RESEARCH ARTICLE

AN ANALYSIS OF ENERGY DEPENDENCE PARAMETER AS MEAN CHARGED MULTIPLICITY FOR PROTON-ANTIPROTON INTERACTIONS

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ARTICLE INFO	ABSTRACT
	The theoretical study of energy dependence of charged multiplicity could discriminate among the different theoretical models of particle production. In the present work, an attempt has been made to analyse the data on mean charged multiplicity in Hadron-Hadron interactions and also to modify the earlier parameterization with modified values of different parameters on the basis of some unambiguous phenomenon. The mean charged multiplicity has been calculated at different energies, ranging between 1.0 GeV to 1500 GeV for proton-antiproton interactions and the results are compared with experimental data as well as, with other theoretical results. Proposed parameterization is found to be capable of predicting the entire data, even at threshold energy.

INTRODUCTION

During last several years, the Hadron-Hadron (Zborovsky Proc 1996; Ellis *et al.,* 1986; Abe *et al.,* 1991; Shakeel *et al.,*1982; Gui-Jun Ding and Mu-Lin Yan 2005) and Hadron-Nucleus (Girija and Hufner 1988; Brodsky *et al.,* 1991; Slewart *et al.,* 1990; Koba *et al.,* 1972; Dahiya *et al.,* 2002) interactions have been attracting considerable attention of high energy physicists. In the field of high energy physics, the mean charged Hadron multiplicity has been an important phenomenological parameter to study the properties of particle production. During last few decades, a considerable amount of the data on mean charged multiplicities has become available at high energies (Albini *et al.,* 1976; Thome *et al.,* 1977; Aziz *et al.,* 1979; Lam *et al.,* 1990; Block *et al.,* 1983; Pancheri and Rubbia 1984; Bjorken Fermilab 1982; Elias *et al.,* 1990). And the theoretical studies of energy dependence of charged multiplicity could discriminate among the different theoretical models of particle production. The concept of multiplicity arises from the production in the inelastic events of interaction process. To predict the experimental data, several authors (Feynman *et al.,* 1977; Field *et al.,* 1977; Frazer *et al.,* 1972; Albini *et al.,* 1975) have proposed different parameterizations and fittings of charged multiplicity as a function of centre of mass (c.m.) energy ($\sqrt{s}$). At high energies, the concept of secondary production reveals, that the number of relativistic charged

particles, produced in any collision, increases as the incoming beam energy increases. A comparatives slow increasement in the charged multiplicity is also found, with the increasement in the size of target nucleus. Such an increase in the multiplicity occurs only in the central region of the rapidity space of particle production and it is believed that quarks and gluons should play an important role in the process of particle production in the central region. The available data (Albini *et al.,* 1976; Thome *et al.,* 1977; Aziz *et al.,* 1979; Pancheri and Rubbia 1984) on multiplicities as a function of energy indicates that at high energies, the mean charged multiplicity $\langle n_{ch} \rangle$ in different Hadron-Hadron collisions (p-p, p-$\bar{p}$, $\pi^{+}p$ etc.) tends to become independent of the type and the charge of the incident and target particles. This behavior of $\langle n_{ch} \rangle$ seems to be valid also at lower energies and also for photon-Hadron (17-19), lepton-Hadron (Albini *et al.,* 1975; Shibata *et al.,* 1980; Bhattacharya *et al.,* 2005) and lepton-lepton (Feynman *et al.,* 1977; Field *et al.,* 1977; Frazer *et al.,* 1972) collisions. Thus it would seem that the entire mean charged multiplicities will follow a universal curve.

Different authors have given different fittings (Albini *et al.,* 1976; Thome *et al.,* 1977; Aziz *et al.,* 1979; Lam *et al.,* 1990; Albini *et al.,*1975) for multiplicity, produced in Hadron-Hadron collisions. But there is a large discrepancy in different fittings. In the present work, an attempt has been made to analyse the experimental observations on mean charged multiplicity at various energies for Hadron-Hadron

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interactions. Different parameterizations proposed by different authors, have been studied and an efforts is done to modify the earlier parameterization (Thome *et al.,* 1977; Aziz *et al.,* 1979; Lam *et al.,* 1990; Block *et al.,* 1983; Pancheri and Rubbia 1984; Albini *et al.,*1975). The values of different parameters have been fixed on the basis of some unambiguous phenomenon. The present parameterization has been used to calculate the mean charged multiplicity as a function of incident c.m. energy for proton-antiproton interactions.

