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RESEARCH ARTICLE

ON SOLVING MULTI OBJECTIVE FRACTIONAL LINEAR PROGRAMMING PROBLEMS

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ABSTRACT

Here, the concept of fractional linear programming problem is extended to Multi objective Fractional Linear programming problem (MOFLPP) in crisp as well as fuzzy sense. Procedures for also solving Multi objective fractional linear programming problems are provided for both crisp and fuzzy cases. A new idea for solving a MOFLPP in both crisp and fuzzy sense is also given. A numerical example is provided to illustrate the proposed methods.

INTRODUCTION

Linear fractional programming (LFP) problems are a special type of non-linear programming problems in which the objective function is a ratio of linear functions and the constraints are linear functions. In real life situations, linear fractional models arise in decision making such as construction planning, economic and commercial planning, health care and hospital planning. Several methods (Bajalinov, 2003; Stancu-Minasian, 1997, 2006) have been recommended to solve LFP Problems. Isbell and Marlow (1956) first identified an example of LFP Problems and solved it by a sequence of linear programming problems. Charnes and Cooper (1962) considered variable transformation method to solve LFP and the updated objective function method were developed for solving the LFP problem by Bitran and Novaes (1973).Gilmore and Gomory (1963), Martos (1964), Swarup (1965),Wagner and Yuan (1968), Pandey and Punnen (2007) and Sharma *et al*. (1980) solved the LFP problem by various types of solution procedures based on the Simplex method developed by Dantzig (1962). Tantawy (2007, 2008) proposed two different approaches namely; a feasible direction approach and a duality approach to solve the LFP problem. Mojtaba Borza *et al*. (2012) solved the LFP problem with interval coefficients in objective function which is based on Charnes and Cooper technique (1962). Odior (2012) solved the LFP problem by algebraic approach which depends on the duality concept and the partial fractions. If more than one objective function involves in LFP problem then Multi objective Linear fractional programming problem arises.

In most of real life models, the possible values of coefficients of a linear programming problem are obviously unclear and vague. In fuzzy decision making problems, the idea of maximizing decision was anticipated by Bellman and Zadeh (1970). The theory of fuzzy linear programming on general level was initially proposed by Taneka *et al*. (1973). Fuzzy set theory introduced by Zadeh (1965) is generalization of conventional set theory to represent vagueness or imprecision in everyday life in strict mathematical framework (Garcia 2008). There are many kinds of formulations to the objective function of problems, may be linear programming, quadratic programming, multi objective and fractional linear programming and all of this kinds it is possible, that some coefficient of the problem in objective function, technical coefficient or decision making variable be fuzzy numbers. In this work, we focus on multi objective fractional linear programming problem with symmetric trapezoidal fuzzy numbers in the

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objective function. Nassirie *et al.* (2007) proposed a method for solving linear programming problem with fuzzy numbers. Here, we first explain the multi objective fractional linear programming problems in crisp case and then solved the multi objective fractional linear programming problems in the fuzzy sense. Complementary development method (1) is used to solve for both the crisp and fuzzy cases. This method is used to transform the fractional linear programming into linear programming problem.

This paper is organized as follows. In section 2, preliminaries are given. An algorithm for solving a multi objective fractional linear programming problem is developed in section 3. In section 4, a numerical example is provided to illustrate its feasibility. The last section draws some concluding remarks.

2. Preliminaries

In this section, we will discuss some definitions on fractional linear programming problems, and symmetric trapezoidal fuzzy numbers.

2.1 Fractional linear programming Problem

A Fractional linear programming problem is defined as

$$\text{Max } Z = \frac{cx + p}{dx + q}$$

Subject to $AX \le B$, $X \ge 0$

where $c = (c_1, c_2, \ldots c_n)$, $d = (d_1, d_2, \ldots d_n)$, $B = (b_1, b_2, \ldots b_m)^T$, $X \in R^n$, $x \in X$, p and q are scalar and $A = (a_{ij})_{nxm}$

2.2 Multi objective Fractional linear programming Problem

A Multi objective fractional linear programming problem is defined as

$$\text{Max } Z_i = \frac{f_i(x)}{g_i(x)} \qquad i = 1,2\ldots..k$$

Subject to

$AX \le B$, $X \ge 0$

Where $B = (b_1, b_2, \ldots b_m)^T$, $X \in R^n$, $x \in X$, and $A = (a_{ij})_{nxm}$ $f_i(x) = cx + p$ and $g_i(x) = dx + q$, $c = (c_1, c_2, \ldots c_n)$, $d = (d_1, d_2, \ldots d_n)$, p and q are scalar

