



RESEARCH ARTICLE

ANGULAR DISTRIBUTION IN HEAVY-ION INDUCED FISSION

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ABSTRACT

Statistical scission model has been applied to study fission fragment angular distributions induced in heavy-ion reaction on ^{232}Th and ^{237}Np target. The ratio of spherical to effective moments of inertia as a function of spin I has been investigated for $^{16}\text{O} + ^{232}\text{Th}$ fission reactions.**Key words:**Fission Anisotropies for heavy-ion reactions, Ratio of moments of inertia $\mathfrak{I}_{\text{sph}}/\mathfrak{I}_{\text{eff}}$ for $^{16}\text{O} + ^{232}\text{Th}$.

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INTRODUCTION

It is recognized that Statistical Transition Model (TSM) provides a good representation of experimental angular distributions of fragments from low-energy fission of nuclei (Rossener 1986; Behkami and Nazarzadeh 1998). The fundamental assumption of TSM is that: the spin projection K on the nuclear symmetry axis remains unchanged during the subsequent descent from saddle to scission. A number of studies have focused on extending TSM into the domain of fission, such as heavy-ion reactions producing intermediate nuclei for which a saddle point is not well defined or is absent (Schmitt *et al.*, 1984; Huizenga *et al.*, 1985). In such cases, the transition state theory is incapable for nuclear spin I in excess of the Rotated Liquid Drop Model (RLDM); where an equilibrium point in the potential energy no longer exists. For heavy reaction systems with small or negligible fission barriers, fission fragment angular distributions are calculated with a Statistical Scission Model (SSM), first suggested by Ericson (Ericson 1960). Versions of this model have been published by several authors (Rossener *et al.*, 1993; Rahimi *et al.*, 2003). We found it appropriate to apply this model to calculate several experimental heavy-ion reactions produced by ^{11}B , ^{19}C and ^{16}O projectiles on ^{232}Th and ^{237}Np targets. In section II, the basic assumptions and model formulization are presented. The variance of the spin distribution for selected heavy-ion induced fission reactions are reported in section III. The results on $\mathfrak{I}_{\text{sph}}/\mathfrak{I}_{\text{eff}}$ obtained for $^{16}\text{O} + ^{232}\text{Th}$ fission reaction is presented in section III.

Transition State Theory

In fission reactions, fragments are found in different directions with a range of probabilities. Statistical models are used to explain the fission phenomena. There is considerable evidence that TSM provides a good representation of experimental fission fragment angular distribution at low spin values, and moderate excitation energies (Rahimi *et al.*, 2005). However it has been shown that TSM is not applicable for heavy ion reactions, and those with a low fission barrier (Back *et al.*, 1985; Ramaurthy *et al.*, 1990). In the present work, we have applied SSM to heavy-ion fission reactions. SSM assumes a statistical partition of the initial angular momentum I of fissioning nucleus in to orbit angular momentum l and channel spin S of two primary fission fragments, where $I = l \oplus S$. This model explains fragment angular distributions with a series of parameters. In this model $W(\theta)$ is given by (Mahata *et al.*, 2002):

$$W(\theta) \propto \sum_{I_{\text{min}}}^{I_{\text{max}}} (2I + 1) T_I \sum_{K=-I}^I (2I + 1) |d_{M=0,K}^I(\theta)|^2 \exp\left(\frac{-K^2}{2K_0^2}\right) / \sum_{K=-I}^I \exp\left(\frac{-K^2}{2K_0^2}\right) \quad (1)$$

Where I , is the total angular momentum; K and M , are its projection along the symmetry and space fixed axis respectively. The d-function is given by (Behkami *et al.*, 2000):

$$d_{M,K}^I(\theta) = [(I + M)! (I - M)! (I + K)! (I - K)!]^{1/2}$$

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$$\sum_x (-1)^x \frac{(\sin\theta/2)^{K-M+2x} (\cos\theta/2)^{2I+K-M-2x}}{(I-K-x)!(I+M-x)!(x+K-M)!} \quad (2)$$

Where the sum is over $x=0, 1, 2, \dots$, and contains all terms in which no negative value appears in the denominator of the sum for any quantity in parentheses.