Earlier Parameterization

To predict the experimental data, different authors (Albini *et al.,* 1976; Thome *et al.,* 1977; Aziz *et al.,* 1979; Lam *et al.,* 1990; Block *et al.,* 1983; Pancheri and Rubbia 1984; Frazer *et al.,* 1972; Albini *et al.,* 1975; Zichichi 1983) have given different fittings, depending upon the c.m. ($\sqrt{s}$). In experimental studies of particle production in Hadron-Hadron interactions, some following general features have been considered,

a) Limited transverse momentum of the secondaries.
b) Leading particle effect i.e. the incident Hadron retains on an average half of its initial energy.
c) Slow increase in the multiplicities of produces particles, consisting mainly of pions, with increase in energy and these are on an average 2/3 charged and 1/3 neutral particles in the final state.
d) The multiplicity depends on the centre of mass energy.

Some models predict, at asymptotic energies, an energy dependence of mean charged multiplicity $\langle n_{ch} \rangle$ of the 'ln s' type and the others, a dependence faster than 'ln s' or power dependence on 'ln s' and 's', also more specifically,

i) The Cheng-wu model (Albini *et al.,* 1975) gives $\langle n_{ch} \rangle \sim S^B$ i.e. $\langle n_{ch} \rangle = A\,S^B$ Where $A = 1.42 \pm 0.01$, $B = 0.28 \pm 0.01$
ii) The multiperipheral model (Frazer *et al.,* 1972) and the Mueller-Ragge analysis predict $\langle n_{ch} \rangle \sim \ln s$ i.e. $\langle n_{ch} \rangle = A + B \ln s$; $A = -0.67 \pm .04$, $B = 1.32 \pm 0.01$ The Thermodynamical model (Thome *et al.,* 1977; Pancheri and Rubbia 1984)
iii)) predict an energy dependent multiplicity faster than 'ln s' i.e. $\langle n_{ch} \rangle = A + B (\ln s)^2$; $A = 1.73 \pm 0.02$, $B = 0.17 \pm 0.01$ and $\langle n_{ch} \rangle = A + B + C (\ln s)^2$; $A = 1.73 \pm 0.02$, $B = 0.17 \pm 0.01$ and $C = 0.13 \pm 0.01$Fermi's old statistical model (Shyam *et al.,* 1985) and Belenjki and Landau's Hydrodynamical model (Albini *et al.,* 1975) predict that $\langle n_{ch} \rangle = A + B \ln s + C\,S^{-1/2}$; with $A = -4.55 \pm 0.20$, $B = 1.99 \pm 0.04$ and $C = 8.16 \pm 0.42$

Short Coming of Earlier Parameterizations

• *Lack of Global Fitting*

The earlier parameterizations have been done over a limited energy range, for example, the parameterization (i) and (ii) were considered only for energy range 3 GeV to 14 GeV for proton-antiproton interactions and the parameterization (iii)

and (iv) were considered for energy range 3 GeV to 152 GeV for the same types of the interactions. This reduces the applicability of parameterization.

• *Lack of Phenomenology*

The parameterization (i) has only one parameter A (ii) has two parameter A and B, while the third parameterization consists of three parameters A, B and C thus these parameterization has one, two and three terms respectively. The individual term is found to be incapable of explaining any physical phenomena to which the term is associated, also, if we consider threshold c.m. energy (i.e. the minimum energy required to particle productions), the mean charged multiplicity should be two, but no parameterization can predict this value. It is important for any parameterization that its each term should explain the related physical concept and also should predict the value of $\langle n_{ch} \rangle$ at threshold c.m. energy for particle production.

