

Example 3.2 Let $X = \{0,a,b\}$ with the binary operation $*$ defined by

$*$	0	a	b
0	0	0	0
a	a	0	0
b	b	a	0

$*$	0	$-a$	$-b$
0	0	0	0
$-a$	$-a$	0	0
$-b$	$-b$	$-a$	0

Then X is a d-algebra. Let $f^+: X \to X$ and $f^-: X \to X$ be defined by

$$f^+(x) = \begin{cases} 0 \; if \; x = 0, a \\ a \; if \; x = b \end{cases}$$

$$f^-(x) = \begin{cases} 0 \; if \; x = 0, -a \\ -a \; if \; x = -b \end{cases}$$ Then (f^+,f^-) is a multipliers on bipolar-valued fuzzy d-algebra on X.

Proposition 3.3 Let X be a d-algebra and (f^+,f^-) is a multipliers on bipolar-valued fuzzy d-algebra on X, then

1) $f^+(0) = 0, f^-(0) = 0$.
2) $f^+(x) \leq x, f^-(x) \geq x \forall x \in X$.
3) If $x \leq y$ then $f^+(x) \leq y \forall x,y \in X$ and $f^-(y) \geq x$.

Proof: Let X be a d-algebra and let (f^+,f^-) multipliers on bipolar-valued fuzzy d-algebra on X, then we have $f^+(x*y) = f^+(x)*y$ and $f^-(x*y) = x*f^-(y)$ To prove, (1) Let$x = 0, y = 0, f^+(x*y) = f^+(0*0) = f^+(0)*0 = 0*0 = 0$. Therefore $f^+(0) = 0$ and $f^-(x*y) = x*f^-(y) \Rightarrow f^-(0*0) = 0*f^-(0) = 0*0 = 0$.

Therefore $f^-(0) = 0$. Hence prove the (1)

To prove (2) Let $x \in X$, then by (1) we have $0 = f^+(0) = f^+(x*x) = f^+(x)*x$ so $f^+(x) \leq x$. Also $f^-(0) = 0 \Rightarrow f^-(x*x) = 0 = x*f^-(x) \Rightarrow x \leq f^-(x)$ so $f^-(x) \geq x$. hence prove (2).

To prove (3) Let $x,y \in X, x \leq y$. Then $x*y = 0$ by (1) we have $0 = f^+(0) = f^+(x*y) = f^+(x)*y$. Thus $f^+(x) \leq y$. and we also have by (1) $0 = f^-(0) = f^-(x*y) = x*f^-(y)$. Thus $x \leq f^-(y) \Rightarrow f^-(y) \geq x$. Hence proves the (3).

Proposition 3.4 Let (f^+,g^+) and (f^-,g^-) be a multipliers on bipolar-valued fuzzy d-algebra on X. Then their composition $(f^+ \circ g^+), (f^- \circ g^-)$ is a multipliers on bipolar-valued fuzzy d-algebra on X. Proof: Let X be a d-algebra, let $(f^+,g^+),(f^-,g^-)$ be multipliers on bipolar-valued fuzzy d-algebra on X. Let $x,y \in X$. Then

$$\begin{aligned}(f^+ \circ g^+)(x*y) &= f^+(g^+(x*y)) \\ &= f^+(g^+(x)*y)) \\ &= f^+(g^+(x))*y \\ &= (f^+ \circ g^+)(x)*y\end{aligned}$$

Similarly, we can prove that

$(f^- \circ g^-)(x*y) = x*(f^- \circ g^-)(y)$.

Definition 3.5 A d-algebra X is said to be positive implicative if $(x*y)*z = (x*z)*(y*z)$. for all $x,y,z \in X$.

Definition 3.6 Let $M(X)$ denotes the collection of all multipliers on X. Obviously $O: X \to X$ defined by $O(x) = 0$ for all $x \in X$and $I: X \to X$ defined by $I(x) = x$ for all $x \in X$ are in $M(X)$. So $M(X)$ is nonempty.

Definition 3.7 Let X be a positive implicative d-algebra and $M(X)$ be the collection of all multipliers on X we define a binary operation $*$ on $M(X)$ by

$(f^+ * g^+)(x) = f^+(x)*g^+(x)$, For all $x \in X$ and $f^+,g^+ \in M(X)$.

$(f^- * g^-)(x) = f^-(x)*g^-(x)$, For all $x \in X$ and $f^-,g^- \in M(X)$.

Theorem 3.6 Let X be a positive implicative on d-algebra. Then $(M(X),*,0)$ is a positive implicative bipolar-valued fuzzy d-algebra.

Proposition 3.7 Let X be a d-algebra and (f^+,f^-) a multipliers on X. If (f^+,f^-) is one-to-one. Then (f^+,f^-) is the identity map on X.

Proof: Let f^+ be one-to-one. Let $x \in X$ then $f^+(x * f^+(x)) = f^+(x) * f^+(x) = 0 = f^+(0)$. Thus $x * f^+(x) = 0 \Rightarrow x \leq f^+(x)$. Since $f^+(x) \leq x$, by proposition 3.3(2) for all x, therefore $f^+(x) = x$. hence f^+ is the identity map.