The anisotropy $W(\theta)/W(90)$ for a specific S_0^2 values can be evaluated from these equations. There are two ways for finding the variance S_0^2 parameter.

1- The anisotropy $W(\theta)/W(90)$ can be computed from equations (1) and (2), and plotted as function of S_0^2 parameter. The intersection of this curve with line $W(\theta)/W(90)$ known from experiment gives the "best fit value" for the parameter S_0^2 .

2- In SSM S_0^2 follow the relation (12)

$$S_0^2 = 2 \Im_{\parallel} \frac{T}{\hbar^2} \left[\frac{(2\Im_{\perp} + \mu R_C^2)}{(2\Im_{\perp} + \mu R_C^2 - 2\Im_{\parallel})} \right] \quad (3)$$

μ is the reduced mass of fission fragments, $R_C = r_0 \left(A_1^{1/3} + A_2^{1/3} \right) \left(b/c \right)^{2/3}$, $r_0 = 1.225$ fm. Here A_1 and A_2 are the mass numbers of fission fragments, and b, c are the major and minor one half axis of fragments. Where \Im_{\parallel} and \Im_{\perp} are moments of inertia for a single fragment, rotating about an axis parallel and perpendicular to its symmetry axis. If one assumes constant mass density for fragments, the following equation can be used:

$$\begin{cases} 2\Im_{\parallel} = 2/5 A c^2 \\ 2\Im_{\perp} = 1/5 A (b^2 + c^2) \end{cases} \quad (4)$$

Where A is the mass number of fissioning system, the temperature of fission fragments can be estimated as follows:

$$T = \left(\frac{E^*/2}{a} \right)^{1/2} \quad (5)$$

Where E^* is the total intrinsic excitation energy of the two fission fragments, "a" is the level density parameter ranging from $A/4$ to $A/16$. The excitation energy is given by

$$E^* = E_{cm} + Q - E_k - E_D - \langle E_{rot} \rangle \quad (6)$$

Here E_{cm} is the center of mass in the entrance channel.

$$E_{cm} = E_{lab} \frac{M_x}{M_x + M_a}, \quad Q \text{ is the Q-value for fission reaction}$$

$$Q = [(M_a + M_x) - (M_{y_1} + M_{y_2})] c^2 \quad (7)$$

And E_k is the total kinetic energy of fission fragments and can be calculated from

$$E_k = 0.107 \frac{Z^2}{A^{1/3}} + 22 \text{ Mev} \quad (8)$$

And E_D is deformation energy of fragments. Average rotational energy can be estimated from (12)

$$\langle E_{rot} \rangle = \frac{[(I + 1/2)^2 - M^2] \hbar^2}{2\mu R_C^2 + \mu \Im_{\perp}} \quad (9)$$

Where I is the total spin of the system at the scission configuration.

RESULTS AND DISCUSSION

The angular distribution of fission fragments in heavy ion induced fission on ^{232}Th and ^{237}Np have been analyzed by means of SSM model. In order to investigate the effect of projectile energy and spin on the fragment angular distributions, we have studied the fission reactions $^{10}\text{B} + ^{237}\text{Np}$ ($E_{lab} = 62 \text{ Mev}$), $^{12}\text{C} + ^{232}\text{Th}$ ($E_{lab} = 79 \text{ Mev}$), $^{16}\text{O} + ^{232}\text{Th}$ ($E_{lab} = 88 \text{ Mev}$), $^{12}\text{C} + ^{237}\text{Np}$ ($E_{lab} = 64 \text{ Mev}$) and $^{16}\text{O} + ^{237}\text{Np}$ ($E_{lab} = 64 \text{ Mev}$). Variances S_0^2 for these reactions have been obtained by two different methods as out lined in section II. In the first method the variance S_0^2 was obtained by the graphical method outlined above, and in the second method variance S_0^2 was obtained using equations (3 to 9). The results are listed in Table I.