Present Parameterization

An analysis of the available data on mean charged multiplicity has been considered, in the present work, with a view to finding whether,

i) The mean charged multiplicity $\langle n_{ch} \rangle$ data can be parameterized low as well as at high values of the energy, by the same parameterization.
ii) Each term may explain the related physical concept or phenomena of interaction process. For it, we have considered that all the parameters A, B and C should be energy dependent so that they may be consistent with the associated phenomenology.
iii) Any regular feature of mean charged multiplicity $\langle n_{ch} \rangle$ as a function of c.m. energy may be inferred.

In the present work, an attempt is made to modify the earlier parameterization, $\{\langle n_{ch} \rangle = A + B \ln s\, C\,(\ln s)^2\}$ on the basis of some phenomenological concepts. The various terms of the present parameterization have their own identity and may express the contribution of particular type of interaction process. To predict the experimental data, the value of the parameter A is considered to be constant, but the values of the parameters B and C are considered to be variable, depending upon some other factors, based on the concerned interaction process. The present parameterization for mean charged multiplicity in Proton-Antiproton interaction has the following form,

$$< n_{ch} >_{hh} = A + B \ln \sqrt{S_A} + C\,(\ln \sqrt{S_A})^2 \quad \text{-----(1)}$$

Here $\sqrt{S_A}$ represents the available centre of mass energy (i.e. $\sqrt{S_A} = \sqrt{S} - m_p - m_t$), $\sqrt{S}$ the centre of mass energy, m_p the mass of projectile and m_t the mass of target nucleus. The values of the parameters A, B and C are considered on the basis of interaction process. The different terms of the above equation signifies their individual contribution, viz. the first term {i.e. A} gives the mean charged multiplicity {$\langle n_{ch} \rangle$}, even at threshold energy of particle production in hadronic interaction {i.e. 1GeV}. Since the particle production in any

interaction is the result of the formation of quark-gluon plasma and this state is found near 1 GeV interaction energy, therefore the minimum charged multiplicity, in charged hadronic interactions may be two. The second term {i.e. B ln $\sqrt{S_A}$} represents the contribution due to direct interaction process. The value of parameter B is considered, in this work, to be dependent on the ratio ρ of the real to the imaginary part of the coulomb amplitude, which is related with the total and scattering cross-section as $\rho = \frac{\sigma_{tot.}}{\sigma_{el.}}$ and is energy dependent. The parameter B is related with ρ by the relation B = (1+ ρ). The various values of ρ at different c.m. energies may be obtained by its above relation with $\sigma_{tot.}$ and $\sigma_{el.}$ (26-27), as shown in Fig.2 for proton-proton and proton-antiproton interactions. And finally the third term {i.e. C (ln $\sqrt{S_A}$)^2} gives the contribution due to fire-ball formalism in the interaction process. The value of parameter C is considered, in the present work, to be dependent on the absorption coefficient α_S as C = $\frac{4\alpha_S}{3\pi}$ = 0.4246 α_S(25) where π = 3.14 also α_S = (1- α) where α is inelastic coefficient and has different values at different incident energies. The different parameters are supposed to have their origins from Quantum Chromo dynamics (QCD). For the various values of α at different c.m. energies, we have used our earlier work (Kumar *et al.,* 2004) to obtain the different values of the parameter C.

due to the annihilation of proton and antiproton, their rest mass energy is also included in the available energy, and hence in the case of p-$\bar{p}$, the subtraction of 1.862 GeV (the rest mass energy of target and projectile) from the total energy is not required. These mean charged multiplicities are plotted in the Fig.1.
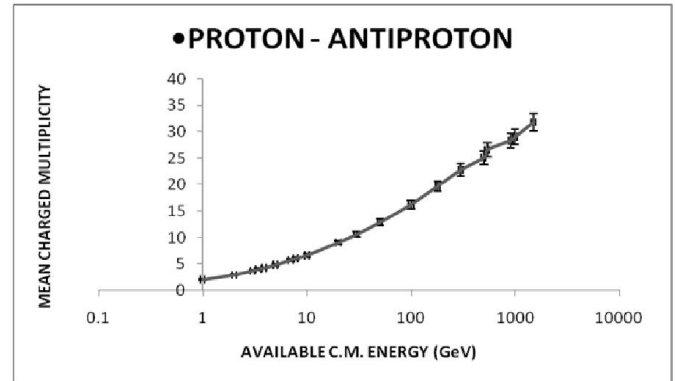