2.3 Symmetric trapezoidal fuzzy number

Let us consider a symmetric trapezoidal fuzzy number $\tilde{a} = (a_1, a_2, h, h)$ whose membership function is given by

$$\tilde{a}(x) = \begin{cases} \dfrac{x}{h} + \dfrac{h - a_1}{h}, & x \in [a_1 - h, a_1] \\ 1, & x \in [a_1, a_2] \\ \dfrac{-x}{h} + \dfrac{a_2 + h}{h}, & x \in [a_2, a_2 + h] \\ 0, & \text{otherwise} \end{cases}$$

where $a_1 \le a_2$ and $h \ge 0$ in the real line R.

2.4 Ranking function

Let $\mathcal{F}$ (R) be the set of all symmetric trapezoidal fuzzy numbers.

For $\widetilde{a}$ = (a_1 , a_2 , h, h) $\in \mathcal{F}$ (R), we define a ranking function F : $\mathcal{F}$(R)→ R by

F ($\widetilde{a}$) = $\dfrac{(a_1 - h) + (a_2 + h)}{2} = \dfrac{a_1 + a_2}{2}$ as in (2)

2.5 Arithmetic operations on symmetric trapezoidal fuzzy numbers

For $\widetilde{x} = (x_1, x_2, h, h)$ and $\widetilde{y} = (y_1, y_2, k, k)$ in $\mathcal{F}$(R), we define

1. **Addition** : $\widetilde{x} + \widetilde{y} = (x_1, x_2, h, h) + (y_1, y_2, k, k)$ = ((F($\widetilde{x}$) + F($\widetilde{y}$)) – s , (F($\widetilde{x}$) + F($\widetilde{y}$)) + s , h + k, h + k)

where s = $\dfrac{(y_2 + x_2) - (y_1 + x_1)}{2}$

2. **Subtraction:** $\widetilde{x} - \widetilde{y} = (x_1, x_2, h, h) - (y_1, y_2, k, k)$ = ((F($\widetilde{x}$) - F($\widetilde{y}$)) – s , (F($\widetilde{x}$) - F($\widetilde{y}$)) + s , h + k, h + k)

where s = $\dfrac{(y_2 + x_2) - (y_1 + x_1)}{2}$

3. **Multiplication:** $\widetilde{x}\widetilde{y} = (x_1, x_2, h, h)\ (y_1, y_2, k, k)$ = ((F($\widetilde{x}$) F($\widetilde{y}$)) – s , (F($\widetilde{x}$) F($\widetilde{y}$)) + s , $|x_2 h + y_2 k|$, $|x_2 h + y_2 k|$)

where s = $\dfrac{\beta - \alpha}{2}$

$\alpha = \min \{ x_1y_1, x_1y_2, x_2y_1, x_2y_2 \}$, $\beta = \max \{ x_1y_1, x_1y_2, x_2y_1, x_2y_2 \}$

4. Division: $\dfrac{1}{(x_1, x_2, h, h)} = \left[\dfrac{1}{F(\widetilde{x})} - s, \dfrac{1}{F(\widetilde{x})} + s, h, h \right]$ where **s** = $\dfrac{1}{2}\left(\dfrac{1}{x_1} - \dfrac{1}{x_2} \right)$

5. Scalar multiplication

$$\lambda \widetilde{x} = \begin{cases} (\lambda x_1, \lambda x_2, \lambda h, \lambda h), & \text{for } \lambda \geq 0 \\ (\lambda x_2, \lambda x_1, -\lambda h, -\lambda h), & \text{for } \lambda < 0 \end{cases}$$

2.6 Fractional linear programming problem with fuzzy numbers

A fuzzy number Fractional linear programming problem is defined as

$$\text{Max } \widetilde{Z} = \frac{\widetilde{c}x + \widetilde{p}}{\widetilde{d}x + \widetilde{q}}$$

Subject to

$AX \leq B,\ X \geq 0$

where B= $(b_1, b_2, \ldots b_m)^T$, $X \in R^n$, $x \in X$, and $A = (a_{ij})_{nxm}$ $\widetilde{c}$, $\widetilde{d}$, $\widetilde{p}$, $\widetilde{q}$ $\in \mathcal{F}$ (R)

2.7 Multi objective Fractional linear programming problem with fuzzy numbers

A Multi objective fuzzy number fractional linear programming problem is defined as

$$\text{Max } \tilde{Z}_i = \frac{\tilde{f}_i(x)}{\tilde{g}_i(x)} \qquad i= 1,2\ldots..k$$

Subject to

$AX \leq B, \quad X \geq 0$

where $B= (b_1, b_2, \ldots. b_m)^T$, $X \in R^n$, $x \in X$, and $A=(a_{ij})_{nxm}$,

$\tilde{f}_i(x) = \tilde{c}x + \tilde{p}$ and $\tilde{g}_i(x) = \tilde{d}x + \tilde{q}$ $\tilde{c}, \tilde{d}, \tilde{p}, \tilde{q}$ are symmetric trapezoidal fuzzy numbers.