Table I. Variance S_0^2 deduced with SSM for the reactions under study

Reaction	Composite	E_{lab}	$W(\theta)/W(170)$	S_0^{2a}	S_0^{2b}
$^{10}\text{B} + ^{237}\text{Np} \rightarrow ^{247}\text{Cf}$	62	1.325		56.0	58.00
$^{12}\text{C} + ^{232}\text{Th} \rightarrow ^{244}\text{Cm}$	79	1.234		64.0	56.20
$^{12}\text{C} + ^{237}\text{Np} \rightarrow ^{249}\text{Es}$	72	1.263		58.0	56.52
$^{16}\text{O} + ^{232}\text{Th} \rightarrow ^{244}\text{Cm}$	88	1.610		58.0	54.58
$^{16}\text{O} + ^{237}\text{Np} \rightarrow ^{253}\text{Md}$	64	2.115		57.0	63.01

S_0^{2a} the best fit value. S_0^{2b} the theoretical value.

Table II. Parameters relevant for the analysis of the fission fragment angular distribution data

Reaction	$E_{lab} \langle I^2 \rangle$	$W(\theta)/W(90)$	T_{saddle}	$K_0 \Im_{sph}/\Im_{eff} \Im_{sph}/\Im_{eff}$	(h)
$^{16}\text{O} + ^{232}\text{Th}$	905951.751.23	14.5	0.7987	0.76	
	95	640		1.93	1.29
13.3	0.9956	0.75			
	120	1695		2.32	1.52
18.3	0.6196	0.63			
	140	2575		2.54	1.69
20.8	0.5333	0.53			
	160	3335		2.71	1.84
22.3	0.5051	0.45			

These values of the variance S_0^2 are used in equation (1), and the full fragment angular distributions are calculated accordingly. The results are plotted in Figure 1 and Figure 2. Examination of these figures shows that both values of the variance S_0^2 result in fragment angular distributions in good agreement with experiment. However, the fission fragment angular distributions deduced using the theoretical S_0^2 values produce much more superior results. In summary, it is of interest to report that fission fragment angular distributions induced by heavy ion reactions can be described using SSM. Finally, the calculated values of the variance S_0^2 have been used to study the characteristics of a nucleus undergoing the fission process. The ratio of $\mathfrak{I}_{sph}/\mathfrak{I}_{eff}$, for $^{16}\text{O} + ^{232}\text{Th} \rightarrow ^{248}\text{Cf}$ reaction has been deduced using equations (3 to 9). The results obtained are listed in Table II.

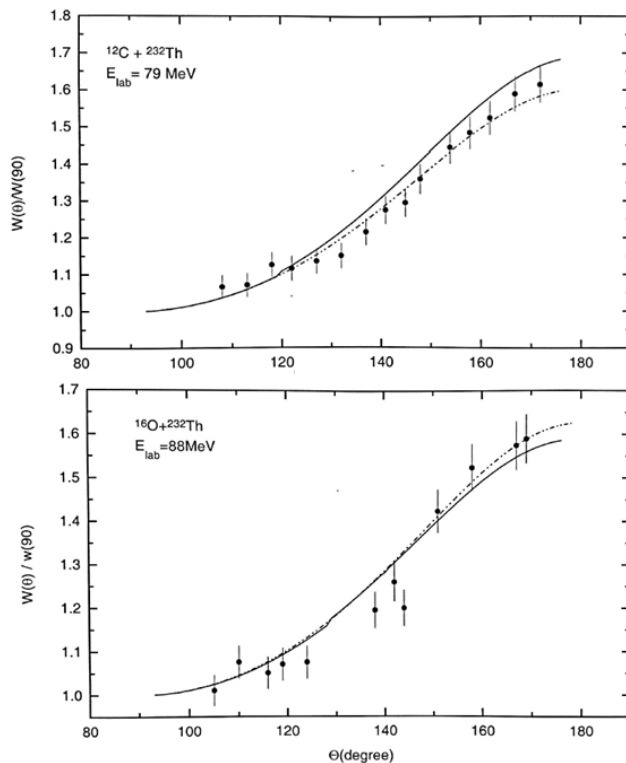


Figure 1. Fission fragment angular distributions produced by ^{12}C and ^{16}O on ^{232}Th target. The experimental values are taken from ref. (10). The angular distributions are calculated using the “best fit” values of S_0^2 , and are shown by dashed lines. The ones calculated with theoretical S_0^2 are shown by solid lines