Fig. 1. Variation of mean charged multiplicity (< n_{ch} >$_{p\bar{p}}$) as a function of energy ($\sqrt{S}$). The curve shows the present work. The experimental data has been taken from ref. [Albini *et al.,* 1976; Aziz *et al.,* 1979; Elias *et al.,* 1990; Zichichi *et al.,* 1983]

Table: Mean Charged Multiplicity < n_{ch} >$_{p\bar{p}}$ at different c.m. energies in the proton-antiproton interactions. Here A = 2, B and c are calculated as a function of c.m. energy

Energy ($\sqrt{S}$) GeV	Parameter (B)	Parameter(C)	< n_{ch} >$_{p\bar{p}}$ (Present Work)	< n_{ch} >$_{p\bar{p}}$ (Exp.)
1.0	0.97	0.52	2.0	--
2.0	0.97	0.52	2.92	2.9 ±0.20
3.0	0.97	0.52	3.7	3.4 ±0.30
3.42	0.97	0.52	3.98	3.64 ±0.40
3.87	0.98	0.48	4.21	3.7 ±0.40
5.0	0.98	0.48	4.82	4.05 ±0.50
7.0	0.98	0.48	5.73	5.05 ±0.40
7.75	0.98	0.48	6.02	5.55 ±0.30
10.0	0.99	0.44	6.61	--
20.0	1.03	0.44	9.04	--
30.0	1.03	0.44	10.59	9.8 ±0.80
50.0	1.05	0.44	12.84	--
100.0	1.07	0.44	16.26	--
180.0	1.13	0.44	19.73	19.5 ±0.70
300.0	1.13	0.44	22.76	--
500.0	1.14	0.44	25.07	--
540.0	1.14	0.44	26.59	27.8 ±0.80
900.0	1.15	0.40	28.32	31.2 ±1.10
1000.0	1.15	0.40	29.03	--
1500.0	1.15	0.40	31.80	--

RESULTS AND DISCUSSION

The mean charged multiplicities < n_{ch} >$_{p\bar{p}}$ for proton-antiproton interactions is calculated in this work, at different energies, using present parameterization (eq. 1). Various values of < n_{ch} >$_{p\bar{p}}$ at different c.m. energy are given in the Table along with the corresponding experimental data. As a function of incident energy for proton-proton interactions, the available energy has been calculated by the subtraction of rest mass energy of the projectile and target protons {i.e. $\sqrt{S_A}$ = $\sqrt{S}$ − 1.862} GeV. But in the case of proton-antiproton interactions,

The energy range, to calculate $\langle n_{ch} \rangle$ for proton-antiproton interaction, is considered between 1.0 GeV to 1500 GeV. The results of the present parameterization are well in consistent with the experimental data. A slight deviation of experimental data from the proposed fitting is due to some experimental errors. At low energies viz. $\sqrt{S}$ = 5.0 GeV the value of $\langle n_{ch} \rangle$ in proton-antiproton interaction is found to be 4.82 and this difference increases continuously with the increase in the c.m. energy. At high energies viz. $\sqrt{S}$ = 1500 GeV, the values of $\langle n_{ch} \rangle$ for proton-antiproton is found to be 31.80. In the proton-antiproton interactions the available c.m. energy is due

to the relative motion of the interacting particles, as well as due to the annihilation of proton and antiproton also.
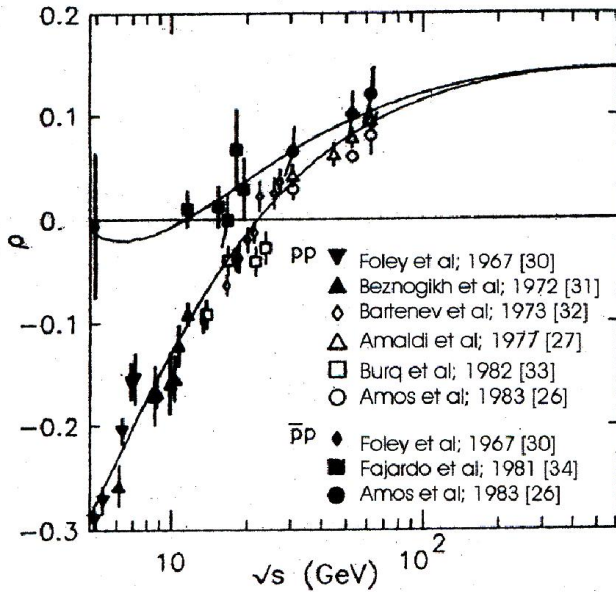