Similarly, let f^- be one-to-one let $x \in X$. Then $f^-(x * f^-(x)) = f^-(x) * f^-(x) = 0 = f^-(0)$. Thus $x * f^-(x) = 0 \Rightarrow x \geq f^-(x)$. Since $f^-(x) \geq x$, by proposition 3.3(2) for all x, therefore $f^-(x) = x$. Hence f^- is the identity map.
Definition 3.8 Let (f^+, f^-) be a multipliers on X. We define $Ker(f^+)$ by $\ker(f^+) = \{x: x \in X \; and \; f^+(x) = 0\}$, and $\ker(f^-) = \{x: x \in X \; and \; f^-(x) = 0\}$

Proposition 3.9 Let X be a d-algebra and (f^+, f^-) be a bipolar-valued fuzzy d-algebra is a multipliers on X. Then (1) Ker (f^+) and Ker (f^-) is a sub algebra of X and (2) If (f^+, f^-) is one-to-one then Ker(f^+) =ker$(f^-) = \{0\}$.

Proof: (1) Let $x, y \in \ker(f^+)$. Then $f^+(x) = 0$ and $f^+(y) = 0$ so $f^+(x * y) = f^+(x) * y = 0 * y = 0$. Then $x * y \in$ ker (f^+) which implies ker (f^+) is sub algebra of X. $f^-(x) = 0, f^-(y) = 0$ so $f^-(x * y) = x * f^-(y) = x * 0 = 0$. Thus $x * y \in$ ker (f^-) which implies that ker (f^-) is a sub algebra of X.
(2) Let f be one-to-one. Let $x \in \ker(f^+)$. so $f^+(x) = 0 = f^+(0)$. Thus $x = 0$. so $\ker(f^+) = \{0\}$. Also we can prove $\ker(f^-) = \{0\}$.

Definition 3.10 A d-algebra X is called commutative if $x * (x * y) = y * (y * x)$ for all $x, y \in X$.

Proposition 3.11 Let X be a commutative d-algebra satisfying $x = 0 = x, x \in X$. Let bipolar-valued fuzzy d-algebra (f^+, f^-) be a multipliers on X. If $x \in \ker(f^+)$, if $y \leq x$, then $y \in \ker(f^+)$ and if $x \in \ker(f^-)$, $y \geq x$, then $y \in \ker(f^-)$.

Proof: Let $x \in$ ker (f^+) and $y \leq x$. Then $f^+(x) = 0$ and $y * x = 0$. Now $f^+(y) = f^+(y * 0) = f^+(y * (y * x)) = f^+(x * (x * y)) = f^+(x) * (x * y) = 0 * (x * y) = 0$. So $y \in \ker(f^+)$.

Also, Let $x \in \ker(f^-)$, $y \geq x$. Then $f^-(x) = 0, y * x = 0$. Now $f^-(y) = f^-(y * 0) = f^-(y * (y * x)) = f^-(x * (x * y)) = f^-(x) * (x * y) = 0 * (x * y) = 0$. So $y \in \ker(f^-)$.

Theorem 3.12 Let X be a d-algebra satisfying $x = 0 = x$ for all $x \in X$. Let bipolar-valued d-algebra (f^+, f^-) be a multipliers on X, which is also an endomorphism on X. Then Ker(f^+) and Ker(f^-) is a d-ideal of X.

Proof: Let $x * y \in \ker(f^+)$, $y \in \ker(f^+)$. Then $f^+(y) = 0$. Also $f^+(x * y) = 0$, which implies that $0 = f^+(x) * f^+(y) = f^+(x) * 0 = f^+(x)$. Thus $x \in \ker(f^+)$. Let $x \in$ ker (f^+) ad $y \in X$. Then

$$f^+(x * y) = f^+(x) * y = 0 * y = 0.$$

So $x * y \in \ker(f^+)$. Hence Ker(f^+) is a d-ideal of X.

Similarly, Let $x * y \in \ker(f^-)$, $y \in \ker(f^-)$. Then $f^-(y) = 0$ and $f^-(x * y) = 0$, which implies that $0 = f^-(x) * f^-(y) = f^-(x) * 0 = f^-(x)$. Thus $x \in \ker(f^-)$. Let $x \in$ ker (f^-) and $y \in$ X.Then $f^-(x * y) = x * f^-(y) = x * 0 = 0$. So $x * y \in \ker(f^-)$. Hence Ker(f^-) is a d-ideal of X.

Definition 3.13 Let X be a d-algebra and bipolar-valued fuzzy d-algebra (f^+, f^-) is a multipliers on X. Then the set

Fix$(f^+) = \{x: x \in X \; and \; f^+(x) = x\}$.
Fix$(f^-) = \{x: x \in X \; and \; f^-(x) = x\}$. is called the set of fixed points of (f^+, f^-).

Proposition 3.14 Let X be a d-algebra and bipolar-valued d-algebra (f^+, f^-) a multipliers on X. Then Fix(f^+) and Fix(f^-) is a sub algebras of X.

Proof: Since $f^+(0) = 0, f^-(0) = 0$, so Fix(f^+) and Fix(f^-) is non-empty. Let $x, y \in Fix(f^+)$. Then $f^+(x) = x, f^+(y) = y$. Thus $f^+(x * y) = f^+(x) * y = x * y$. So $x * y \in Fix(f^+)$. Hence Fix(f^+) is a Sub algebra of X. Also Let $x, y \in Fix(f^-)$. Then $f^-(x) = x, f^-(y) = y$. Thus $f^-(x * y) = x * f^-(y) = x * y$. So $x * y \in Fix(f^-)$. Hence Fix(f^-) is a sub algebra of X.

Conclusion

We have initiated a study of multipliers on bipolar-valued fuzzy d-algebras. I have shown that the collection M(X) of multipliers on a bipolar-valued fuzzy d-algebra. We have also investigate the conditions under which Ker(f^+) and Ker(f^-) of a multipliers $(f^+, f^-) \in M(X)$ is an ideal.

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