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International Journal of Current Research
Vol. 6, Issue, 08, pp.8087-8088, August, 2014

INTERNATIONAL JOURNAL
OF CURRENT RESEARCH

ISSN: 0975-833X

RESEARCH ARTICLE

AN EOQ MODEL FOR ITEMS WITH TIME-PROPORTIONAL BACKLOGGING

*[1]Selvaraju P. and [2]Kumara Ghuru S.

[1]Department of Mathematics, RVS College of Engineering and Technology, Coimbatore, Tamilnadu, India
[2]Department of Mathematics, Chikkanna Government Arts College, Tirupur, India

ARTICLE INFO

Article History:
Received 06th May, 2014
Received in revised form
25th June, 2014
Accepted 20th July, 2014
Published online 31st August, 2014

Key words:
Inventory, Deterioration,
Time-Proportional Backlogging.

ABSTRACT

In this paper, a deteriorating inventory model with time-varying demand, time-varying deterioration is analyzed. The general time-proportional backlogging rate and general form deterioration rate is also discussed.

INTRODUCTION

An inventory model for deteriorating items with time-varying demand has undergone extensively mathematical developments in the past 30 years. The existing literature in this subject generally deals with the shortages with two sets of time-proportional backlogging rates: (i) linear time-proportional backlogging rate and (ii) exponential time-proportional backlogging rate. In this connection, one may refer to Chang and Dye (Papachristos and Skouri 2000; Skouri and Papachristos 2003) and Papachristos and Skouri (2000) (Chang *et al.,* 1999; Chang *et al.,* 2001). Recently, Lee and Wu (2002) depleted not only by power demand but also by Weibull distributed deterioration, in which shortages are allowed. They also prove the uniqueness for the solution of the system of first derivative. However, their model is limited only to completed backlogged and should be amended to make the model more complete and more applicable in practice. In this paper, we propose a general class time-proportional backlogging rate to make the theory more complete and more applicable in practice. The backlogging rate is defined to be a general function of waiting time, says $B(x)$, with the conditions $B(0) = 1$, $0 \le B(x) \le 1, B'(x) < 0$ and $C_2\, B(x) + C_2 x\, B'(x) - C_4 B'(x) > 0$ for all $x \ge 0$, where x is the customer's waiting time to the next replenishment and C_4 is the opportunity cost due to lost sales Rs.per unit.

Corresponding author: Selvaraju P.
Department of Mathematics, RVS College of Engineering and Technology, Coimbatore, Tamilnadu, India.

Mathematical Model

Since the backlogging rate is $B(T\text{-}t)$, the shortage cost and the opportunity cost due to lost sales during the period can be computed as

$$C_2 \int_{t_1}^{T} (T-t)\, B(T-t)\, D(t)\, dt \quad \text{and} \quad C_4 \int_{t_1}^{T} [1 - B(T-t)]\, D(t)\, dt$$

respectively.

Conjunct with other relevant costs mentioned in Lee and Wu (Lee and Wu 2002), the average total cost per unit time leads to

$$C(t_1) = \frac{C_1}{T} \int_0^{t_1} \left[e^{-g(t)} \int_t^{t_1} [e^{g(u)} - 1]\, D(u)\, du \right] dt + \frac{C_3}{T} \int_0^{t_1} [e^{g(t)} - 1]\, D(t)\, dt$$

$$+ \frac{C_2}{T} \int_{t_1}^{T} (T-t)\, B(T-t)\, D(t)\, dt + \frac{C_4}{T} \int_{t_1}^{T} [1 - B(T-t)]\, D(t)\, dt \quad ..(1)$$

where $g(t) = \int_0^t \theta(s)\, ds$ and $g'(t) = \theta(t)$. To minimize the average total cost per unit time, the optimal value of t_1 (denoted by t_1^*) can be obtained by solving the following equation:

$$\frac{dC(t_1)}{dt_1} = \frac{D(t_1)}{T} \left\{ C_1 \int_0^{t_1} e^{g(t_1) - g(t)} dt + C_3 [e^{g(t_1)} - 1] - C_2 (T - t_1) B(T - t_1) - C_4 [1 - B(T - t_1)] \right\} = 0$$