3. Complementary development method to solve Multi objective fractional linear Programming Problem

The method proposed here is used to transform the fractional linear programming problem into a linear programming problem and it can be applied for both the crisp and fuzzy cases. An algorithm for solving a multi objective fractional linear programming problem is developed here.

3.1 Algorithm of complementary development method for solving multi objective fractional linear programming problem:

Step 1: Dividing the first objective function into linear functions in which the first function represents the numerator function and the second one is the denominator function. The value of the objective function is taken as maximum (max $z_1(x)$) for the numerator and minimum (min $z_2(x)$) for the denominator function.

Step 2: Reclamation a function max $z^*(x)$ from subtracting the denominator function from the numerator function and this function is putting in mathematical module made up of original restriction of problem in addition to non negative conditions and to solve this linear system go to step 3.

Step 3: Enervating the mathematical module to the standard form by adding slack Variable (s_i) then solve the system by using Simplex method gives a solution for x_j.

Step 4: The same procedure is repeated for the second objective Function.

Step 5: Including the prior objective function as one of the constraints and then solved by using the Preemptive optimization method.

Step 6: The same procedure is repeated until all the objective functions are optimized.

3.2 Algorithm of complementary development method for solving multi objective fuzzy number fractional linear programming problem

Step 1: Dividing the first objective function into linear functions in which the first function represents the numerator function and the second one is the denominator function. The value of the objective function is taken as maximum (max $\tilde{z}_1$(x)) for the numerator and minimum (min $\tilde{z}_2$(x)) for the denominator function.

Step 2: Reclamation a function max $\tilde{z}^*$(x) from subtracting the denominator function from the numerator function using the arithmetic operation of symmetric trapezoidal fuzzy numbers and this function is putting in mathematical module made up of original restriction of problem in addition to non negative conditions and to solve this linear system go to step 3.

Step 3: Enervating the mathematical module to the standard form by adding slack variable (s_i) and then solve the system to obtain the solution for x_j by using Simplex method and also using the ranking function of symmetric trapezoidal fuzzy numbers

Step 4: The same procedure is repeated for the second objective Function.

Step 5: Including the prior objective function as one of the constraints (use ranking function to convert it into crisp one) and then solved by using the Preemptive optimization method.

Step 6: The same procedure is repeated until all the objective functions are optimized.

4. Numerical example

Consider a multi objective fractional linear programming problem

Max Z_1 $= \dfrac{6x_1 + 5x_2}{2x_1 + 7}$

Max Z_2 $= \dfrac{2x_1 + 3x_2}{x_1 + x_2 + 7}$

Subject to the constraints

$x_1 + 2x_2 \le 3$
$3x_1 + 2x_2 \le 6$
$x_1, x_2 \ge 0$

We first separate the first objective function into sub-functions as given below

Max $Z_1 = 6x_1 + 5x_2$
Subject to the constraints

$x_1 + 2x_2 \le 3$
$3x_1 + 2x_2 \le 6$
$x_1, x_2 \ge 0$

and

Min $Z_2 = 2x_1 + 7$
Subject to the constraints

$x_1 + 2x_2 \le 3$
$3x_1 + 2x_2 \le 6$
$x_1, x_2 \ge 0$

Now the new objective function max Z^* is constructed as per the algorithm developed here.

Max $Z_1^* = 4x_1 + 5x_2 - 7$
Subject to the constraints

$x_1 + 2x_2 \le 3$
$3x_1 + 2x_2 \le 6$
$x_1, x_2 \ge 0$

We will transform the above in standard form by introducing the slack variable.