The ratio of $\mathfrak{I}_{sph}/\mathfrak{I}_{eff}$, as a function of the mean square spin of the composite system for $^{16}\text{O} + ^{232}\text{Th}$ reaction is listed in Table II, and plotted in Figure 3. Solid curve represents the prediction of the rotating drop model. In summary, for heavy-ion induced fission reactions, which the fission barrier is small, it is not unreasonable to expect that the significance of the saddle point in controlling the angular distributions is lost. We have shown that with different parameters of the statistical scission model it is possible to fit the measured fission fragment anisotropies. In regards to spin distribution shown in Figure 3, it's clear that for $^{16}\text{O} + ^{232}\text{Th}$ system, the $\mathfrak{I}_{sph}/\mathfrak{I}_{eff}$ value decreases at higher angular moments, which results in a reduction in fission

anisotropies. Study of this effect for other reactions is in progress.

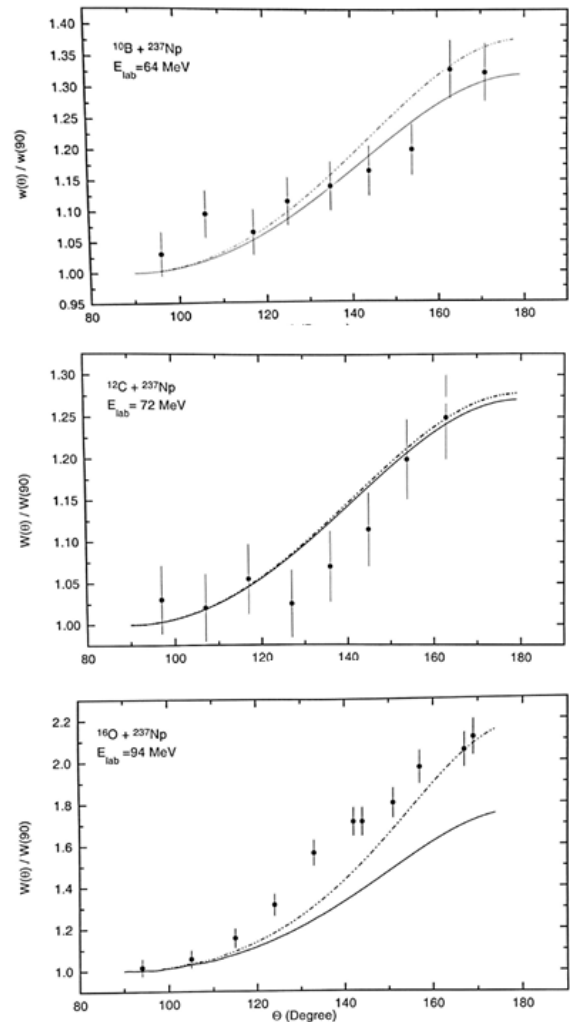


Figure 2. The same as Figure 1, except anisotropies are produced by ^{10}B , ^{12}C and ^{16}O on ^{237}Np target

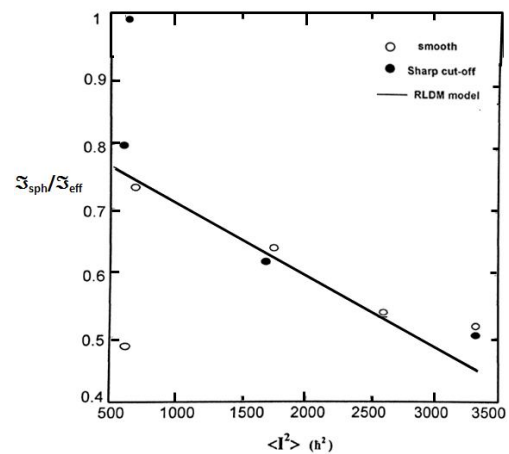


Figure 3. The ratio of $\mathfrak{I}_{sph}/\mathfrak{I}_{eff}$ are shown as a function of the mean square $\langle I^2 \rangle$, for sharp cut-off (solid points) and smooth T_1 values (open circles)

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