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RESEARCH ARTICLE

SOME EQUIVALENT CONDITIONS ON SECONDARY k-NORMAL MATRICES

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ABSTRACT

Concept of secondary k-unitary(s-k unitary) equivalent matrices is introduced. Some equivalent conditions on secondary k-normal matrices(s-k normal) are given.

INTRODUCTION

The concept of secondary k-normal was introduced in [Krishnamoorthy and Bhuvaneswari 2013]. Equivalent conditions on normal matrices are given in [David W. Lewis 1991]. In this paper, our intention is to define s-k unitarily equivalent matrices and prove some equivalent conditions on s-k normal matrices. Also we prove some results on s-k normal matrices. Let C_{nxn} be the space of nxn complex matrices. Throughout, let 'k' be a fixed product of disjoint transpositions in S_n the set of all permutations on {1,2,3,….n} (hence involutory) and 'K' be the associated permutation matrix and 'V' is the permutation matrix with units in the secondary diagonal. Clearly 'K' and 'V' satisfies the following properties. $\overline{K}=K^T=K^S=K^*=\overline{K}^S$; $K^2=I$

$\overline{V}=V^T=V^S=V^*=\overline{V}^S=V$; $V^2=I$.

A matrix $A\in C_{nxn}$ is said to be s-k hermitian matrix if $KVA^*VK= A$.

2. Definitions: In this section, we define s-k normal, s-k unitary and s-k unitary equivalent matrices.

Definition 2.1: A matrix $A\in C_{nxn}$ is said to be secondary k-normal (s-k normal) if $A(KVA^*VK)= (KVA^* VK)A$.

Example 2.2: $A = \begin{pmatrix} 1+3i & 0 & 1+i \\ 0 & 1+3i & 0 \\ 0 & 0 & 1+3i \end{pmatrix}$ is an s-k normal matrix for k=(1,2)(3) the permutation matrix be

$K=\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ and $V=\begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$

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Definition 2.3: A matrix $A \in C_{nxn}$ is said to be s-k unitary if $A(KVA^{*}VK) = (KVA^{*}VK)A = I$.

Example 2.4: $A = \begin{pmatrix} i & 0 & i \\ 0 & i & 0 \\ 0 & 0 & i \end{pmatrix}$ is an s-k unitary matrix for k=(1,2)(3) the permutation matrix be $K = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ and $V = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$.

Definition 2.5: Let $A,B \in C_{nxn}$. The matrix B is said to be secondary k-unitarily equivalent (s-k unitarily equivalent) to A if there exists an s-k unitary matrix U such that $B = (KVU^{*}VK)AU$.

3. Equivalent conditions on secondary k-normal matrices

Theorem 3.1: Let $A \in C_{nxn}$. If A is secondary k-unitarily equivalent to a diagonal matrix, then A is secondary k-normal.

Proof: Let $A \in C_{nxn}$. If A is secondary k-unitarily equivalent to a diagonal matrix D, then there exists an secondary k-unitary matrix P such that $(KVP^{*}VK)AP = D$. Say $KVP^{*}VK = P^{\Phi}$. Similarly A^{Φ} and D^{Φ}.

Therefore $A = PDP^{\Phi}$, since P is s-k unitary.

Now $AA^{\Phi} = (PDP^{\Phi})(PDP^{\Phi})^{\Phi} = PDP^{\Phi}PD^{\Phi}P^{\Phi} = PDD^{\Phi}P^{\Phi}$

Since D and D^{Φ} are each diagonal, $DD^{\Phi} = D^{\Phi}D$

Therefore $AA^{\Phi} = PD^{\Phi}DP^{\Phi} = PD^{\Phi}P^{\Phi}PDP^{\Phi}$ $AA^{\Phi} = A^{\Phi}A$, Since $A = PDP^{\Phi}$. Hence A is s-k normal.

Remark 3.2: It can be shown that A is secondary k-normal Û $A^{-1}(KVA^{*}VK)$ is s-k unitary.

Theorem 3.3 Let $H,N \in C_{nxn}$ be invertible. If $B = HNH$, where H is s-k hermitian and N is s-k normal then $B^{-1}(KVB^{*}VK)$ is similar to an s-k unitary matrix.

Proof: Let $H,N \in C_{nxn}$ be invertible. If $B = HNH$, then $B^{-1}(KVB^{*}VK) = H^{-1}N^{-1}H^{-1}\,KV(HNH)^{*}VK$ $= H^{-1}N^{-1}H^{-1}(KVH^{*}VK)(KVN^{*}VK)(KVH^{*}VK) = H^{-1}N^{-1}H^{-1}H(KVN^{*}VK)H$

Since N is s-k normal from remark (3.2), $N^{-1}(KVN^{*}VK)$ is s-k unitary and hence the result follows.

Theorem 3.4: If $A \in C_{nxn}$ is s-k unitary and if λ is an eigen value of A, then $|\lambda| = 1$.