Max $Z_1^* = 4x_1 + 5x_2 - 7$
Subject to the constraints

$x_1 + 2x_2 + x_3 = 3$
$3x_1 + 2x_2 + x_4 = 6$
$x_1, x_2, x_3, x_4 \ge 0$

		4	5	0	0		
C_j	Basis	x_1	x_2	x_3	x_4	RHS	Ratio
0	x_3	1	2	1	0	3	3/2
0	x_4	3	2	0	1	6	3
	$Z_j - C_j$	-4	-5	0	0		
5	x_2	1/2	1	1/2	0	3/2	3
0	x_4	2	0	-1	1	3	3/2
	$Z_j - C_j$	-3/2	0	5/2	0		
5	x_2	0	1	3/4	-1/4	3/4	
4	x_1	1	0	-1/2	1/2	3/2	
	$Z_j - C_j$	0	0	7/4	3/4		

Since $Z_j - C_j \geq 0$, the optimal solution is obtained at $x_1=3/2$ and $x_2=3/4$

$$\text{Max } Z_1^* = 11/4$$

Next, we consider the second objective function and following the same procedure to obtain

$$\text{Max } Z_2^* = x_1 + 2x_2 - 7$$

Subject to the constraints

$$\begin{aligned} x_1 + 2x_2 &\leq 3 \\ 3x_1 + 2x_2 &\leq 6 \\ 4x_1 + 5x_2 &\geq 39/4 \\ x_1, x_2 &\geq 0 \end{aligned}$$

Using TORA Software, the formulated problem is solved and the optimal solution is obtained as $x_1=3/2$ and $x_2=3/4$ with Max $Z_2^* = -4$.

Proposed new method

Based on the Complementary development method given in Aws Nidhar Dheyab (2012), we proposed a new method to solve multi objective linear fractional programming problems in which odd numbered fractional objective functions of the problem are converted into linear objective functions by subtracting the denominator function from the numerator function and even numbered fractional objective functions of the problem are converted into linear objective functions by adding the denominator function and the numerator function and then solve by the same procedure to obtain an optimal solution. This method is applicable for both the crisp and fuzzy cases. TORA software is used for the crisp case linear programming problems.

The same problem is solved in our proposed new method and we got the better results as compared with the Complementary development method proposed by Aws Nidhar Dheyab (2012). In the first objective function of the above problem, we subtracted the denominator function from the numerator function and for the second objective function, we added both the numerator and the denominator functions and then solve by the same procedure developed by us and optimal solution is obtained.

$$\text{Max } Z_1^* = 4x_1 + 5x_2 - 7$$

Subject to the constraints

$$\begin{aligned} x_1 + 2x_2 &\leq 3 \\ 3x_1 + 2x_2 &\leq 6 \\ x_1, x_2 &\geq 0 \end{aligned}$$

Solving this as in table 4.1 and we obtain the optimal solution as $x_1=3/2$ and $x_2=3/4$ with Max $Z_1^* = 11/4$

$$\text{Max } Z_2^* = 3x_1 + 4x_2 + 7$$

Subject to the constraints

$$\begin{aligned} x_1 + 2x_2 &\leq 3 \\ 3x_1 + 2x_2 &\leq 6 \\ 4x_1 + 5x_2 &\geq 39/4 \\ x_1, x_2 &\geq 0 \end{aligned}$$

Using TORA Software, solving this problem and we obtain the optimal solution as $x_1=3/2$ and $x_2=3/4$ with Max $Z_2^* = 58/4$.

We observed that, however the values of x_i's are same in both the methods, the objective function values are differed and better results are obtained by the proposed new method.

The same problem is considered in the fuzzy sense. (Fuzziness in the objective function only)
Consider a multi objective fractional fuzzy number linear programming problem

$$\textbf{Max } \mathbf{Z_1} = \frac{\tilde{6}x_1 + \tilde{5}x_2}{\tilde{2}x_1 + \tilde{7}}$$

$$\textbf{Max } \mathbf{Z_2} = \frac{\tilde{2}x_1 + \tilde{3}x_2}{\tilde{1}x_1 + \tilde{1}x_2 + \tilde{7}}$$

Subject to the constraints

$$x_1 + 2x_2 \le 3$$
$$3x_1 + 2x_2 \le 6$$
$$x_1, x_2 \ge 0$$

here $\tilde{6}$ =(5,7,2,2) $\tilde{5}$ = (4,6,2,2) $\tilde{2}$ =(1,3,2,2) $\tilde{7}$ =(6,8,2,2) $\tilde{3}$ = (2,4,2,2) $\tilde{1}$ = (0.25,0.75,1.25,1.75) are symmetric trapezoidal fuzzy numbers.

$$\textbf{Max } \mathbf{Z_1} = \frac{(5,7,2,2)x_1 + (4,6,2,2)x_2}{(1,3,2,2)x_1 + (6,8,2,2)}$$

First, we separate the first objective function into sub-function as mentioned below