………...(2)

After rearranging the terms in (2), if D(t)>0 for all t ∈ (0, T), we find that

$$C_1\int_0^{t_1} e^{g(t_1)-g(t)}dt + C_3[e^{g(t_1)}-1] - C_2(T-t_1)B(T-t_1) - C_4[1-B(T-t_1)] = 0$$

…………(3)

It is easily shown that the left hand side of (3) is a strictly increasing function of t_1 because its first derivative with respective to t_1 is

$$C_1 + [C_3 e^{g(t)} + C_1\int_0^{t_1} e^{g(t_1)-g(t)}\,dt]\,\theta(t_1) + C_2 B(T-t_1) + [C_2(T-t_1) - C_4]\,B'(T-t_1) > 0$$

Besides, it is not difficult to check from (2) that

$$\left[\frac{dC(t_1)}{dt_1}\right]_{t_1=0} = \frac{-D(0)\{C_2 TB(T) + C_4[1-B(T)]\}}{T} < 0$$

and

$$\left[\frac{dC(t_1)}{dt_1}\right]_{t_1=T} = \frac{D(T)\{C_1\int_0^T e^{g(T)-g(t)}dt + C_3[e^{g(T)}-1]\}}{T} > 0$$

Therefore, there exists an unique solution t_1^*, so that $\frac{dC(t_1)}{dt_1}|_{t_1=t_1^*} = 0$ in the interval (0, T) if $D(t) > 0$. However, since $D(t) = dq\ (1/T)^q\ t^{q-1} = 0$ when q ≥ 1 and $t = 0$ in Lee and Wu's (5) model, it is obvious to see that $t_1^* = 0$ is another optimal solution so that $dC(t_1)\ /\ dt_1 = 0$. From those existing solutions to the (2), we only need the minimizing solution.

Conclusion

In this paper, a deteriorating inventory model with time-varying demand, time-varying deterioration and general time-proportional backlogging rate is developed. In practice, one can observe periodically the proportion of demand which would like to accept back-logging and the corresponding waiting time for the next replenishment. Then the statistical techniques, such as the non-linear regression method, can be used to estimate the backlogging rate. As long as the backlogging pattern belongs to the class,

$$S = \left\{ B(x) \middle| \begin{array}{l} B(0)=1,\ 0 \le B(x) \le 1, B'(x) < 0 \text{ and} \\ C_2 B(x) + C_2 x B'(x) - C_4 B'(x) > 0,\ \ x \ge 0 \end{array} \right\}$$

We can show that the optimal solution not only exists but also is unique and is independent on the form of the demand rate. If we define the backlogging rate as $B(x) = \lambda/(1 + \delta_1 x) + (1-\lambda)\ e^{-\delta_2 x}$, where $\delta_1 > 0$ and $0 < \delta_2 < 1/T$ and λ, 0 ≤ λ ≤ 1, is a weight parameter, then the condition which leads to a minimum is

$$\lambda\,(C_2 + C_4\beta)\,/\,(1+\beta x)^2 + (1-\lambda)\,(C_2 - C_2\delta_2 x + C_4\delta_2)\ e^{-\delta_2 x} > 0.$$

That is, we can easily see that it will always be satisfied. Consequently, the proposed model is in a general framework that includes numerous previous models such as in (Lee and Wu 2002; Wu *et al.,* 2000; Wu 2002) as special cases. Furthermore, we can also see that any deterioration rate can be applied to this model such as the three-parameter Weibull deterioration rate (Chakrabarthy *et al.,* 1998) and Gamma deterioration rate (Tadikamalla 1978). Hence, if $D(t)$ and $\theta(t)$ are continuous and differentiable on an open interval $1{:}0 < t < T$, and $D(t) > 0$ for all $t \in (0, T)$, then there is exist a unique solution that satisfies $dC(t_1)/\ dt_1 = 0$.

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