Proof: Since $A \in C_{nxn}$ is s-k unitary, A is s-k normal. If λ is an eigen value of A there exists an eigen vector U ¹ 0 such that $AU = \lambda U$ which implies $(KVA^{*}VK)U = \bar{\lambda}U$ as A is s-k normal .Now $U = IU = ((KVA^{*}VK)A)U$ which leads to $U(1-\lambda\bar{\lambda}) = 0$ .Since U ¹ 0, $1-\lambda\bar{\lambda} = 0$ which implies that $|\lambda| = 1$.

Theorem 3.5: Let $A \in C_{nxn}$. Assume that $A = UP$ where U is s-k unitary and P is non singular and s-k hermitian such that if P^2 commutes with U ,then P also commutes with U. Then the following conditions are equivalent.

(i) A is s-k normal

(ii) $UP = PU$

(iii) $AU=UA$

(iv) $AP=PA$

(i) Û (ii): If A is s-k normal, then $A(KVA^*VK) = (KVA^*VK)A$

Since $A=UP$, $(UP)(KV(UP)^*VK) = (KV(UP)^*VK)UP$

Þ $UP\,KVP^*U^*VK = KVP^*U^*\,VKUP$

Þ $UP\,KVP^*VK\,KVU^*VK = KVP^*VK\,KVU^*VK\,UP$

Þ $UPPU^{-1} = PU^{-1}UP$,

Þ $UP = PU$

Conversely if $UP = PU$ then $KV(UP)^*VK = KV(PU)^*VK$

Now, $A(KVA^*VK) = (UP)KV(UP)^*VK$

$=UP\,KV(PU)^*VK$

$= U\,KVP^*VK\,KVU^*VK\,P$ since P is s-k hermitian

$= U(KVU^*VK)KVP^*VK\,P$

$= (KVU^*VK)U\,PP$ since P is s-k hemitian and s-k unitary

$= (KVU^*VK)PUP$ since $PU=UP$

$= (KVU^*VK)(KVP^*VK)UP$

$= (KV(PU)^*VK)UP$

$A(KVA^*VK) = (KV(A)^*VK)A$

Hence A is s-k normal

(i) Û (iii):

If A is s-k normal, then $AU=(UP)U=U(PU)=U(UP)$ by (ii).

Conversely, if $AU=UA$, then $(UP)U=U(UP)$

$(KVU^*VK)(UP)U = (KVU^*VK)U(UP)$

Þ $((KVU^*VK)U)PU=((KVU^*VK)U)UP$

Þ $PU = UP$. Therefore A is s-k normal.

(i) Û (iv): If A is s-k normal $AP=(UP)P = PUP = PA$.

Conversely, if $AP = PA$, then $(UP)P = P(UP)$

Post multiplying by P^{-1}, we have $UP = PU$ and so A is s-k normal.

Theorem 3.6: Let $A \in C_{nxn}$. Assume that $A=UP$ where U is s-k unitary and P is non singular and secondary k-hermitian such that P^2 commutes with U, then P also commutes with U then P also commutes with U. Then the following conditions are equivalent.

(i) A is secondary k-normal.
(ii) Any eigen vector of U is an eigen vector of P (as long as U has distinct eigen values).
(iii) Any eigen vector of P is an eigen vector of U (as long as P has distinct eigen values).
(iv) Any eigen vector of U is an eigen vector of A (as long as U has distinct eigen values).
(v) Any eigen vector of A is an eigen vector of U (as long as A has distinct eigen values).

Proof: (i) $\Leftrightarrow$ (ii):

Let U have distinct eigen values. If we prove $UP = PU \Rightarrow$ any eigen vector of U is an eigen vector of P, then (i) $\Leftrightarrow$ (ii): follows by theorem (3.5) . Assume that any eigen vector of U is an eigen vector of P. If X is an eigen vector of U, then X is also an eigen vector of P. Therefore there exist eigen values λ and μ such that $UX=\lambda X$ and $PX=\mu X$. Now $UX=\lambda X$ implies $PUX=P\lambda X=\lambda\mu X$. Similarly $PX=\mu X$ implies $UPX= \lambda\mu X$. Therefore $PUX = UPX \Rightarrow (PU-UP)X=0$ which implies $PU = UP$ as $X \neq 0$.

Conversely, assume that $UP = PU$. If X is an eigen vector of U, then there exists an eigen value λ such that $UX=\lambda X$. Let μ be an eigen value of U such that $UX = \mu X$ Therefore $\lambda \neq \mu$. Now $UP = PU$ implies $(UP - PU)X = 0$ which shows that $UPX = \lambda PX$. Similarly $UX = \mu X$ implies $UPX=\mu PX$. Therefore $\lambda PX = \mu PX \Rightarrow (\lambda - \mu)PX=0 \Rightarrow PX=0$ as $\lambda - \mu \neq 0$. Therefore $PX = 0 X$ and hence X is an eigen vector of P corresponding to the eigen value 0. In general, if μ is an eigen value of U, then we can prove that X is also an eigen vector of P. Therefore any eigen vector of U is also eigen vector of P.
Similar proof holds for other equivalent conditions.

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