Max Z_1 = (5,7,2,2) x_1 + (4,6,2,2) x_2

Subject to the constraints

$$x_1 + 2x_2 \le 3$$
$$3x_1 + 2x_2 \le 6$$
$$x_1, x_2 \ge 0$$

and

Min Z_2 = (1,3,2,2) x_1 + (6,8,2,2)

Subject to the constraints

$$x_1 + 2x_2 \le 3$$
$$3x_1 + 2x_2 \le 6$$
$$x_1, x_2 \ge 0$$

Now we construct max Z_1^*

Max Z_1^* = (0,8,4,4) x_1 + (4,6,2,2) x_2 – (6,8,2,2)

Subject to the constraints

$$x_1 + 2x_2 \le 3$$
$$3x_1 + 2x_2 \le 6$$
$$x_1, x_2 \ge 0$$

		(0,8,4,4)	(4,6,2,2)	(0,0,0,0)	(0,0,0,0)		
C_j	Basis	x_1	x_2	x_3	x_4	RHS	Ratio
(0,0,0,0)	x_3	1	2	1	0	3	3/2
(0,0,0,0)	x_4	3	2	0	1	6	3
	$Z_j - C_j$	(-8,0,4,4)	(-6,-4,2,2)	(0,0,0,0)	(0,0,0,0)		
	$\Re(Z_j - C_j)$	-4	-5	0	0		
(4,6,2,2)	x_2	1/2	1	1/2	0	3/2	3
(0,0,0,0)	x_4	2	0	-1	1	3	3/2
	$Z_j - C_j$	(-6,3,5,5)	(0,0,0,0)	(2,3,1,1)	(0,0,0,0)		
	$\Re(Z_j - C_j)$	-3/2	0	5/2	0		
(4,6,2,2)	x_2	0	1	3/4	-1/4	3/4	
(0,8,4,4)	x_1	1	0	-1/2	1/2	3/2	
	$Z_j - C_j$	(0,0,0,0)	(0,0,0,0)	(-1,9/2,7/2,7/2)	(-3/2,3, 5/2,5/2)		
	$\Re(Z_j - C_j)$	0	0	7/4	3/4		

Since $\Re(Z_j - C_j) \ge 0$, the optimal solution is obtained at x_1=3/2 and x_2 =3/4 Max Z_1^* = (-5, 10.5, 9.5, 9.5) $\Re(Z_1^*)$ = 2.75

Next, we consider the second objective function

$$\text{Max } Z_2^* = \frac{\tilde{2}x_1 + \tilde{3}x_2}{\tilde{1}x_1 + \tilde{1}x_2 + \tilde{7}}$$

(i.e.) $$\text{Max } Z_2^* = \frac{(1,3,2,2)x_1 + (2,4,2,2)x_2}{(0.75,1.25,0.5,0.5)x_1 + (0.75,1.25,0.5,0.5)x_2 + (6,8,2,2)}$$

Subject to the constraints

$$x_1 + 2x_2 \le 3$$
$$3x_1 + 2x_2 \le 6$$
$$(0,8,4,4)\,x_1 + (4,6,2,2)\,x_2 - (6,8,2,2) \ge (-5, 10.5, 9.5, 9.5)$$
$$\Rightarrow (0,8,4,4)\,x_1 + (4,6,2,2)\,x_2 \ge (-5, 10.5, 9.5, 9.5) + (6,8,2,2)$$

Using ranking functions, we got

$$4x_1 + 5x_2 \ge 9.75$$
$$x_1, x_2 \ge 0$$

Solving this as the same procedure, we get

$$\text{Max } Z_2^* = (-0.25,2.25,2.5,2.5)\,x_1 + (0.75,3.25,2.5,2.5)\,x_2 - (6,8,2,2)$$

Subject to the constraints

$$x_1 + 2x_2 \le 3$$
$$3x_1 + 2x_2 \le 6$$
$$4x_1 + 5x_2 \ge 9.75$$
$$x_1, x_2 \ge 0$$

We obtain the optimal solution as

$x_1 = 3/2$ and $x_2 = 3/4$ with Max $Z_2^* = (-7.8125, -0.1875, 7.625, 7.625)$

$\Re(Z_2^*) = -4$ which is same as that of crisp linear programming problem.

Conclusion

In this paper, we proposed two methods of solving Multi objective fractional Linear programming problems in crisp case and also comparing the results are same or not. Further, multi objective fuzzy number fractional programming problems are solved in which the cost coefficients of the objective function are considered as symmetric trapezoidal fuzzy numbers. The optimal solution is obtained by using the ranking functions of the symmetric trapezoidal fuzzy numbers. The results are verified by means of a numerical example.

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