Fig. 2. The ratio ρ of real to imaginary part of the scattering amplitude for p-p and p-$\bar{p}$ interactions as a function of energy ($\sqrt{S}$) [Amos *et al.*, 1983; Amaldi *et al.*, 1977]
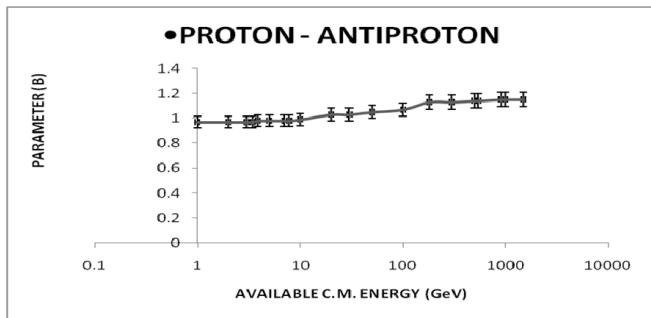

Fig. 3. Variation of parameter B for proton-antiproton interactions as a function of c.m.energy ($\sqrt{S}$)
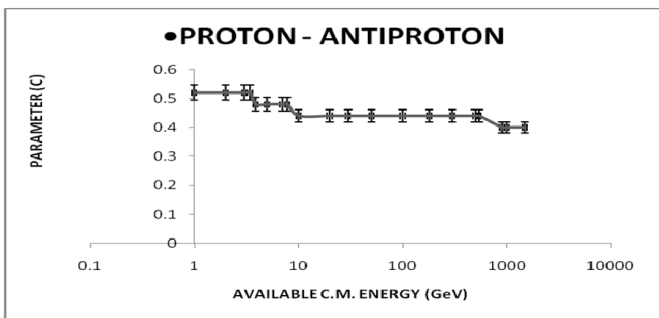

Fig. 4. Variation of parameter C for proton-antiproton interactions as a function of c.m.energy ($\sqrt{S}$)

In the present work, we have proposed the values of the parameters B and C, to be energy dependent. The different values of B are calculated by the relation (B= 1+ ρ) at different energies and are presented in the Table for proton-antiproton. Also the different values of C are calculated by the relation (C

$= \frac{4\alpha_S}{3\pi}$) at different values of $\sqrt{S}$, and are presented in the respective tables for proton-antiproton interactions. The variation of the parameter B, with c.m. energy $\sqrt{S}$, for proton-antiproton is shown in Fig.3. Since the second term of the present parameterization gives the inelastic contribution and thus the parameter B is related to the ratio ρ of the real to the imaginary part of the scattering amplitude, therefore the values of B are found to be increasing slowly in the case of proton-antiproton interactions in comparison to that in the case of proton-proton interactions. The variations in B, in both the cases, may also be inferred from the variation in ρ with c.m. energy (Amos *et al.,* 1983; Amaldi *et al.,* 1977) as given in Fig.2. The variation of the parameter C with $\sqrt{S}$ for proton-antiproton is shown in Fig.4. Since the third term gives the contribution due to fireball formalism in the interaction process and thus the parameter C is related to the absorption coefficient α_S, therefore the value of C is found to be decreasing regularly. The variation in parameter C in the case of proton-antiproton may also be inferred from the variation in α_S (Shyam *et al.,* 1985).

Conclusion

1) The present parameterization of mean charged multiplicity $\langle n_{ch} \rangle$ in Hadron-Hadron interactions provides simple and unambiguous results, which are in well consistent with experimental data for the entire range of incident energy.

2) The present parameterization is capable of explaining the mean charged multiplicity even at threshold c.m. energy (i.e. 1.0 GeV). It may be 2, one for projectile Hadron and one for target Hadron.

Acknowledgement

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